Phantom crossing dark energy in Horndeski's theory

Jiro Matsumoto[*]

Leung Center for Cosmology and Particle Astrophysics, National Taiwan University, Taipei 10617, Taiwan and Institute of Physics, Kazan Federal University, Kremlevskaya Street 16a, Kazan 420008, Russia

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The ΛCDM model is a remarkably successful model which is consistent with the observations of cosmic microwave background radiation (CMB), baryon acoustic oscillation (BAO), and the large scale structure of the Universe. However, the discrepancy in the value of H_0 between the local observations and PLANCK observation of CMB was recently pointed out. One of the ways to ease the discrepancy is to introduce phantom dark energy instead of the cosmological constant. However, phantom dark energy often suffers from instabilities. We will investigate the general solution to overcome the difficulty of phantom dark energy and construct some particular models which have a phantom crossing and can be consistent with the observations.

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I. INTRODUCTION

The accelerated expansion of the Universe was discovered by the observations of type Ia supernovae in the late 1990s [1,2], and it is now also supported by the other observations: cosmic microwave background (CMB) radiation [3–5], baryon acoustic oscillations (BAO) [6–10], and so on. To realize the accelerated expansion in the homogeneous and isotropic Universe, it is necessary to introduce some energy components with the equation of state parameter $w = p/\rho$ which is less than $-1/3$ in the Einstein equations. Dark energy is a hypothetical energy which has such a property. The most famous model of dark energy is the cosmological constant Λ, and the Λ cold dark matter (ΛCDM) model is known as the standard model in cosmology. The ΛCDM model is a simple model but it is consistent with almost all of the astronomical observations. However, the recent observations of supernovae and Cepheid variables [11,12] and the observation of CMB by PLANCK satellite [4,5] show that there is more than a 2σ discrepancy in the obtained values of H_0. This result would imply that dark energy is not constant but dynamical [13,14]. Moreover, it is also shown that dynamical dark energy which has a phantom crossing is favored if we take the other observations into account [15]. Here, phantom crossing is the phenomenon that the equation of state parameter w dynamically crosses over the value -1.

A famous model of dynamical dark energy, the quintessence model [16–19], cannot realize phantom crossing without instability, because phantom crossing is only realized when the sign of the kinetic term flips [20]. Horndeski's theory [21] is known as a general theory of scalar-tensor theory including the quintessence model as a special case. In Horndeski's theory, it is also known that phantom dark energy is, in some cases, realized without instability (e.g., see [22,23]). We will investigate the general conditions for realizing stable phantom dark energy in Horndeski's theory in this paper. The contents of the paper are as follows: general background equations and sound speeds in Horndeski's theory are given in Sec. II, the general conditions for stable phantom dark energy are derived in Sec. III, some examples of phantom crossing dark energy are given in Sec. IV, and concluding remarks are in Sec. V. We use natural units, $\hbar = c = k_\mathrm{B} = 1$, and the gravitational constant $8\pi G$ is denoted by $\kappa^2 \equiv 8\pi/{M_\mathrm{Pl}}^2$ with the Planck mass of $M_\mathrm{Pl} = G^{-1/2} = 1.2 \times 10^{19}$ GeV in the following.

II. HORNDESKI'S THEORY

The action in Horndeski's theory is given by [21,24,25]

$$S_H = \sum_{i=2}^{5} \int d^4x \sqrt{-g}\mathcal{L}_i, \tag{2.1}$$

where

$$\mathcal{L}_2 = K(\phi, X), \tag{2.2}$$

$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi, \tag{2.3}$$

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4X}[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2], \tag{2.4}$$

[*]jmatsumoto@ntu.edu.tw

$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5X}}{6} \times [(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3]. \quad (2.5)$$

Here, K, G_3, G_4, and G_5 are generic functions of ϕ and $X = -\partial_\mu\phi\partial^\mu\phi/2$, and the subscript X means derivative with respect to X. The total action we will consider is the sum of S_H and the action of matter fluid $S_{\rm matter}$, which contain baryons and cold dark matter. The background equations of the Universe are given by assuming homogeneity and isotropy of the metric. Substituting $\phi = \phi(t)$ and the metric $ds^2 = -N^2(t)dt^2 + a^2(t)dx^2$ into the action, and subsequently varying the action with respect to $N(t)$ gives [25]

$$\rho_{\rm matter} + \sum_{i=2}^{5}\mathcal{E}_i = 0, \quad (2.6)$$

where

$$\mathcal{E}_2 = 2XK_X - K, \quad (2.7)$$

$$\mathcal{E}_3 = 6X\dot\phi HG_{3X} - 2XG_{3\phi}, \quad (2.8)$$

$$\mathcal{E}_4 = -6H^2G_4 + 24H^2X(G_{4X} + XG_{4XX}) - 12HX\dot\phi G_{4\phi X} - 6H\dot\phi G_{4\phi}, \quad (2.9)$$

$$\mathcal{E}_5 = 2H^3X\dot\phi(5G_{5X} + 2XG_{5XX}) - 6H^2X(3G_{5\phi} + 2XG_{5\phi X}), \quad (2.10)$$

