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ABSTRACT

It has been proposed that laser-induced relativistic plasma mirror can accelerate if the plasma has a properly tailored density profile. Such accelerating plasma mirrors can serve as analog black holes to investigate Hawking evaporation and the associated information loss paradox. Here we reexamine the underlying dynamics of mirror motion in a graded-density plasma to provide an explicit trajectory as a function of the plasma density and its gradient. Specifically, a decreasing plasma density profile (down-ramp) along the direction of laser propagation would in general accelerate the mirror. In particular, a constant-plus-exponential density profile would generate the Davies–Fulling trajectory with a well-defined analog Hawking temperature, which is sensitive to the plasma density gradient but not to the density itself. We show that without invoking nano-fabricated thin-films, a much lower density gas target at, for example, $\sim 1\times 10^{17}\,\mathrm{cm}^{-3}$, would be able to induce an analog Hawking temperature, $k_B T_H \sim 3.1\times 10^{-2}\,\mathrm{eV}$, in the far-infrared region. We hope that this would help to better realize the experiment proposed by Chen and Mourou.

I. INTRODUCTION

The black hole Hawking evaporation[1] has triggered the wonder whether it would result in the loss of information.[2] If yes, it would signalize a severe conflict between general relativity and quantum field theory; but if no, then how would the information be preserved?[3–10] This information loss paradox has been under debate for more than 40 years, which is essentially theoretical, without a firm conclusion. It is, however, generally agreed that the final verdict relies heavily on our knowledge about the end-stage of Hawking evaporation. (For an overview, see, for example, Ref. 7 and references therein.) It can be easily verified that the Hawking lifetime of a stellar-size black hole is much, much longer than the age of the universe. Therefore, the resolution to this paradox through direct astrophysical observations seems hopeless.

Giving the fundamental importance of this issue, and that physics is essentially an experimental science, the idea of creating analog black hole in the laboratory for such investigation has become attractive. (For a review on analog gravity, see, for example, Ref. 11.) Unruh first proposed to investigate the Hawking evaporation through what he called the "dump hole."[12] Additional proposals include electromagnetic waveguides,[13] traveling index of refraction in media,[14] and Bose–Einstein condensates.[15] There is also the idea to induce spontaneous thermal emissions by the interaction between vacuum fluctuations and a periodically accelerated electron "detector" driven by an ultra-intense laser[16] to test the closely related Unruh effect.[17]

The idea of a relativistic flying mirror as an analog black hole has been theoretically studied ever since Hawking's seminal discovery in 1975.[18,19] It was Wilczek who first pointed out[20] that a flying mirror can further serve to investigate the information loss paradox. In order to preserve the "black hole unitarity," Wilczek argued, based on the moving mirror model, that the partner modes of the Hawking particles would be trapped by the horizon until the end of the evaporation, where they would be released and the black hole initial pure state recovered with essentially zero cost of energy. In analogy, the partner modes that are trapped on the mirror's horizon would be released when the flying mirror suddenly comes to a stop; the quantum entanglement between these partner modes and the emitted Hawking particles would recover the initial pure state and, thus, the preservation of the unitarity and the information.

It, however, remains a gedanken experiment over the years until recently. In 2017, it was proposed[21] that such an experiment can indeed be realized through an accelerating plasma mirror induced by an intense laser that impinges a plasma with a graded density. The feasibility of this proposed scheme relies heavily on a more precise description of the mirror trajectory as a function of the local plasma density and gradient.

It is well-known that in the plasma wakefield acceleration (PWFA)[22] and the laser wakefield acceleration (LWFA)[23] mechanisms, the phase velocity of the plasma wakefield v_{ph} equals the group velocity of the driver, v_g, i.e., $v_{ph} = v_g$, be it a charged particle beam (in PWFA) or a short laser pulse (in LWFA). This *wakefield principle* is true, however, only for a uniform-density plasma. In a more general situation where the plasma ambient density is nonuniform, the interplay between the laser group velocity and the wakefield phase velocity becomes more intricate and v_{ph} and v_g are not necessarily equal. A generalized treatment of the plasma mirror dynamics should therefore break away from the conventional wakefield principle for uniform plasmas.

In addition, in Ref. 21, the acceleration of plasma mirrors was derived based on the eikonal equation,[24] which is the foundation of geometric optics and corresponds to the zero-wavelength limit. The validity of the eikonal approximation to treat the acceleration in a nonuniform plasma lies in the assumption that the density gradient is minute. For that reason, an adiabatic condition was imposed, where the characteristic length of the plasma density gradient D must be much larger than the local plasma wavelength λ_p, i.e., $D \gg \lambda_p$. To apply to more general situations, a different, less restrictive formulation is desirable. As we will see, our new approach in this paper is void of the adiabatic condition.

Perhaps the most counter-intuitive fact of our new approach to the flying mirror dynamics is that the adiabatic condition introduced in our 2017 paper, which is required for the eikonal equation method, is now found unnecessary. The reason is that being a phase wave, the plasma oscillation frequency is governed by the motion of the electrons within a length scale that is much smaller than the nominal plasma wavelength. It is therefore perfectly fine that the plasma density varies sizably within one plasma wavelength.

