Mimetic gravity

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Can we find alternative explanations for the CDM and DE phenomena by considering a different theory of gravity (other than GR)?

Maybe there are several components of DM. **Mimetic dark matter** could be one of them.

The mimetic DM is of **gravitational origin**.

Consider a **conformal transformation** of the type:

\[ g_{\mu\nu} = -w \ell_{\mu\nu} \]

\[ w \equiv \ell^{\rho\sigma} \partial_{\rho} \Psi \partial_{\sigma} \Psi \]

Then it follows that:

\[ g^{\mu\nu} \partial_{\mu} \Psi \partial_{\nu} \Psi = -1 \]

**Kinematical constraint**

and

The theory is invariant under **Weyl** rescaling as:

\[ \ell_{\mu\nu} \rightarrow \Omega^2(x) \ell_{\mu\nu} \]

\[ g_{\mu\nu} \rightarrow g_{\mu\nu} \]
Let us start with the modified Einstein-Hilbert action as:

\[ S = \frac{1}{2} \int d^4x \sqrt{-g} \left[ R(g_{\mu\nu}, \Psi) + \mathcal{L}_m \right] \]

The equations of motion (eom) are:

\[ G_{\mu\nu} - T_{\mu\nu} = (G - T) \partial_\mu \Psi \partial_\nu \Psi, \quad \nabla_\rho [(G - T) \partial^\rho \Psi] = 0 \]

\( \ell_{\mu\nu} \) does not appear explicitly in the equations but \( \Psi \) does.
The eom for the metric is traceless because of the constraint. It can be written as

\[ G_{\mu\nu} = T_{\mu\nu} + \tilde{T}_{\mu\nu} \]

\[ \tilde{T}_{\mu\nu} = (G - T) \partial_\mu \Psi \partial_\nu \Psi \]

Cf. \( \tilde{T}_{\mu\nu} = (\rho + P) u_\mu u_\nu + Pg_{\mu\nu} \)

\[ P = 0 \]

The scalar field imitates dust! With \( \rho = G - T \neq 0 \) even if \( T_{\mu\nu} = 0 \)

A. H. Chamseddine, V. Mukhanov and A. Vikman, JCAP 1406 (2014) 017

By generalizing the previous model by including a potential one can have almost any expansion history. One can produce quintessence, inflation and a nonsingular bouncing universe.

The speed of sound is zero so one needs to introduce higher-derivative terms to have successful inflation.
Other works

- Why do we have more solutions than in GR? A. Barvinsky, JCAP 1401 (2014) 014

  We are doing a non-invertible transformation. The theory is a conformal extension of GR. The theory is free from ghost instabilities if the energy density of the fluid is positive. There may exist caustic instabilities caused by the geodesic flow.

  He proposed a modification including a vector field. The constraint implies $a_\mu = u^\alpha \nabla_\alpha u_\mu = 0$


- The theory has a dual formulation in terms of a Lagrange multiplier field.

  N. Deruelle and J. Rua, JCAP 1409 (2014) 002

- Identified a more general disformal transformation (DT) that leads to mimetic DM. Generically, if we are not in that particular case, Einstein’s gravity is invariant under DT.


- How can we obtain dust from a scalar field? And how can we obtain “dust with pressure”? They discovered the mimetic model, called $\lambda \phi$ - fluid.

- And several more...
Non-invertibility condition

Disformal transformation

\[ g_{\mu\nu} = A(\Psi, w) \ell_{\mu\nu} + B(\Psi, w) \partial_{\mu} \Psi \partial_{\nu} \Psi \]

\[ w \equiv \ell^{\rho\sigma} \partial_{\rho} \Psi \partial_{\sigma} \Psi \]

\( g_{\mu\nu} \) - “physical” metric

\( \ell_{\mu\nu} \) - “auxiliary” metric

\( \Psi \) - mimetic scalar field

\( A, B \) Free functions obeying some conditions: \( A > 0 \), should preserve the Lorentzian signature, causal and the inverse metric should exist

- When can we invert the transformation (for a fixed \( \Psi \))? i.e. find \( \ell_{\mu\nu}(g_{\alpha\beta}) \)
  - This is equivalent to ask when can we write \( w(g_{\mu\nu}) \)?