and $\rho_{\rm matter}$ is the energy density of matter. Here, $H = \dot a/a$ is the Hubble rate function and the dot means derivative with respect to time. Variation with respect to $a(t)$ yields

$$p_{\rm matter} + \sum_{i=2}^{5}\mathcal{P}_i = 0, \quad (2.11)$$

where

$$\mathcal{P}_2 = K, \quad (2.12)$$

$$\mathcal{P}_3 = -2X(G_{3\phi} + \ddot\phi G_{3X}), \quad (2.13)$$

$$\mathcal{P}_4 = 2(3H^2 + 2\dot H)G_4 - 4H^2X\left(3 + \frac{\dot X}{HX} + 2\frac{\dot H}{H^2}\right)G_{4X} - 8HX\dot XG_{4XX} + 2(\ddot\phi + 2H\dot\phi)G_{4\phi} + 4XG_{4\phi\phi} + 4X(\ddot\phi - 2H\dot\phi)G_{4\phi X}, \quad (2.14)$$

$$\mathcal{P}_5 = -2X(2H^3\dot\phi + 2H\dot H\dot\phi + 3H^2\ddot\phi)G_{5X} - 4H^2X^2\ddot\phi G_{5XX} + 4HX(\dot X - HX)G_{5\phi X} + 2H^2X\left(3 + 2\frac{\dot X}{HX} + 2\frac{\dot H}{H^2}\right)G_{5\phi} + 4HX\dot\phi G_{5\phi\phi}, \quad (2.15)$$

and $p_{\rm matter}$ is the pressure of matter. The above two Eqs. (2.6) and (2.11) correspond to the Friedmann equations. The equation of motion of the scalar field is given by varying the action with respect to $\phi(t)$:

$$\frac{1}{a^3}\frac{d}{dt}(a^3J) = P_\phi, \quad (2.16)$$

where

$$J = \dot\phi K_X + 6HXG_{3X} - 2\dot\phi G_{3\phi} + 6H^2\dot\phi(G_{4X} + 2XG_{4XX}) - 12HXG_{4\phi X} + 2H^3X(3G_{5X} + 2XG_{5XX}) - 6H^2\dot\phi(G_{5\phi} + XG_{5\phi X}), \quad (2.17)$$

$$P_\phi = K_\phi - 2X(G_{3\phi\phi} + \ddot\phi G_{3\phi X}) + 6(2H^2 + \dot H)G_{4\phi} + 6H(\dot X + 2HX)G_{4\phi X} - 6H^2XG_{5\phi\phi} + 2H^3X\dot\phi G_{5\phi X}. \quad (2.18)$$

Equations (2.6), (2.11), and (2.16) control the background evolution of the Universe. In the same manner as the quintessence model, Eqs. (2.11) and (2.16) are equivalent when Eq. (2.6) holds. Equations (2.6) and (2.11) can be rewritten in the well-known form

$$3H^2 = \kappa^2(\rho_{\rm matter} + \rho_\phi), \quad (2.19)$$

$$-3H^2 - 2\dot H = \kappa^2(p_{\rm matter} + p_\phi), \quad (2.20)$$

if we define ρ_ϕ and p_ϕ as

$$\rho_\phi \equiv \sum_{i=2}^{5}\mathcal{E}_i + \frac{3H^2}{\kappa^2}, \qquad p_\phi \equiv \sum_{i=2}^{5}\mathcal{P}_i - \frac{1}{\kappa^2}(3H^2 + 2\dot H). \quad (2.21)$$

We will use Eq. (2.21) as the definitions of effective energy density and effective pressure.

The perturbative behavior of the Universe can be described if we employ metric perturbations, the perturbation of ϕ, and that of the energy-momentum tensor of matter. Sound speed is an important quantity for understanding the dynamics of perturbation quantities, because the propagating speed of perturbation quantities is determined by sound speed if it is not zero. The sound speed for tensor perturbations is expressed as [25]

$$c_T^2 = \frac{G_4 - XG_{5\phi} - XG_{5X}\ddot{\phi}}{G_4 - 2XG_{4X} - X(G_{5X}\dot{\phi}H - G_{5\phi})}. \tag{2.22}$$

Equation (2.22) shows that the sound speed for tensor perturbations is independent from the functions $K(\phi, X)$, $G_3(\phi, X)$, and matter components. If the terms $XG_{5\phi}$, $XG_{4X}, \ldots$ are relevant for the evolution of the Universe, then they should be the same order as G_4 as seen from Eqs. (2.6) and (2.11). In this case, c_T^2 deviates from 1 except for some special cases. The recent observation of gravitational wave GW170817 [26] and its electromagnetic counterparts [27–29] showed that the speed of gravitational waves should satisfy

$$|c_T^2 - 1| \lesssim 10^{-15} \tag{2.23}$$

in the relatively recent Universe. This bound means that the speed of gravitational waves should be almost the same as that of an electromagnetic wave not only around stellar objects but also in the void region. Therefore, it is natural to think that the terms proportional to G_{4X}, $G_{5\phi}$, and G_{5X} are not relevant for the current accelerated expansion of the Universe. In the following, we treat $G_4(\phi, X)$ and $G_5(\phi, X)$ as $G_4(\phi)$ and $G_5(\phi, X) = 0$. Here, $G_5(\phi, X)$ is not expressed as constant but as 0 because constant G_5 does not contribute to Eqs. (2.6), (2.11), and (2.16). Further discussions for the constraints from gravitational wave detection GW170817 for Horndeski's theory are given in Refs. [30–37].