Another motivation of this work is to search for a new and simplified operation regime for the proposed flying plasma mirror as analog black hole experiment.[21] In the original Chen–Mourou proposal, a two-stage laser–plasma interaction was invoked, where the second stage assumes a nano-fabricated thin-film target with graded density to induce the analog Hawking radiation. One practical concern is that such nano-target would generate excessive backgrounds that compete against the rare Hawking signals. The preparation of the high-intensity x-ray pulse is another technical challenge. Such high intensity, high frequency x-ray pulse is necessary for penetrating the high density nano-target to induce the flying plasma mirror inside. One convenient way to prepare such x-ray pulse is to reflect an optical laser pulse against a relativistic plasma mirror,[24] which, however, has a rather low reflectivity. With these considerations in mind, it is highly desirable to explore the possibility of a single-stage, gaseous plasma target that could deliver the proposed analog black hole experiment.

Since the analog Hawking temperature and radiation depend sensitively on the mirror's detailed trajectory, it is desirable to invoke as short a laser pulse as possible. This is because in the nonlinear regime long laser pulses are prone to the plasma back-reaction, which tends to deform the laser pulse. In turn, the mirror's subsequent dynamics would deviate from the designed trajectory based on the prescribed plasma density profile.

Motivated by these considerations, in this paper we re-derive the acceleration of a plasma mirror, which in essence is a plasma wakefield, in the strongly nonlinear perturbation regime, induced by an ultra-short laser pulse based on the generalized wakefield principle. The shortest possible laser pulse conceivable, both in principle and in practice, is a single-cycle laser,[25] which, however, is not a prerequisite for our proposed experiment. It suffices our purpose to consider state-of-the-art few-cycle lasers, as long as the pulse length is sufficiently short that the plasma back-reaction is negligible. As a consequence, which is fortuitous, the analytic formulation becomes simplified.

The driving laser pulse, through dumping its energy into the plasma to create the wakefields, would suffer a frequency redshift and therefore would gradually slow down, or decelerate. This, however, is a slow process. In the case of a uniform plasma, such a slow-down will result in the deceleration of the plasma wakefield or mirror. The situation is different when the laser traverses a nonuniform plasma.

There are several factors that determine whether a relativistic flying plasma mirror would accelerate or decelerate when traversing a plasma with a density gradient. In the case of an increasing/decreasing density along the direction of the laser propagation, two effects would happen. One, in addition to the red-shift mentioned above, the driving laser will face an additional slow-down/speed-up mechanism by entering a denser/less-dense plasma region. Namely, as the local refractive index for the same laser frequency becomes smaller/larger, its group velocity decreases/increases. Two, upon entering a denser/less-dense region, the ambient plasma wavelength becomes shorter/longer and therefore the mirror, i.e., the wakefield, behind the driving pulse catches up/falls behind. Such dichotomy surely breaks the conventional wisdom of $v_{ph} = v_g$, which stems from the uniform plasma situations. Whether the net effect results in the acceleration or deceleration of the plasma wakefield or mirror depends on the interplay between the laser pulse and the local plasma conditions.

Note that both the driving laser and the plasma mirror move with a velocity already very close to the speed of light. Since by construction the mirror accelerates, its velocity may at some point exceed the speed of light. No physical principle is violated, of course, since the plasma mirror moves with a phase velocity. With the motivation to mimic the black hole evaporation, in this paper we consider truncating the mirror motion before it turns superluminal, so as to release Hawking radiation's partner modes properly.[6,20,21] It is actually fine that the plasma mirror enters the superluminal regime before it stops. The partner modes would still be trapped until the mirror eventually stops. This superluminal regime may provide extra interesting black hole physics.

This paper is structured as follows. Section II sets up the notations. In Sec. III, we analyze the equation of motion of a plasma mirror traversing a nonuniform plasma. In Sec. IV, we investigate the variation of the ion bubble size. In Sec. V, we derive the plasma velocity and trajectory as a function of the plasma density gradient. In Sec. VI, we derive the mirror acceleration. We then apply our results to the physics of analog black holes in Sec. VII. In Sec. VIII, we discuss the Hawking temperature induced by such accelerating mirror with a set of physical parameters. Finally, in Sec. X, we briefly comment on some experimental considerations for such an analog black hole experiment.

II. COMPETITION BETWEEN TWO PARTIES

In the nonlinear (blowout) regime of plasma perturbations, the back-end of the "ion bubble" with the length x_B that trails behind the driving pulse is where the plasma mirror locates. So the position of the plasma mirror is

$$x_M = x_L - x_B, \tag{1}$$

where x_M and x_L are the position of the mirror and the laser, respectively, in the lab frame. Let $\dot{x} \equiv dx/dt$ and $\ddot{x} \equiv d^2x/dt^2$. Then the velocity and the acceleration of the mirror are

$$\dot{x}_M = \dot{x}_L - \dot{x}_B, \tag{2}$$
$$\ddot{x}_M = \ddot{x}_L - \ddot{x}_B, \tag{3}$$

respectively. $\dot{x}_L$ is what we usually identify as the laser group velocity, $v_g \equiv \eta c$, where the index of refraction η of a plasma in the nonlinear perturbation regime is

$$\eta = \sqrt{1 - \frac{\omega_p^2}{\omega_0^2}\frac{1}{1+\phi}}, \tag{4}$$

where ϕ is the normalized dimensionless scalar potential of the laser--plasma system.