Mimetic disformal transformation

- We found that one cannot invert the transformation if

\[ B(\psi, w) = -\frac{A(\psi, w)}{w} + b(\psi) \]

This is the mimetic transformation. \( b(\psi) \) is an integration constant.

- Then one can find

\[ b(\psi)g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi = 1 \]

This is called the mimetic constraint.

- This is the same condition as the one found by Deruelle and Rua for the system of eom of mimetic dark matter (i.e. conformal trans. on GR) to be indeterminate as we will see.

- This is a very general result which does not depend on the theory. The mimetic constraint is a kinematical constraint valid independently of the dynamics.

- Because the transformation is not invertible it is not surprising that the new theory may contain new solutions.
Disformal transformation method

- Let us perform a DT on a very general scalar-tensor theory and generalize N. Deruelle and J. Rua, JCAP 1409 (2014) 002

(See our paper for the case when the field in the DT is a new field).

\[ S = \int d^4x \sqrt{-g} \mathcal{L}[g_{\mu\nu}, \partial_{\lambda_1} g_{\mu\nu}, \ldots, \partial_{\lambda_1} \ldots \partial_{\lambda_p} g_{\mu\nu}, \psi, \partial_{\lambda_1} \psi, \ldots, \partial_{\lambda_1} \ldots \partial_{\lambda_q} \psi] + S_m[g_{\mu\nu}, \phi_m] \]

Matter action

One can write the (contracted) eom for the metric as

\[ M \left( \alpha_1, \alpha_2 \right) = 0, \quad \text{where} \quad M = \begin{pmatrix} A - w \frac{\partial A}{\partial w} & -w \frac{\partial B}{\partial w} \\ w^2 \frac{\partial A}{\partial w} & -A + w^2 \frac{\partial B}{\partial w} \end{pmatrix} \]

where \( \alpha_1 \equiv (E^{\rho\sigma} + T^{\rho\sigma}) \ell_{\rho\sigma} \) and \( \alpha_2 \equiv (E^{\rho\sigma} + T^{\rho\sigma}) \partial_{\rho} \psi \partial_{\sigma} \psi \)

\[ \Omega_{\psi} = \frac{\delta (\sqrt{-g} \mathcal{L})}{\delta \psi}, \quad E^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta (\sqrt{-g} \mathcal{L})}{\delta g_{\mu\nu}}, \quad T^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta (\sqrt{-g} \mathcal{L}_m)}{\delta g_{\mu\nu}}, \quad \Omega_m = \frac{\delta (\sqrt{-g} \mathcal{L}_m)}{\delta \phi_m} \]
Solving the system: generic case

- If \( \det(M) \neq 0 \) then the only solution is: \( \alpha_1 = \alpha_2 = 0 \)

And the eom reduce to:

\[
E^{\mu\nu} + T^{\mu\nu} = 0, \quad \Omega_\psi = 0, \quad \Omega_m = 0
\]

- We get the same equations of motion by taking the variation with respect to \( \ell_{\mu\nu} \) or \( g_{\mu\nu} \)

- Generically, the theory is invariant under disformal transformations.