The sound speed for the scalar mode is written as

$$c_s^2 = \frac{1}{A}\Big[G_4(K_X - 2G_{3\phi} + 2\ddot{\phi}G_{3X} + \dot{\phi}^2 G_{3\phi X} + \dot{\phi}^2\ddot{\phi}G_{3XX} + 4H\dot{\phi}G_{3X}) + 3G_{4\phi}^2 - \dot{\phi}^2 G_{3X}G_{4\phi} - \frac{1}{4}\dot{\phi}^4 G_{3X}^2\Big], \tag{2.24}$$

$$A \equiv G_4[K_X + \dot{\phi}^2 K_{XX} - 2G_{3\phi} - \dot{\phi}^2 G_{3\phi X} + 3H\dot{\phi}(2G_{3X} + \dot{\phi}^2 G_{3XX})] + 3\left(G_{4\phi} - \frac{1}{2}G_{3X}\dot{\phi}^2\right)^2. \tag{2.25}$$

The expression (2.24) is obtained by substituting $\rho_{\mathrm{matter}} + p_{\mathrm{matter}} = -\rho_\phi - p_\phi - 2\dot{H}/\kappa^2$ into Eq. (3.12) in Ref. [38].

III. GENERAL PROPERTIES OF STABLE PHANTOM DARK ENERGY

The no-ghost condition for scalar mode and that for tensor mode are given by $A > 0$ and $G_4 > 0$, respectively [38]. Therefore, non-negativeness of the sound speeds and no-ghost conditions give the following stability conditions:

$$G_4(K_X - 2G_{3\phi} + 2\ddot{\phi}G_{3X} + \dot{\phi}^2 G_{3\phi X} + \dot{\phi}^2\ddot{\phi}G_{3XX} + 4H\dot{\phi}G_{3X}) + 3G_{4\phi}^2 - \dot{\phi}^2 G_{3X}G_{4\phi} - \frac{1}{4}\dot{\phi}^4 G_{3X}^2 \geq 0, \tag{3.1}$$

$$G_4\Big[K_X + \dot{\phi}^2 K_{XX} - 2G_{3\phi} - \dot{\phi}^2 G_{3\phi X} + 3H\dot{\phi}(2G_{3X} + \dot{\phi}^2 G_{3XX})\Big] + 3\left(G_{4\phi} - \frac{1}{2}G_{3X}\dot{\phi}^2\right)^2 > 0, \tag{3.2}$$

$$G_4 > 0. \tag{3.3}$$

The conditions for realizing phantom dark energy, which are $\rho_\phi > 0$ and $w_\phi = p_\phi/\rho_\phi < -1$, are explicitly written as

$$\dot{\phi}^2 K_X - K - \dot{\phi}^2 G_{3\phi} + 3H\dot{\phi}^3 G_{3X} + 3H^2\left(\frac{1}{\kappa^2} - 2G_4\right) - 6H\dot{\phi}G_{4\phi} > 0, \tag{3.4}$$

$$\dot{\phi}^2 K_X - \dot{\phi}^2(2G_{3\phi} + \ddot{\phi}G_{3X} - 3H\dot{\phi}G_{3X}) - 2\dot{H}\left(\frac{1}{\kappa^2} - 2G_4\right) + 2(\ddot{\phi} - H\dot{\phi})G_{4\phi} + 2\dot{\phi}^2 G_{4\phi\phi} < 0. \tag{3.5}$$

In the following, we will evaluate the conditions (3.1)–(3.5) by using case analysis.

A. The case $G_3(\phi,X) = G_3(\phi)$ and $G_4(\phi) = 1/(2\kappa^2)$

If $G_3(\phi, X)$ only depends on ϕ and $G_4(\phi) = 1/(2\kappa^2) > 0$, the conditions (3.1)–(3.5) are written as

$$K_X - 2G_{3\phi} \geq 0, \tag{3.6}$$

$$K_X + \dot{\phi}^2 K_{XX} - 2G_{3\phi} > 0, \tag{3.7}$$

$$\dot{\phi}^2 K_X - K - \dot{\phi}^2 G_{3\phi} > 0, \tag{3.8}$$

$$\dot{\phi}^2 K_X - 2\dot{\phi}^2 G_{3\phi} < 0. \tag{3.9}$$

There is a contradiction between inequalities (3.6) and (3.9). Therefore, phantom dark energy cannot be realized without instability in this case. This result shows that stable phantom dark energy can only be realized if there is a ϕ dependence in G_4 or a X dependence in G_3. The reason why we do not consider constant G_4 which is different from $1/(2\kappa^2)$ is in order to be consistent with the solar system tests and the laboratory experiments of gravitation. In fact, an $O(10^{-5})$ difference in the value of G_4 is only allowed by the experiments [39]; however, such a small difference does not affect the conditions (3.6)–(3.9).