III. SPEED CHANGE OF DRIVING LASER

The rate of change of the laser group velocity $\ddot{x}_L = \dot{v}_g$ is directly related to that of the index of refraction, which in general should depend on the variations of ω_p, ω_0, and ϕ

$$\dot{\eta}(x) = \frac{1}{\eta}\frac{\omega_p^2}{\omega_0^2}\left\{\left(\frac{\dot{\omega}_0}{\omega_0} - \frac{\dot{\omega}_p}{\omega_p}\right)\frac{1}{1+\phi} + \frac{\dot{\phi}}{2(1+\phi)^2}\right\}. \tag{5}$$

Modeling the laser pulse envelop with a sine function, it can be shown[28,29] that

$$\phi \simeq \frac{a_L^2 k_p^2}{8}\left\{\chi^2 - 2\left(\frac{l}{2\pi}\right)^2[1 - \cos(2\pi\chi/l)]\right\}, \tag{6}$$

where a_L is the (Lorentz invariant) laser strength parameter, $l \sim n\lambda_0 \ll \lambda_p$ is the laser pulse length, and $-l \le \chi \le 0$ is the comoving coordinate relative to the head of the laser envelope. We can see that $\phi \ll 1$ within the envelope of a very short laser pulse. Furthermore, the change of the potential within the laser pulse, $\Delta\phi/\Delta\chi \ll \phi/l$, is also minute. We are therefore safe to ignore the contributions of ϕ and $\dot{\phi}$ to η when dealing with the acceleration or deceleration of the laser pulse, i.e.,

$$\eta(x) \simeq \sqrt{1 - \frac{\omega_p^2}{\omega_0^2}}. \tag{7}$$

We emphasize that this approximation applies only to the refractive index within the laser envelop even though the laser is ultra-intense. This is a fortuitous situation due to the fact that the laser pulse is ultra-short.

Let us examine the contributions to the change of the refractive index separately below.

A. *Change of* $\eta(x)$ *due to local density gradient*

Holding the laser frequency fixed, the acceleration (or deceleration) of the laser group velocity due to the variation of plasma density is

$$\dot{v}_{g1} = c\dot{\eta}|_{\omega_0} \simeq -\frac{c}{\eta}\frac{\omega_p^2}{\omega_0^2}\left(\frac{\dot{\omega}_p}{\omega_p}\right). \tag{8}$$

We see that for an increasing plasma frequency in time, which is directly related to an increasing plasma density along the laser propagation direction, the laser pulse tends to decelerate.

B. Laser frequency redshift due to wakefield excitation

The driving laser frequency redshift was well described in the concept of *photon accelerator* by Wilks *et al.*[26] In typical laser–plasma interactions, the laser photon number is roughly conserved. As a consequence, the conservation of energy demands that, by exciting the plasma wakefields, the energy loss of the laser is manifested through a frequency redshift of all photons. In the limit where the laser frequency ω_0 is much larger than that of the plasma, i.e., $\omega_0 \gg \omega_p$, the laser frequency redshift per unit distance of propagation is[26]

$$\frac{\delta\omega_0}{\delta x} = -\frac{\omega_p^2 k_p}{2\omega_0}\frac{\delta n_p}{n_p}. \tag{9}$$

Since

$$\frac{\delta n_p}{n_p} \simeq \frac{eE}{m\omega_p c} = a_L\frac{\omega_0}{\omega_p}, \tag{10}$$

where $\omega_p \equiv \sqrt{4\pi r_e n_p}$, we have, in the nonlinear laser–plasma interaction regime (i.e., $a_L \ge 1$)

$$\frac{\delta\omega_0}{\delta x} \simeq -\frac{a_L}{2}\omega_p k_p. \tag{11}$$

Holding ω_p constant, that is, for a uniform plasma, the rate of change of the laser group velocity is

$$\dot{v}_{g2} = c\dot{\eta}|_{\omega_p} \simeq \frac{c}{\eta}\frac{\omega_p^2}{\omega_0^2}\left(\frac{\dot{\omega}_0}{\omega_0}\right). \tag{12}$$

With $\dot{\omega}_0 = c\eta\partial\omega_0/\partial x$, and identifying $\partial\omega_0/\partial x$ with $\delta\omega_0/\delta x$, we find

$$\dot{v}_{g2} \simeq -c\frac{a_L}{2}k_p\frac{\omega_p^3}{\omega_0^3}. \tag{13}$$

Once again, this frequency redshift effect also slows down the laser.