- Not surprising because all we did was a well-behaved invertible change of variables.
Solving the system: mimetic case

- If \( \det(M) = w^2 A \frac{\partial}{\partial w} \left( B + \frac{A}{w} \right) = 0 \)

  then \( B(\psi, w) = -\frac{A(\psi, w)}{w} + b(\psi) \)

  This leads to the same mimetic transformation

  and the solution is: \( \alpha_2 = w\alpha_1 \)

The eom now read:

\[
\begin{align*}
E_{\mu\nu} + T_{\mu\nu} &= (E+T) b \partial_\mu \psi \partial_\nu \psi, \\
\nabla_\rho [(E + T)b \partial^\rho \psi] - \frac{\Omega_\psi}{\sqrt{-g}} &= \frac{1}{2} (E+T) \frac{1}{b} \frac{db}{d\psi} \\

b(\psi) g^{\mu\nu} \partial_\mu \psi \partial_\nu \psi &= 1, \quad \Omega_m = 0
\end{align*}
\]

- These eom are different from the eom resulting from taking the variation \( \text{wrt} \ g_{\mu\nu} \)

✓ These are the new eom of mimetic gravity that generalize the mimetic dark matter model.
Formulation with a Lagrange multiplier

$$S_\lambda = \int d^4x \sqrt{-g} L[g_{\mu\nu}, \partial_{\lambda_1}g_{\mu\nu}, \ldots, \partial_{\lambda_1} \ldots \partial_{\lambda_p}g_{\mu\nu}, \psi, \partial_{\lambda_1}\psi, \ldots, \partial_{\lambda_1} \ldots \partial_{\lambda_q}\psi] + S_m[g_{\mu\nu}, \phi_m]$$

$$+ \int d^4x \sqrt{-g}\lambda (b(\psi)g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - 1)$$

For GR+conformal trans.  
A. Barvinsky, JCAP 1401 (2014) 014

$\lambda$ - the Lagrange multiplier field which enforces the kinematical constraint  
b($\psi$) is a given potential function

- One obtains the same eom as the DT method before.

  The LM can be found using the eom to be $2\lambda = E + T$

- Using Horndeski’s identity, we showed the field equation is not independent from the other eom.  

- The mimetic theory has the same number of derivatives as the original theory if written in terms of $g_{\mu\nu}$.

  No higher-derivative ghosts if they did not exist already.
Mimetic Horndeski: the simplest example

\[ S_H = \int d^4x \sqrt{-g} \left( \frac{R}{2} + c_2 X \right) \]

\[ X = -\frac{1}{2} \nabla_\mu \psi \nabla^\mu \psi \]

No other matter

For a flat FLRW universe the (independent) eom are:

\[ b(\psi) \dot{\psi}^2 + 1 = 0, \quad 6H^2 + 4\dot{H} + c_2 \dot{\psi}^2 = 0 \]

A solution is:

\[ a(t) = t^{\frac{2}{3(1+\omega)}}, \quad \psi(t) = \pm \sqrt{-\frac{\alpha}{c_2}} \log \frac{t}{t_0}, \quad b(\psi) = \frac{c_2}{\alpha} t^2_0 e^{\pm \frac{c_2}{\alpha} \psi} \]

\[ \alpha = -\frac{8\omega}{3(1+\omega)^2} \]

Constant equation of state (eos)

\[ \text{Integration constant} \]

The mimetic field can \textit{mimic} the expansion history of a \textit{perfect fluid} with eos \( \omega \)!

Cf. usual case \( \omega = 1 \)

✔ By adjusting, the function \( b(\psi) \) we can \textit{mimic} the expansion history of a \textit{perfect fluid} with a fixed sign for the pressure \( 6H^2 + 4\dot{H} = -2p \)

✓ This is a similar feature as the simple generalization of the original model where one can have almost \textit{any} expansion history desired.

A. H. Chamseddine, V. Mukhanov and A. Vikman, JCAP 1406 (2014) 017
E. A. Lim, I. Sawicki and A. Vikman, JCAP 1005 (2010) 012
Mimetic cubic Galileon

\[ S_H = \int d^4x \sqrt{-g} \left( \frac{R}{2} + c_2 X - 2c_3 X \Box \psi \right) \]

The eom are:

\[ b(\psi) \dot{\psi}^2 + 1 = 0, \quad 6H^2 + 4\dot{H} + \dot{\psi}^2(c_2 - 4c_3 \ddot{\psi}) = 0 \]

Again one can have almost any desired expansion history by choosing an appropriate \( b(\psi) \).