B. The case G_3 has an X dependence

If G_3 has an X dependence and $G_4(\phi) = 1/(2\kappa^2)$, the conditions (3.1)–(3.5) are written as

$$K_X - 2G_{3\phi} + 2\ddot{\phi}G_{3X} + \dot{\phi}^2 G_{3\phi X} + \dot{\phi}^2\ddot{\phi}G_{3XX} + 4H\dot{\phi}G_{3X} - \frac{\kappa^2}{2}\dot{\phi}^4 G_{3X}^2 \geq 0, \quad (3.10)$$

$$K_X + \dot{\phi}^2 K_{XX} - 2G_{3\phi} - \dot{\phi}^2 G_{3\phi X} + 3H\dot{\phi}(2G_{3X} + \dot{\phi}^2 G_{3XX}) + \frac{3\kappa^2}{2}\dot{\phi}^4 G_{3X}^2 > 0, \quad (3.11)$$

$$\dot{\phi}^2 K_X - K - \dot{\phi}^2 G_{3\phi} + 3H\dot{\phi}^3 G_{3X} > 0, \quad (3.12)$$

$$\dot{\phi}^2 K_X - \dot{\phi}^2(2G_{3\phi} + \ddot{\phi}G_{3X} - 3H\dot{\phi}G_{3X}) < 0. \quad (3.13)$$

Both inequalities (3.10) and (3.13) can be satisfied only if

$$\left(3\ddot{\phi} + H\dot{\phi} - \frac{\kappa^2}{2}\dot{\phi}^4 G_{3X}\right)G_{3X} + \dot{\phi}^2(G_{3\phi X} + \ddot{\phi}G_{3XX}) > 0. \quad (3.14)$$

Inequalities (3.10)–(3.13) are so complicated that it is difficult to find appropriate function forms of $K(\phi, X)$ and $G_3(\phi, X)$ which satisfy all of the conditions; however, we can find the large/small relations between the functions in the following manner. In the case of potential driven slow-roll accelerated expansion i.e., the case $K(\phi, X) = X - V(\phi)$, $X \ll V(\phi) \sim 3H^2/\kappa^2$, and $|\ddot{\phi}| \ll |H\dot{\phi}|$, inequality (3.12) is automatically satisfied because $V(\phi)$ is much larger than the other terms. Inequalities (3.10) and (3.13) imply that $H\dot{\phi}G_{3X}$ or the other terms proportional to G_{3X} or $G_{3\phi X}$ should be the same order as $K_X - 2G_{3\phi} = 1–2G_{3\phi}$, because the terms $K_X - 2G_{3\phi}$ commonly exist in inequalities (3.10) and (3.13) but the signs of the inequalities are different. In particular, if $G_{3\phi X}$ and G_{3XX} are negligible and $K_X - 2G_{3\phi}$ is negative, inequalities (3.10) and (3.13) can be satisfied by positive $H\dot{\phi}G_{3X}$ which is larger than $-(K_X - 2G_{3\phi})/4$ and less than $-(K_X - 2G_{3\phi})/3$.

C. The case $G_3(\phi,X) = 0$ and $G_4(\phi)$ has a ϕ dependence

If we only take $K(\phi, X)$ and $G_4(\phi)$ into account, then the conditions (3.1)–(3.5) are given as

$$G_4(K_X G_4 + 3G_{4\phi}^2) \geq 0, \quad (3.15)$$

$$G_4[G_4(K_X + \dot{\phi}^2 K_{XX}) + 3G_{4\phi}^2] > 0, \quad (3.16)$$

$$G_4 > 0, \quad (3.17)$$

$$3\left(\frac{1}{\kappa^2} - 2G_4\right)H^2 + \dot{\phi}^2 K_X - K - 6H\dot{\phi}G_{4\phi} > 0, \quad (3.18)$$

$$-2\left(\frac{1}{\kappa^2} - 2G_4\right)\dot{H} + \dot{\phi}^2 K_X + 2(\ddot{\phi} - H\dot{\phi})G_{4\phi} + 2\dot{\phi}^2 G_{4\phi\phi} < 0. \quad (3.19)$$

In this case, to realize stable dark energy is much easier than in the case that G_3 has an X dependence and $G_4(\phi) = 1/(2\kappa^2)$, because inequalities (3.15)–(3.17) are always established as long as $G_4 > 0$, $K_X > 0$, and $K_{XX} \geq 0$. In particular, in the case of the canonical kinetic term, which is $K(\phi, X) = X - V(\phi)$, inequalities (3.15)–(3.17) are completed if $G_4 > 0$; moreover, sound speed of scalar mode c_s always satisfies $c_s^2 = 1$. Then, we can adjust two arbitrary functions $V(\phi)$ and $G_4(\phi) > 0$ to make the conditions (3.18) and (3.19) true.

