

# Constraining parity and charge-parity violating varying-alpha theory through laboratory experiments

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(Received 11 May 2011; published 22 July 2011)

In this report we have studied the implication of a parity and charge-parity (PCP) violating interaction in varying-alpha theory. Because of this interaction, the state of photon polarization can change when it passes through a strong background magnetic field. We have calculated the optical rotation and ellipticity of the plane of polarization of an electro-magnetic wave and tested our results against different laboratory experiments. Our model contains a PCP violating parameter  $\beta$  and a scale of alpha variation  $\omega$ . By analyzing the laboratory experimental data, we found the most stringent constraints on our model parameters to be  $1 \leq \omega \leq 10^{13} \text{GeV}^2$  and  $-0.5 \leq \beta \leq 0.5$ . We also found that with the existing experimental input parameters it is very difficult to detect the ellipticity in the near future.

DOI: 10.1103/PhysRevD.84.026008

PACS numbers: 98.80.Cq, 03.50.-z, 11.30.Er

## I. INTRODUCTION

Parity violation is one of the simplest straightforward extension of the standard model of physics. So far, violation of parity (P) and charge-parity (CP) has been observed only in the electroweak sector of the standard model particle physics. Considering this as a guiding principle, in recent years several models of parity and charge-parity (PCP) violation have been constructed [1–5]. The basic idea of all these models is to add an explicit parity-violating term in the Lagrangian. Because of its nature, this parity-violating term leads to cosmic birefringence [1,2] and left-right asymmetry in the gravitational wave dynamics [3,4]. String theory inspired models with non-standard parity-violating interactions have also been discussed [5]. Recently, we have constructed a parity and charge-parity (PCP) violating model [6] in the framework of “varying-alpha theory”. Some aspects of our model are similar to that proposed by Carroll [1]. But, as we have argued, our model has the advantage over that of Carroll’s in that the origin of the parity violation may be more physically motivated.

String theory has given us sufficient theoretical motivation to consider theories of varying fundamental constants in nature. As is well known, string theory is fundamentally a higher dimensional theory. In principle, therefore, all the so-called fundamental constants in our four dimensional world may actually be spacetime dependent as a result of the dimensional reduction. Meanwhile, increasingly high precision cosmological as well as laboratory experiments give us hope that signature of new physics, including those that give rise to variation of fundamental constants, may emerge in the near future.

A consistent, gauge-invariant, and Lorentz-invariant framework of  $\alpha$  variability was first proposed by Bekenstein [6]. Subsequently, this subject has been studied quite extensively from the theoretical side [7–9] as well as from the observational side [10–14].

Apart from verifying this notion from cosmological/astrophysical and high energy collider experiments, it is also important to study various purely laboratory-based experiments which can provide complementary results. In this report, we focus on its connection to a particular class of laboratory experiments which make use of the conversion of axion or any other low-mass (pseudo)scalar particle into photon in the presence of an electro-magnetic field. These include the Brookhaven-Fermilab-Rutherford-Trieste (BFRT) experiment [15], the Italian Polarizzazione del Vuoto con Laser (PVLAS) experiment [16] and several other experiments such as QED and Axion (Q&A) [17], Biréfringence Magnétique du Vide (BMV)[18], etc., which are either already in progress or ready to be built. All these experiments are expected to produce (pseudo)scalars from polarized laser beams, which are allowed to propagate in a transverse, constant, and homogeneous magnetic field. In addition to the direct production of (pseudo)scalar particles, modification of the polarization of light can in principle be induced by its coupling with pseudoscalar axions as it propagates through a transverse magnetic field [19,21–27]. The model that we recently introduced also exhibits this effect induced from the PCP violating term in our varying fine-structure constant theory [6]. This motivates us to use a different class of experiments to constrain the parameters of a given varying fine-structure constant theory. Such approach has not been explored before. So far, varying-alpha theory has been constrained mostly based upon the observations on the possible variation of fine-structure constant. As we mentioned therefore by introducing PCP violation in a varying-alpha theory, we actually

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unified different class of experimental observation in a single framework. The birefringence and the dichroism of the vacuum induced from the nontrivial coupling of photon and the varying-alpha scalar can be tested in those class of experiments, which in turn will constrain the parameter space of a given varying-alpha model. In this paper, we aim to address these issues and test it against above mentioned various laboratory-based experiments to constrain our model parameters.