The expansion history of a universe with cold dark matter and a positive cosmological constant \( \Lambda \), \( a = a_\star \sinh^{\frac{2}{3}}(Ct) \), is a solution for:

\[ \frac{4c_3}{c_2} \left[ -\arctan \left( \pm \sqrt{\frac{3c_2}{8C^2}} \dot{\psi} \right) \pm \sqrt{\frac{3c_2}{8C^2}} \ddot{\psi} \right] = t \]

\[ C = \sqrt{3\Lambda/4} \]
Linear scalar perturbations in Mimetic Horndeski

We work in the Poisson gauge $g_{00} = -a^2(\tau)(1 + 2\Phi)$, $g_{0i} = 0$, $g_{ij} = a^2(\tau)(1 - 2\Psi)\delta_{ij}$

$$\varphi(\tau, x) = \varphi_0(\tau) + \delta\varphi(\tau, x)$$

The first order eom can be simplified to:

\[
\begin{align*}
2b(\varphi_0)\delta\varphi' + \varphi_0' b(\varphi_0)\delta\varphi - 2b(\varphi_0)\varphi_0' \Phi &= 0 \quad \text{Mimetic constraint} \\
f_{18}\psi' + f_{19}\delta\varphi' + \left(f_{20} + \frac{a^2E(0)}{\varphi_0'}\right)\delta\varphi + f_{21}\Phi &= 0 \quad 0i\ eq. \\
f_7\psi + f_8\delta\varphi + f_9\Phi &= 0 \quad \text{ij eqs.} \\
f_{10}\psi'' + f_{11}\delta\varphi'' + f_{12}\psi' + f_{13}\delta\varphi' + f_{14}\Phi' + f_{15}\psi + f_{16}\delta\varphi + f_{17}\Phi &= 0
\end{align*}
\]

$f_i$ - are given in terms of the Horndeski functions and their derivatives (long expressions)

It can be shown this eq. is redundant.

- No spatial derivatives appear this implies that the sound speed is also zero even for mimetic Horndeski.
Evolution equations for the perturbations

- By introducing the co-moving curvature perturbation defined as 
  \[ -\zeta = \Psi + \frac{H}{\varphi_0} \delta \varphi \]

  The independent set of eom can be written as
  \[
  2b(\varphi_0)\delta \varphi' + \varphi_0 b,\varphi(\varphi_0)\delta \varphi - 2b(\varphi_0)\varphi_0' \Phi = 0 \\
  -f_7 \zeta + \left( f_8 - \frac{H}{\varphi_0'} f_7 \right) \delta \varphi + f_9 \Phi = 0 \\
  \zeta' = 0
  \]

  Note that the curvature pert. satisfies a simple first order ode. The solution is just a constant!

The equation of motion for the Newtonian potential is

\[ \Phi'' + \left( \frac{B_2}{B_3} + \left( \ln \frac{B_3}{B_1} \right)' + H - \frac{\varphi_0''}{\varphi_0'} \right) \Phi' + \left( \frac{B_1}{B_3} \varphi_0' + \frac{B_1}{B_3} \left( \frac{B_2}{B_1} \right)' + \frac{B_2}{B_3} \left( H - \frac{\varphi_0''}{\varphi_0'} \right) \right) \Phi = 0 \]

\[ B_i \text{ are given in terms of the } f_i \text{ and their derivatives} \]

No spatial Laplacian term \[ \Rightarrow C_S = 0 \]

- We also showed that the sound speed is zero on all backgrounds and therefore the system does not have any wave-like scalar dof.
Solutions for the simple models

For the two previous simple models, the previous eq. can be written as:

\[ \Phi'' + \Phi' \left( 3\mathcal{H} + \tilde{\Gamma} \right) + \Phi \left( \mathcal{H}^2 + 2\mathcal{H}' + \tilde{\Gamma}\mathcal{H} \right) = 0 \]

\[ \tilde{\Gamma} = \frac{-\mathcal{H}'' + \mathcal{H}\mathcal{H}' + \mathcal{H}^3}{\mathcal{H}' - \mathcal{H}^2} \]

Correction to the standard dust eq.
Exactly vanishes for a LCDM expansion history.