IV. EXAMPLES

A. The case G_3 has an X dependence and $G_4(\phi) = 1/(2\kappa^2)$

As shown in the previous section, a slow-rolling scalar field has the possibility to be a stable phantom dark energy. In the case of

$$K(\phi, X) = X - m_1^2\phi^2 \quad \text{and} \quad G_3(\phi, X) = f\phi + \frac{X}{m_2^3}, \quad (4.1)$$

where m_1, m_2, and f are positive constants, then $G_{3\phi}$, $G_{3X} > 0$ and $G_{3\phi X} = G_{3XX} = 0$ are satisfied, so the large/small relation written in the previous section can be realized by choosing appropriate values of m_2 and f. In Fig. 1, redshift dependence of the Hubble rate parameter and that of the effective equation of state parameter of the scalar field, which is defined as $w_\phi = p_\phi/\rho_\phi$, are depicted. As seen in the right figure, the equation of state of the scalar field crosses over the phantom divide, which is the boundary $w_\phi = -1$, around $z = 3$. By the effect of phantom crossing, the Hubble rate becomes greater than that of the ΛCDM model except for the orange curve. In the case of the orange curve, dark energy density is less than that of the other curves; therefore, the Hubble rate cannot be greater than that of the ΛCDM model in the small redshift region. In Fig. 2, the evolution of c_s^2 and that of A defined in Eq. (2.25) are depicted. Both c_s^2 and A are always positive; therefore, there is no instability at least in the region $0 < z < 5$. However, we should be careful that $w_\phi < -1$ means the breakdown of the null energy condition because the effective equation of state parameter of the scalar field w_ϕ is not only "effective" but also "exact" in the case $G_4(\phi) = 1/(2\kappa^2)$ and $G_5(\phi, X) = 0$.

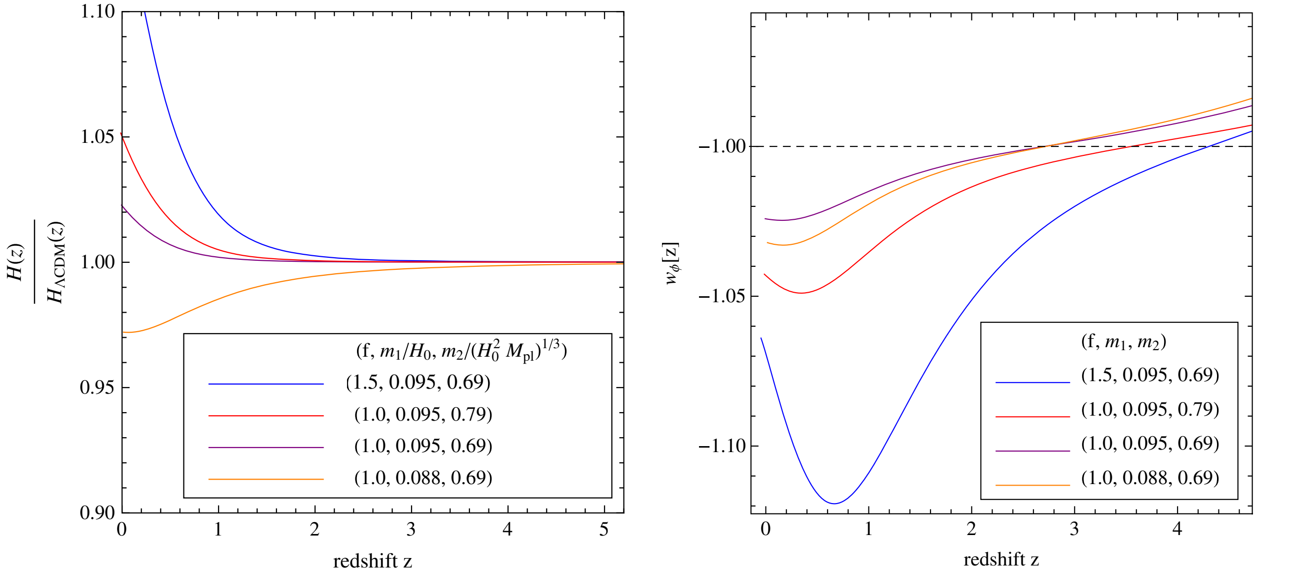

FIG. 1. Redshift dependence of the expansion rate compared to the ΛCDM model (left) and that of the effective equation of state parameter of the scalar field $w_\phi = p_\phi/\rho_\phi$ (right) in the case of $K(\phi, X) = X - m_1^2\phi^2$ and $G_3(\phi, X) = f\phi + X/m_2^3$. The cases of higher value in f, higher value in m_2, and lower value in m_1 are expressed as the blue curve, red curve, and orange curve, respectively. The initial conditions for $\phi(z)$ and $\dot{\phi}(z)$ are assigned as $\phi(10) = 3M_{\rm pl}$ and $\dot{\phi}(10) = 0.04 M_{\rm pl} H_0$, where H_0 means the Hubble constant in the ΛCDM model, which is $H_0 \simeq 68$ (km/s)/Mpc. Also, $\Omega_{\rm matter,0} = 0.31$ is assumed to plot the figures.