The total rate of change of the laser group velocity is the sum due to these two effects. Putting these two contributions together, the net deceleration of the laser pulse is

$$\ddot{x}_L = \dot{v}_g = \dot{v}_{g1} + \dot{v}_{g2} = -\frac{c}{\eta}\frac{\omega_p^2}{\omega_0^2}\left[\frac{\dot{\omega}_p}{\omega_p} + \frac{\dot{\omega}_0}{\omega_0}\right]$$
$$\simeq -\frac{c}{\eta}\frac{\omega_p^2}{\omega_0^2}\left[\frac{\dot{\omega}_p}{\omega_p} + \eta\frac{a_L}{2}k_p\frac{\omega_p}{\omega_0}\right]. \tag{14}$$

Since $\omega_p^2 \ll \omega_0^2$, we will save only terms up to ω_p^2/ω_0^2 in the rest of the discussion. Thus, the deceleration due to the frequency redshift is relatively unimportant and we shall ignore the second term and regard ω_0 is constant. By the same token, η can be replaced by one in this expression.

IV. VARIATION OF ION BUBBLE SIZE

In this section, we turn our attention to the variation of the ion bubble, that is, the change of x_B as the plasma density varies. The plasma wavelength is a subtle issue even in the case of uniform plasmas. In the absence of the driving laser, i.e., the source, the plasma perturbations, and therefore the wavelength, in all regimes (linear and nonlinear) is well described in the Akhiezer–Polovin theory.[27] In the presence of the driving laser, however, the size of the first ion bubble immediately behind the laser is in general different from the Akhiezer–Polovin plasma wavelength due to the interplay between the laser pulse and the plasma.

Laser–plasma interaction in the nonlinear regime has been well studied.[28,29] Relevant to our case of ultra-short laser pulse that covers only a few laser cycles, i.e., $l \sim n/\omega_0$, it was shown that if the condition

$$\frac{\pi}{2} a_L \frac{l}{\lambda_p} \sim \frac{\pi}{2} n a_L \frac{\omega_p}{\omega_0} \ll 1 \tag{15}$$

is satisfied, then the plasma wavelength excited by this short pulse follows the conventional expression in the linear regime, that is,

$$\lambda_p = \frac{2\pi c}{\omega_p}, \quad \text{even if } a_L \gg 1. \tag{16}$$

This fact can be intuitively appreciated by recognizing that when the laser pulse is ultra-short, the scalar potential within the laser pulse, ϕ, is much less than unity [c.f. Eq. (6)]. As a result, the "transverse" Lorentz factor $\gamma_\perp \sim 1$, indicating that the plasma perturbations remain in the linear regime. It is ironic, but true, that in the ultra-short limit of the driving laser pulse the plasma wavelength in the nonlinear regime reduces to that in the linear regime, which significantly simplifies our subsequent derivations.

Furthermore, it can be shown from the Green's function solution to the master equation of the plasma wakefields in the linear regime,[30] which remains true in the nonlinear regime,[31] and from computer simulations[32] that the distance from a short driving laser pulse to the first maximum of the plasma density perturbation behind it; therefore, the size (length) of the first ion bubble equals to 3/4 of the plasma wavelength. In the following, we will assume such a limit in our derivations. That is,

$$x_B = \frac{3}{4}\lambda_p(x) = \frac{3\pi c}{2\omega_p(x)}. \tag{17}$$

The variation of the ion bubble size is therefore

$$\dot{x}_B = -\frac{3\pi c}{2}\frac{\dot{\omega}_p}{\omega_p^2}. \tag{18}$$

We see that $\dot{x}_B$ is negative definite in the situation where $\dot{\omega}_p > 0$, and is positive if $\dot{\omega}_p < 0$. That is, the plasma wavelength, as well as the longitudinal bubble size, shrinks or grows when the laser traverses a plasma with an increasing or decreasing density, respectively.

V. PLASMA MIRROR VELOCITY AND TRAJECTORY

In the concept of a moving mirror as an analog black hole,[18,19,33] the mirror was assumed to be a real object and therefore should move with a velocity that is less than the speed of light. In the Chen–Mourou proposal, however, the mirror is not real but a phase wave, which can in principle have a speed faster than that of light. To qualify the plasma mirror as an analog black hole, we should ensure two conditions. First, the phase velocity of the plasma mirror is less than the speed of light, that is, $\dot{x}_M/c < 1$. Second, it accelerates.

Inserting v_g and $\dot{x}_B$ into Eq. (2), i.e., $\dot{x}_M = \dot{x}_L - \dot{x}_B$, we obtain the velocity of the mirror

$$\frac{\dot{x}_M}{c} = 1 - \frac{1}{2}\frac{\omega_p^2}{\omega_0^2} + \frac{3\pi}{2}\frac{\dot{\omega}_p}{\omega_p^2}. \tag{19}$$

In our conception, the time derivatives of the plasma frequency are induced through the spacial variation of the plasma density via the relation $\omega_p(x) = c\sqrt{4\pi r_e n_p(x)}$. Since $d/dt = \partial/\partial t + (\partial x/\partial t)\partial/\partial x$ and ω_p does not depend on time explicitly, we find that $\dot{\omega}_p = v_{ph}\partial\omega_p/\partial x = \dot{x}_M \partial\omega_p/\partial x \equiv \dot{x}_M \omega_p'$. Substituting it into Eq. (19) and reshuffling terms, we find

$$\frac{\dot{x}_M}{c} = \frac{1 - (1/2)\omega_p^2/\omega_0^2}{1 - (3\pi/2)c\omega_p'/\omega_p^2}. \tag{20}$$

For the case of a decreasing density profile, a "down ramp," i.e., $\omega_p' < 0$, the sub-luminosity requirement is automatically satisfied. On the other hand for an increasing density profile, an "up ramp," i.e., $\omega_p' > 0$, the following constraint must be satisfied:

$$\frac{\omega_p^2}{\omega_0^2} > 3\pi\frac{c\omega_p'}{\omega_p^2}. \tag{21}$$

In the following, we will consider only the down ramp case, where the plasma density and its gradient are *not* constrained by the above condition.