- In the limit \( \tilde{\Gamma} \rightarrow 0 \), the perturbations in these models will behave exactly the same as perturbations in LCDM.

This equation and its solution coincide with the result of found for a model where \( G'_{3} = 0 \)

E. A. Lim, I. Sawicki and A. Vikman, JCAP 1005 (2010) 012

The solution can be found as

\[ \Phi(\tau, x) = C_1(x) + \frac{H}{a} C_2(x) - C_1(x) \frac{H}{a} \int \frac{da}{H} \]

Corresponds to \( \zeta = 0 \)

If \( a \propto t^{\frac{2}{3(1+w)}} \) then \( \frac{H}{a} \propto a^{-\frac{5-3w}{2}} \) which decays for an expanding universe if \( w > -5/3 \)
Living with ghosts

We showed that the mimetic matter scenario is **equivalent** to the IR limit of projectable Horava-Lifshitz gravity. The projectability condition is imposed by the mimetic constraint.

The main result of the paper is to show that one can **live in the ghost branch** of IR HL gravity.

- We estimated the **vacuum decay rate** and found:

\[ \Gamma \sim \frac{\gamma^{17/2}}{M_{Pl}^{13}} \]

Demanding that the **flux of photons** does not exceed the observational result implies:

\[ \sqrt{\gamma} \lesssim 10^9 \text{GeV} \quad \Rightarrow \quad c_s^2 \lesssim 10^{-20} \]

The lower bound from GR local tests is:

\[ c_s^2 \gtrsim 10^{-42} \]

The strong coupling scale is:

\[ \Lambda_p \sim \frac{\gamma^{3/4}}{M_{Pl}^{1/2}} \lesssim 10 \text{TeV} \]

- The mimetic matter scenario is **phenomenologically viable**.
Conclusions 1

- We generalized previous results obtained for GR only and showed that a very general scalar-tensor theory is generically invariant under DT.

- However a particular subset of the DT, when the transformation is not invertible, gives origin to a new theory which is a generalization of the mimetic dark matter scenario.

\[
g_{\mu\nu} = A(\Psi, w)\ell_{\mu\nu} + B(\Psi, w)\partial_{\mu}\Psi\partial_{\nu}\Psi \quad \quad B(\Psi, w) = -\frac{A(\Psi, w)}{w} + b(\Psi)
\]

- We showed that the mimetic theory can also be derived using a Lagrange multiplier field which imposes the mimetic constraint.

- We proposed some simple toy models in the context of mimetic Horndeski theory.
  - The simplest model can mimic the expansion history of a perfect fluid with a constant eos. (the eos cannot change sign)
  - The mimetic cubic Galileon model can easily mimic the expansion history of a dark matter+Lambda universe.
    - In fact these models can mimic almost any desired expansion.

✓ We showed that the mimetic theory does not introduce higher-derivatives when written for \( g_{\mu\nu} \). In terms of \( \ell_{\mu\nu} \) it may introduce them.

✗ We obtained and solved the linear eom for scalar perturbations. We found that the sound speed also exactly vanishes for mimetic Horndeski gravity.
Conclusions 2

- The mimetic matter scenario is equivalent to the IR limit of projectable Horava-Lifshitz gravity, as well as a particular version of the Einstein-Aether theory.

- It is well-known that we cannot live in the gradient unstable branch and we showed that the ghost branch is phenomenologically viable.

- For a sufficiently small scalar sound speed, the photon flux from vacuum decay is compatible with observations.

\[
\left(10^{-42} \lesssim \right) c_s^2 \lesssim 10^{-20}
\]

Thank you for listening!