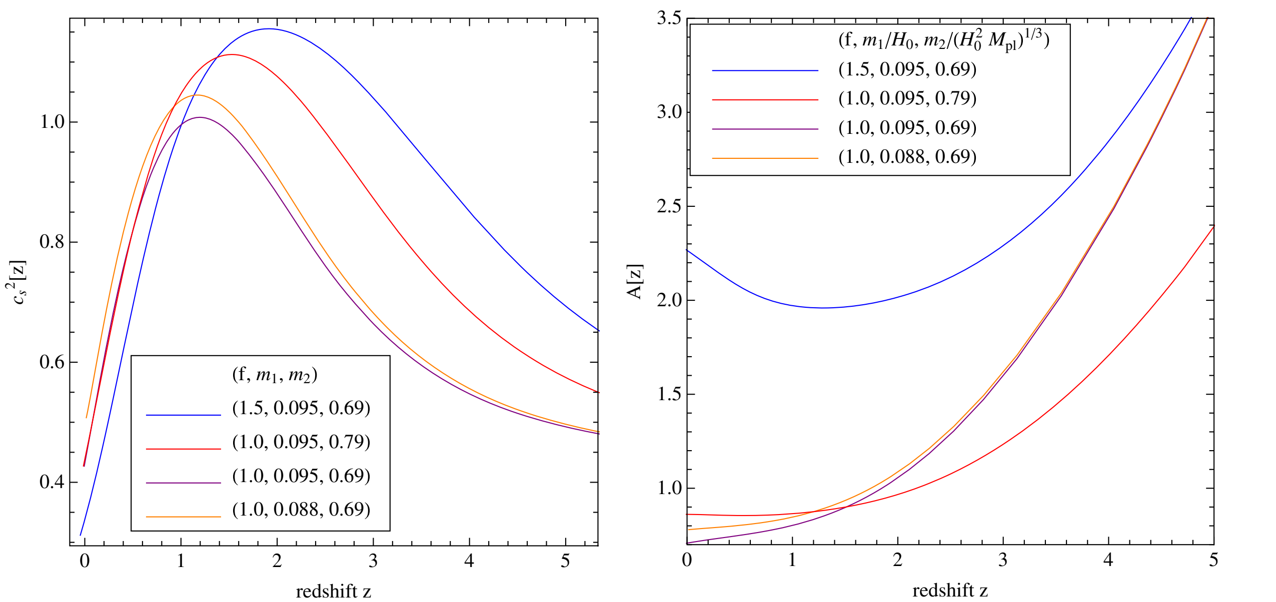

FIG. 2. Redshift dependence of c_s^2 and A in the case of $K(\phi, X) = X - m_1^2\phi^2$ and $G_3(\phi, X) = f\phi + X/m_2^3$. Initial conditions are the same as those in Fig. 1.

B. The case $G_3(\phi, X) = 0$ and $G_4(\phi)$ has a ϕ dependence

Let us first consider the slow-roll accelerated expansion of the Universe. If $K(\phi, X) = X - V(\phi)$, $X \ll V(\phi) \sim 3H^2/\kappa^2$, and $|\ddot{\phi}| \ll |H\dot{\phi}|$, then the inequalities (3.18) and (3.19) can be satisfied for $G_4 \simeq 1/(2\kappa^2)$, $G_{4\phi} > 0$, and $\dot{\phi} > 0$, because the term $V(\phi)$ is the dominant component in the left-hand side of (3.18), and $-2H\dot{\phi}G_{4\phi}$ can be dominant in the left-hand side of (3.19) if $|G_{4\phi\phi}| \lesssim |G_{4\phi}/M_{\rm pl}|$. Such a situation is realized by assuming

$$K(\phi, X) = X - m^2\phi^2 \quad \text{and} \quad G_4(\phi) = \frac{1}{2\kappa^2} e^{\lambda \frac{\phi}{M_{\rm pl}}}, \tag{4.2}$$