The standard method to derive the response function of a particle detector in response to the perturbations induced by a flying mirror is based on the mirror's trajectory.[33] With that purpose in mind, here we separate the x and t variables in Eq. (19) and integrate them separately, that is,

$$\int_0^t \bar{c}dt = \int_0^x dx\left(1 - \frac{3\pi c}{2}\frac{\omega_p'}{\omega_p^2}\right), \quad 0 \le x \le l, \tag{22}$$

where $\bar{c} = (1 - \omega_p^2/\omega_0^2)c = \eta c$ is the speed of light in the plasma medium. We find

$$x(t) = \bar{c}t - \frac{3\pi c}{2}\left[\frac{1}{\omega_p(x)} - \frac{1}{\omega_p(x_0)}\right], \quad 0 \le x \le l. \tag{23}$$

In Sec. VIII, we will use this formula to estimate the Hawking temperature of a flying plasma mirror.

VI. PLASMA MIRROR ACCELERATION

We now examine the acceleration of the plasma mirror. Again, since ω_p does not depend on time explicitly, we have

$$\dot{\omega}_p = \dot{x}_M \omega'_p,$$
$$\ddot{\omega}_p = \ddot{x}_M \omega'_p + \dot{x}_M^2 \omega''_p, \tag{24}$$
$$\dot{\omega}'_p = \dot{x}'_M \omega'_p + \dot{x}_M \omega''_p.$$

Replacing $\dot{\omega}_p$ with $\dot{x}_M \omega'_p$ and inserting $\dot{x}_M$ from Eq. (20), we have

$$\frac{\ddot{x}_M}{c} = \frac{1-(1/2)\omega_p^2/\omega_0^2}{\left[1-(3\pi/2)c\omega'_p/\omega_p^2\right]^3} \times \left\{-\frac{\omega_p^2}{\omega_0^2}\frac{\omega'_p}{\omega_p}\left[1-\frac{3\pi c}{2}\frac{\omega'_p}{\omega_p^2}\right]+\frac{3\pi c}{2}\left[\frac{\omega''_p}{\omega_p^2}-2\frac{\omega_p'^2}{\omega_p^3}\right]\right\}. \tag{25}$$

The second and the fourth terms have the same dependence in $\omega_p'^2/\omega_p^3$, but the former is suppressed by a factor ω_p^2/ω_0^2 relative to the latter. So we may neglect the second term and simplify the formula as

$$\frac{\ddot{x}_M}{c} = \frac{1-(1/2)\omega_p^2/\omega_0^2}{\left[1-(3\pi/2)c\omega'_p/\omega_p^2\right]^3}\left\{-\frac{\omega_p^2}{\omega_0^2}\frac{\omega'_p}{\omega_p}+\frac{3\pi c}{2}\left[\frac{\omega''_p}{\omega_p^2}-2\frac{\omega_p'^2}{\omega_p^3}\right]\right\}. \tag{26}$$

Our desire is to achieve as high an acceleration as possible. To accomplish that, one should design the system in such a way that the characteristic length D and the denominator are minimized.

VII. APPLICATION TO ANALOG BLACK HOLES

For the purpose of investigating Hawking evaporation and the information loss paradox, it is desirable to maximize the mirror acceleration. The laser frequency ω_0 is largely determined by the laser technology and so there is not a large room for tuning. About the plasma frequency ω_p, the range is much larger. Essentially all the plasma based wakefield accelerator[34] experiments have been invoking gaseous plasmas. In the Chen–Mourou proposal,[21] extremely high density plasmas induced by solid targets through nano-fabrication technology was proposed. To be sure, there are pros and cons between these two types of plasmas. One obvious drawback of the nano-target is the inevitable plasma-induced background events that would compete with the Hawking signals.

In addition to the plasma frequency, one other important parameter is the characteristic length of the plasma density gradient, D, associated with ω'_p and ω''_p, which plays an essential role in contributing to the value of mirror acceleration $\ddot{x}_M$. As we will see below, this turns out to be the most sensitive parameter that controls the Hawking temperature. Furthermore, the signs of both ω'_p and ω''_p are evidently critical to the final value of $\ddot{x}_M$. As discussed in Sec. VI, we shall focus on the down ramp case.