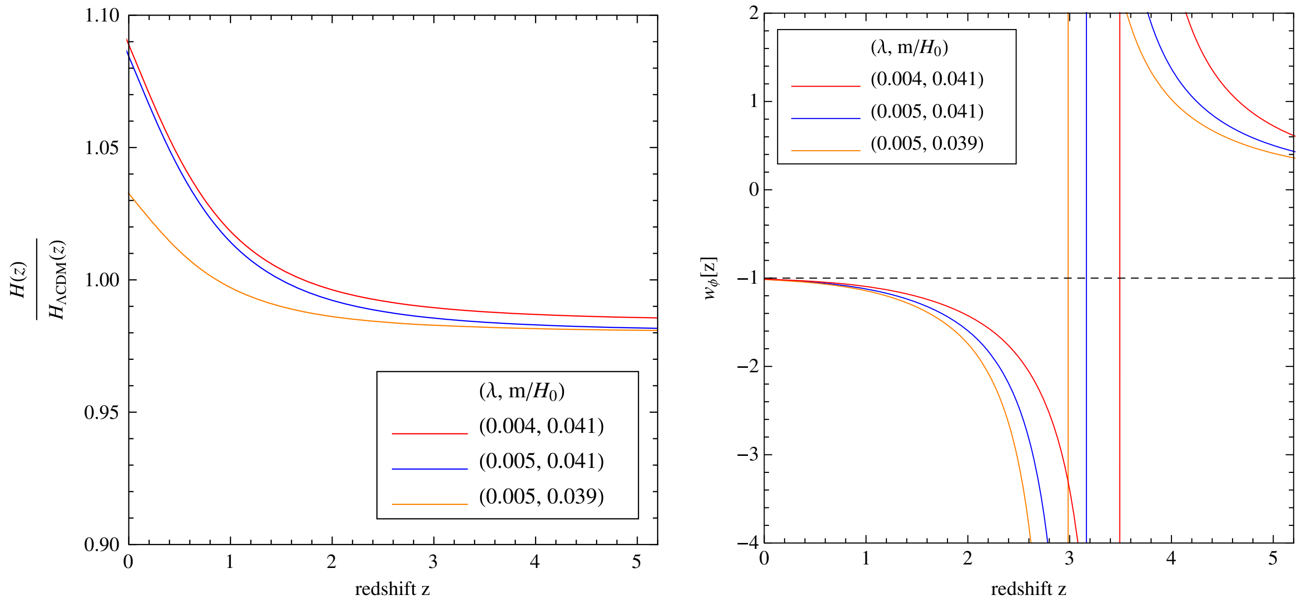

FIG. 3. Redshift dependence of the Hubble rate function (left) and that of the effective equation of state parameter of the scalar field (right) in the case of $K(\phi, X) = X - m^2\phi^2$ and $G_4(\phi) = \frac{1}{2\kappa^2} e^{\lambda\phi/M_{\rm pl}}$. The red curve and orange curve express a lower value in λ and a lower value in m, respectively. The initial conditions for $\phi(z)$ and $\dot{\phi}(z)$ are assigned as $\phi(10) = 8M_{\rm pl}$ and $\dot{\phi}(10) = 0.04M_{\rm pl}H_0$.

with positive λ. In Fig. 3, the evolution of the Hubble rate function and the effective equation of state parameter of the scalar field in the case of Eq. (4.2) are expressed. The phantom crossing is realized around $z = 3$ as seen in the right figure. This phantom crossing is caused not by the change in p_ϕ but by the inversion of the sign of ρ_ϕ. Here, negative ρ_ϕ does not mean the existence of negative energy, because ρ_ϕ not only contains the usual energy density of the scalar field but also contains the deviations from Einstein gravity. This type of phantom crossing is also seen in the other modified gravity models [40,41]. One may think that the discontinuity in the equation of state parameter has a

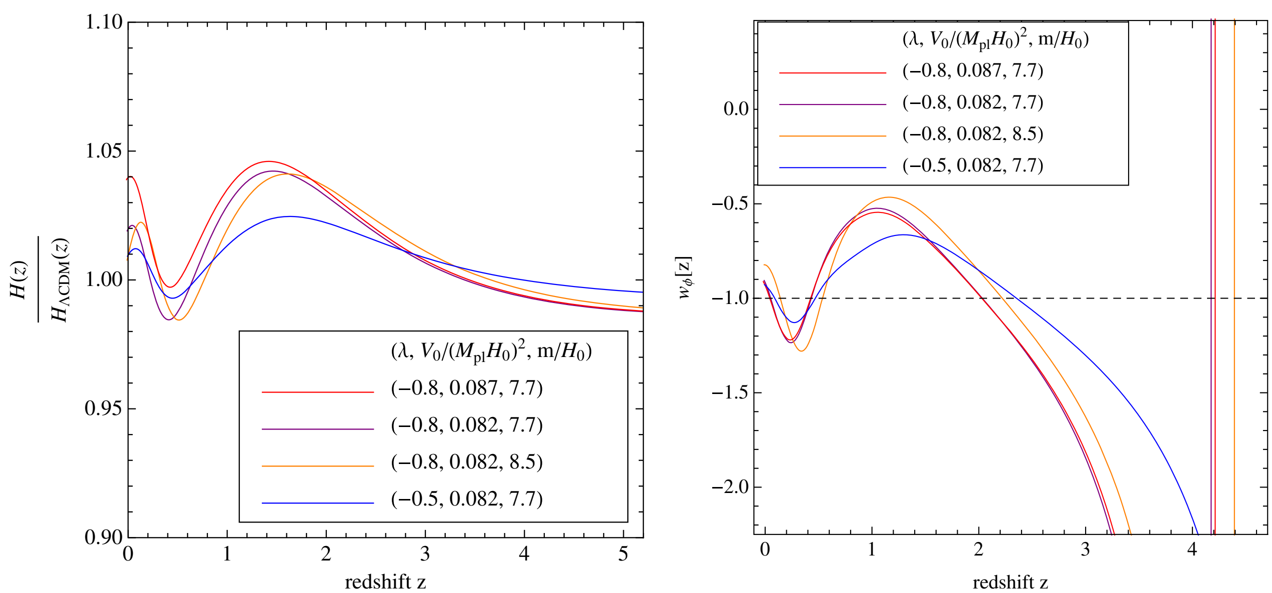

FIG. 4. Redshift dependence of the Hubble rate function (left) and that of the effective equation of state parameter of the scalar field (right) in the case of $K(\phi, X) = X - (V_0 + m^2\phi^2)$ and $G_4(\phi) = \frac{1}{2\kappa^2} e^{\lambda\phi/M_{\rm pl}}$. The red curve, orange curve, and blue curve express a higher value in V_0, a higher value in m, and a higher value in λ, respectively. The initial conditions for $\phi(z)$ and $\dot{\phi}(z)$ are assigned as $\phi(10) = -0.03M_{\rm pl}$ and $\dot{\phi}(10) = 0.04M_{\rm pl}H_0$.