A. Constant-plus-exponential profile

A simple but well motivated plasma density profile is the one that corresponds to the exponential trajectory investigated by Davies and Fulling,[18,19] which is of special geometrical interest because it corresponds to a well-defined horizon.[33] Inspired by that, we consider the following plasma density variation along the direction of the laser propagation inside the plasma target with thickness L:

$$n_p(x) = n_{p0}(a+be^{-x/D})^2, \quad 0 \le x \le L, \tag{27}$$

where n_{p0} is the plasma density at $x=0$ and D is a characteristic length of density variation. Accordingly, the plasma frequency varies as

$$\omega_p(x) = \omega_{p0}(a+be^{-x/D}), \quad 0 \le x \le L, \tag{28}$$

where $\omega_{p0} = c\sqrt{4\pi r_e n_{p0}}$. In our conception, the time derivatives of the plasma frequency are induced through the spacial variation of the plasma density via the relation $\omega_p(x) = c\sqrt{4\pi r_e n_p(x)}$. Thus,

$$\omega'_p(x) = -\frac{b}{D}e^{-x/D}\omega_{p0}(x), \tag{29}$$

$$\omega''_p(x) = \frac{b}{D^2}e^{-x/D}\omega_{p0}(x). \tag{30}$$

Inserting these into Eqs. (20) and (26), we then have, for the exponential distribution,

$$\frac{\dot{x}_M}{c} = \frac{1-(\omega_{p0}^2/2\omega_0^2)(a+be^{-x/D})^2}{1+(3b/4)(\lambda_{p0}/D)e^{-x/D}/(a+be^{-x/D})^2} \tag{31}$$

and

$$\frac{\ddot{x}_M}{c^2} = \frac{1-(\omega_{p0}^2/2\omega_0^2)(a+be^{-x/D})^2}{\left[1+(3b/4)(\lambda_{p0}/D)e^{-x/D}/(a+be^{-x/D})^2\right]^3}\frac{\lambda_{p0}}{D}\frac{be^{-x/D}}{(a+be^{-x/D})} \times \left\{\frac{\omega_p^2}{\omega_0^2}\frac{1}{\lambda_{p0}}+\frac{3}{4D}\left[\frac{1}{a+be^{-x/D}}-\frac{2be^{-x/D}}{(a+be^{-x/D})^2}\right]\right\}. \tag{32}$$

B. Constant-plus-Gaussian profile

A common practice in plasma wakefield acceleration experimentation is to invoke gas jets as the plasma target, where the intersecting laser would instantly ionize the neutral gas into plasma. The density profile of a gas jet is usually in Gaussian distribution. Motivated by the practical experimentation consideration, here we investigate the case of a half-Gaussian plus a constant profile, defined as

$$n_p(x) = n_{p0}(a+be^{-x^2/2D^2})^2, \quad 0 \le x \le L, \tag{33}$$

where n_{p0} is the plasma density at $x=0$, D is a characteristic length of density variation and $x_b > D$ is the location of the plasma front and back boundary. Accordingly, the plasma frequency varies as

$$\omega_p(x) = \omega_{p0}(a+be^{-x^2/4D^2}), \quad 0 \le x \le L, \tag{34}$$

where $\omega_{p0} = c\sqrt{4\pi r_e n_{p0}}$. In our conception, the time derivatives of the plasma frequency are induced through the spacial variation of the plasma density via the relation $\omega_p(x) = c\sqrt{4\pi r_e n_p(x)}$. Thus,

$$\omega'_p(x) = -b\frac{x}{D^2}e^{-x^2/2D^2}\omega_{p0}(x), \tag{35}$$

$$\omega''_p(x) = \frac{b}{D^2}\left(-1+\frac{x^2}{D^2}\right)e^{-x^2/2D^2}\omega_{p0}(x). \tag{36}$$

Inserting these into Eqs. (20) and (25), we then have, for Gaussian distribution,

$$\frac{\dot{x}_M}{c} = \frac{1-\omega_p^2/2\omega_0^2}{1+(3b/2)(\lambda_{p0}x/D^2)e^{-x^2/2D^2}} \tag{37}$$

and

$$\frac{\ddot{x}_M}{c^2} = \frac{1-\omega_p^2/2\omega_0^2}{\left[1+(3b\lambda_{p0}x/4D^2)e^{-x^2/2D^2}/(a+be^{-x^2/2D^2})^2\right]^3}\frac{b\lambda_{p0}}{2D^2} \times \frac{e^{-x^2/2D^2}}{(a+be^{-x^2/2D^2})^2}\left\{\frac{\omega_p^2}{\omega_0^2}\frac{x}{\lambda_{p0}}+\frac{3}{4}\left[-1+\frac{x^2}{D^2}-\frac{2bx^2}{D^2}\frac{e^{-x^2/2D^2}}{a+be^{-x^2/2D^2}}\right]\right\}. \tag{38}$$

VIII. ANALOG HAWKING TEMPERATURE

There exists a wealth of literature on the vacuum fluctuating modes of quantum fields, their reflections from a flying mirror, and the analog "Hawking temperature" of such a flying mirror as an analog black hole.[33] In general, such analog Hawking temperature depends on the actual mirror trajectory. As an example, we invoke the constant-plus-exponential density profile [c.f., Eq. (27)]. Starting with Eq. (22), we obtain

$$\int_0^t \bar{c}dt = \int_0^{x_M} dx\left[1+\frac{3b\lambda_{p0}}{4D}\frac{e^{-x/D}}{(a+be^{-x/D})^2}\right], \tag{39}$$

where $\bar{c} = \eta c = (1-\omega_p^2/2\omega_0^2)c$ is the speed of light in the plasma medium, which is position dependent. In our conception,[21] the plasma target thickness is supposed to be much larger than the characteristic scale of the density variation, i.e., $L \gg D$. In this situation, one is safe to extend the integration to $x_M \to \infty$ (and $t \to \infty$). Taking this approximation, we find

$$x_M(t) = \eta_a ct + Ae^{-\eta_a ct/D} - A, \quad t \to \infty, \tag{40}$$

where $\eta_a = 1 - a^2\omega_{p0}^2/2\omega_0^2$ and $A = \eta_a D[ab(\omega_{p0}^2/\omega_0^2) - (3b/4a^2)(\lambda_{p0}/D)]$. To compare this with the Davies–Fulling trajectory, i.e., Eq. (4.51) of Ref. 33

$$z(t) \to -t - Ae^{-2\kappa t} + B, \quad t \to \infty, \tag{41}$$

where A, B, κ are positive constants and $c \equiv 1$, one is reminded that the setup of our mirror is right-moving while the Davies–Fulling one is left-moving. It is straightforward, by switching our x to $-x$, to verify that our trajectory in Eq. (40) can be recast into

$$x_M(t) = -\eta_a ct - Ae^{-\eta_a ct/D} + A, \quad t \to \infty, \tag{42}$$

which is identical to that of Davies–Fulling safe other than a specific assignment of the constant term, $B = A$, in our case, which does not affect the Hawking temperature and radiation.

Transcribing the $x_M(t)$ coordinates to the (u, v) coordinates, where $u = \eta_a ct - x_M(t)$ and $v = \eta_a ct + x_M(t)$, we see that only null rays with $v < A$ can be reflected. All rays with $v > A$ will pass undisturbed. The ray $v = A$, therefore, acts as an effective horizon.[33] Following the standard recipe,[33] we obtain the Wightman function as:

$$D^+(u,v;u',v') = -\frac{1}{4\pi}\ln\left[2Ae^{2\eta_a c(t+t')/2D} \times \sinh(\eta_a c\Delta t/2D)\right], \tag{43}$$

where $\Delta t = t - t' = \Delta u/2\eta_a c$ in the $t \to \infty$ limit. The constant factors in the argument of the log function in the above equation do not contribute to the nontrivial part of the physics. Note that in our notation t is the time when the ray hits the mirror. Let us denote the observation time and position by T and X. Then $u = \eta_a cT - X = \eta_a ct - x_M$. For large t, $u = \eta_a cT - X = 2\eta_a ct - A$. This leads to $\Delta u = 2\eta_a c\Delta t = \eta_a c\Delta T$ for a static mirror at $X =$ const. Integrating over T and T', we then have, in the asymptotic limit of $t, t' \to \infty$,

$$D^+(u,v;u',v') = -\frac{1}{4\pi}\ln[\sinh(\eta_a c\Delta t/2D)]. \tag{44}$$

This leads to the response function (of the particle detector) per unit time with the form

$$\mathcal{F}(E)/\text{unit time} = \frac{1}{E}\frac{1}{(e^{E/k_BT_H}-1)}, \tag{45}$$

where the analog Hawking temperature of the mirror measured by a stationary particle detector is

$$k_BT_H = \frac{\hbar c}{4\pi}\frac{\eta_a}{D}. \tag{46}$$

Here k_B is the Boltzmann constant. It is interesting to note that the analog Hawking temperature associated with our constant-plus-exponential density profile depends strongly on the characteristic length D and only weakly on the plasma density (through η_a). This points to the possibility of employing gaseous instead of solid plasma targets, which would greatly simplify our proposed experiment.

Consider a flying plasma mirror experiment where the driving laser has the frequency $\omega_0 = 3.5 \times 10^{15}\ \text{s}^{-1}$. For the plasma target, we set $a = b = 1$, and $n_p(x) = n_{p0}(1 + e^{-x/D})^2$, $n_p(x = 0) = 1.0 \times 10^{17}\text{cm}^{-3} = 4n_{p0}$. The corresponding plasma frequency is $\omega_{p0} = 0.9 \times 10^{13}\ \text{s}^{-1}$ and the plasma wavelength $\lambda_{p0} = 200\mu\text{m}$. Next we design the plasma target density profile. Since our formula is not constrained by the adiabatic condition, we are allowed to choose a minute characteristic length $D = 0.5\mu\text{m}$. Then from the above equation we find

$$k_BT_H \sim 3.1 \times 10^{-2}\text{eV}, \tag{47}$$

which corresponds to a characteristic Hawking radiation frequency $\omega_H \sim 4.8 \times 10^{13}\ \text{s}^{-1} > \omega_{p0}$. So the Hawking radiation can propagate through the plasma for detection.