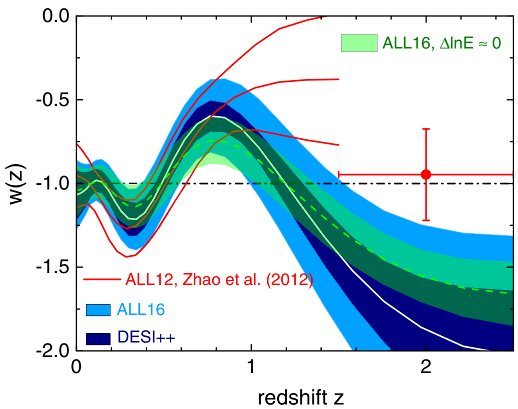

FIG. 5. "The reconstructed evolution history of the dark energy equation of state compared with the 2012 result and the forecasted uncertainty from future data" from Ref. [15].

great effect on the observational quantities. In fact, the effect is small because there are no steep changes in the quantities ρ_ϕ and p_ϕ; moreover, dark energy density is much smaller than that of dark matter around $z = 3$. Therefore, the effects on H and $\dot{H}$ from the discontinuity in the equation of state are negligibly small. In the left figure, we can see that the Hubble rate in this case is a bit smaller than that in the ΛCDM model in large z. This is caused from the modification of gravity, namely, deviation from $G_4(\phi) \equiv 1/(2\kappa^2)$ effectively plays the role of negative energy. However, in the low redshift region, the Hubble rate which is larger than $H_{\Lambda\mathrm{CDM}}$ is realized by the effect of the mass term $m^2\phi^2$.

If we generalize the potential given in Eq. (4.2), we can describe more complicated behavior of w_ϕ. In the case of

$$K(\phi, X) = X - (V_0 + m^2\phi^2) \quad \text{and} \quad G_4(\phi) = \frac{1}{2\kappa^2} e^{\lambda \frac{\phi}{M_{\mathrm{pl}}}}, \tag{4.3}$$

where V_0 is a positive constant of mass dimension four, V_0 causes the accelerated expansion of the Universe in the same way as in the case of the ΛCDM model. However, the effects from $m^2\phi^2$ and $e^{\lambda\phi/M_{\mathrm{pl}}}$ can dramatically change the behavior of $H(z)$ and $w_\phi(z)$ compared to the ΛCDM model. In Fig. 4, the evolution history of $H(z)$ and $w_\phi(z)$ are depicted. In the same manner as in the case (4.2), a phantom crossing caused by the inversion of the sign of ρ_ϕ occurs around $z = 4$. However, several phantom crossings happen after it in this case. This is because the field ϕ oscillates around the stationary point. The oscillation of ϕ is mainly caused by the mass term $m^2\phi^2$ and the phantom crossing is accompanied by the oscillation because of the factor $e^{\lambda\phi/M_{\mathrm{pl}}}$. In the figure, the difference between the purple curve and red curve is only the value of V_0. Therefore, the value of $H(z)$ is simply increased in the red curve compared to the purple one, while the behavior of $w_\phi(z)$ in the red curve is almost the same as that in the purple curve. The reason why there is only a little difference in $w_\phi(z)$ is that the value of $w_\phi(z)$ is not influenced by the total amount of dark energy but by the ratio between V_0 and the other terms. Interestingly, the reconstructed equation of state parameter of dark energy from the observations given by Zhao *et al.* [15] has the same behavior as in Fig. 4 (see Fig. 5). There are phantom crossings in the reconstructed $w(z)$; therefore, the authors of the paper mention that such a behavior could be explained by the Quintom scenario [42,43] or the interaction between dark energy and dark matter [44]. However, Fig. 4 explicitly shows that the behavior of the reconstructed $w(z)$ is realized without instability if we consider Eq. (4.3).

V. CONCLUSION

We have investigated the conditions in order to realize phantom dark energy without instability in Horndeski's theory. First, we have assumed $G_4(\phi, X) = G_4(\phi)$ and $G_5(\phi, X) = 0$ by following the observational results of gravitational wave GW170817 and its electro-magnetic counterparts. Then there are three arbitrary functions $K(\phi, X)$; $G_3(\phi, X)$; and $G_4(\phi)$ in Horndeski's theory. Under this condition, we have derived the following results. X dependence in the G_3 function or ϕ dependence in the G_4 function must exist for realizing stable phantom dark energy, because there is a contradiction among the conditions $c_s^2 \geq 0$, $A > 0$, and $w_\phi < -1$ if $G_3(\phi, X) = G_3(\phi)$ and $G_4(\phi) = \mathrm{const.}$ In both cases $G_3(\phi, X) \neq G_3(\phi)$ and $G_4(\phi) \neq \mathrm{const.}$, slow-roll accelerated expansion with a mass term of the scalar field $m^2\phi^2$ can yield dark energy which crosses the phantom divide. Moreover, it has also been shown that a behavior of $w_\phi(z)$ which is similar to the observationally reconstructed evolution history of the dark energy equation of state [15] can be realized in the model (4.3).

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