IX. KINETIC SIMULATIONS

To verify our analytic formulations for the acceleration of the flying plasma mirror as a function of the plasma density gradient, we performed $(1+1)$ D particle-in-cell (PIC) computer simulations using the fully relativistic electromagnetic code EPOCH[35] that keeps track of the laser–plasma interaction self-consistently. While details of the simulation results, including the mirror reflectivity and the reflection spectrum, will be published separately,[32] here we preview its excerpt on the mirror acceleration. To be sure, such a simulation focuses only on the classical electrodynamic aspect of our analog black hole concept, which does not include the physics of Hawking radiation and the quantum entanglement with their partner modes. Such an all-encompass, end-to-end simulation is our eventual goal that goes beyond the scope of Ref. 32, which would require another sizable effort to carry out. Fortunately, since the Hawking radiation and its associated effects are very weak, they do not induce any significant

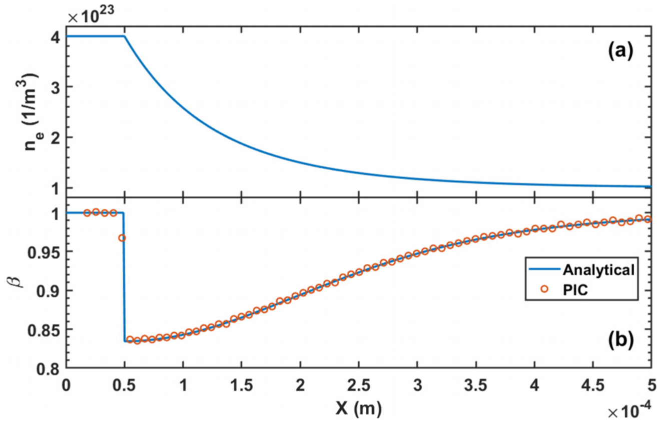

FIG. 1. PIC simulation of the numerical example in Sec. VIII. (a) A plasma target with a constant-plus-exponential density gradient, $n_p(x) = n_{p0}(1 + e^{-x/D})^2$, $n_p(x = 0) = 1.0 \times 10^{17}\,\mathrm{cm}^{-3} = 4n_{p0}$. (b) Comparison of the plasma mirror speed, $\beta = \dot{x}_M/c$, between our analytic formula, Eq. (31), (solid blue curve) and the PIC simulations (orange circles). The two agree extremely well.

back-reaction to the classical laser–plasma dynamics described in Secs. I–VIII.

Figure 1 shows the PIC simulation of the numerical example in Sec. VIII. Specifically, the setting is an intense laser impinging a plasma target with a constant-plus-exponential density gradient, $n_p(x) = n_{p0}(1 + e^{-x/D})^2$, $n_p(x = 0) = 1.0 \times 10^{17}\mathrm{cm}^{-3} = 4n_{p0}$, as shown in Fig. 1(a). The laser impinging from the left induces a plasma mirror (not shown in this figure) that trails behind the leading laser pulse and undergoes an acceleration. Its instantaneous speed, $\beta = \dot{x}_M/c$, as a function of position, computed using the analytic formula in Eq. (31) and through PIC simulations, are plotted in Fig. 1(b). As can be seen, they agree extremely well.

X. CONCLUSION

One key element in the concept of flying plasma mirrors as analog black holes is the dynamics of the mirror acceleration induced by the plasma target density gradient. In this paper, we fill in the gap of such dynamical details left by the original proposal of Chen and Mourou.[21] Different physical effects that cause the plasma mirror to slow down or speed up are explicitly analyzed. Based on such microscopic examination, the mirror speed, trajectory, and acceleration are derived. While the numerical example provided invoked a gaseous plasma, our formulas are generic that can be applied to solid plasmas as well.

As mentioned in the Introduction, from the experimentation's point of view, one of the challenges of the proposed two-stage analog black hole experiment is the excessive backgrounds generated by the nano-thin-film target during the second stage. Another is the preparation of a high-intensity x-ray pulse to induce the flying plasma mirror inside the nano-target. In this work, we have shown that a single-stage, gaseous plasma target is viable. This would help to bypass the challenges of the intense x-ray production and the nano-target fabrication.

This revised approach, however, is not without technical challenges. For one thing, our numerical example requires a plasma density gradient characteristic length D that is 400 times smaller than the plasma wavelength. (Note, however, that although at the first glance this corresponds to a 400-e-folding change, dictated by the constant term the net decrease in the plasma density over one plasma wavelength is only a factor 4, which is mild.) Creating sharp gradients in gas targets has been a topic in laser ion accelerations where supersonic shock waves in gases would temporally and spatially boost the gas density, and thus the plasma density, to near critical so as to slow down the laser for the heavy ions to capture the wakefields for acceleration.[36] Further R&D along this line is evidently needed to further reduce the characteristic length of the density gradient. Another technical challenge is the detection of the rare events of Hawking radiation. Although we managed to raise the Hawking temperature over that corresponds to typical astrophysical black holes by many orders of magnitude, the expected event rate per shot of laser remains low. For this reason and for the purpose of tracking quantum entanglement between Hawking particles and their partner modes, it is important to detect the Hawking radiation photon-by-photon. Single photon detection technology in the far-infrared frequency range, corresponding to our specific numerical example, has been developed in recent years (See, for example, Ref. 37) Again, more R&D is needed to suit our specific demand.

It is hoped that the results in this paper would provide guidance to the design and operation of experiments using flying plasma mirrors as analog black holes.

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DATA AVAILABILITY

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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