We organize this paper as follows: in Sec. II, we review our PCP violating ‘‘varying-alpha theory’’ constructed in [6]. In Sec. III, we will calculate the effect of background magnetic field on the plane of polarization of electro-magnetic field. As we have mentioned before, we will particularly focus on various laboratory-based experiments which measure the rotation of the plane of polarization and the ellipticity of electro-magnetic field through scalar-photon conversion mechanism in the strong background magnetic field. We will analytically calculate the expression for rotation and ellipticity. Then, in Sec. IV, we will use the laboratory-based experimental bounds on rotation and ellipticity to constrain our model parameter. Concluding remarks and future prospects are provided in Sec. V.

## II. PARITY-VIOLATING VARYING-ALPHA THEORY

In this section we will review our previous construction of PCP violating varying-alpha theory [6]. A varying-alpha theory [6–8] is usually referred to as a theory of spacetime variation of the electric charge of any matter field, parameterized by  $e = e_0 e^{\phi(x)}$ , where  $e_0$  denotes the coupling constant and  $\phi(x)$  is a dimensionless scalar field. The fine-structure constant in such a theory is, therefore,  $\alpha = e_0^2 e^{2\phi(x)}$ . This theory has been constructed based upon the shift symmetry in  $\phi$  i.e.  $\phi \rightarrow \phi + c$  and the modified U(1) gauge transformation  $e^\phi A_\mu \rightarrow e^\phi A_\mu + \chi_{,\mu}$ . From the above symmetry considerations, the unique gauge-invariant and shift-symmetric Lagrangian for the modified electro-magnetic field and the scalar field can be written as

$$S = -\frac{1}{4} \int d^4x \sqrt{-g} e^{-2\phi} F_{\mu\nu} F^{\mu\nu} - \frac{\omega}{2} \int d^4x \sqrt{-g} \partial_\mu \phi \partial^\mu \phi, \quad (1)$$

where electro-magnetic field strength tensor can be expressed as

$$F_{\mu\nu} = (e^\phi \mathbf{a}_\nu)_{,\mu} - (e^\phi \mathbf{a}_\mu)_{,\nu} = \mathbf{A}_{\nu,\mu} - \mathbf{A}_{\mu,\nu} \quad (2)$$

with  $\mathbf{A}_\mu = e^\phi \mathbf{a}_\mu$  as the new electro-magnetic gauge potential.

In the above action, and for the rest of this paper, we set  $e_0 = 1$  for convenience. As can be easily seen, the above action reduces to the usual form when  $\phi$  is constant. The coupling constant  $\omega$  is related to a characteristic mass scale of the theory above which the Coulomb force law is valid for a point charge. From the present experimental con-

straints, the energy scale has to be above a few tens of MeV to avoid conflict with experiments. At this point, we to want mention that because of the underlying shift symmetry, we cannot add any arbitrary potential in our Lagrangian. This essentially says that the scalar field responsible for the variation of fine-structure constant should be massless. Of course, one can break this shift symmetry by introducing a potential term which has recently been studied in [28]. We will keep this for our future study in the context of PCP violating varying-alpha theory.

One of the natural assumptions in constructing the above Lagrangian is time-reversal invariance. We have relaxed this assumption and try to analyze its implications based on various laboratory-based experiments. An obvious term that is consistent with the varying-alpha framework yet violates PCP is  $\tilde{F}_{\mu\nu} F^{\mu\nu}$ , where  $\tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\sigma\rho} F_{\sigma\rho}$  is the Hodge dual of the electro-magnetic field tensor. In the conventional electromagnetism, this does not contribute to the classical equation of motion. But in the present framework this is no longer true because of its coupling with the scalar field  $\phi(x)$ . As we have explained in the introduction, at the present level of experimental accuracy, PCP violation in the electro-magnetic sector may not be ruled out, and if the PCP in this EM sector is indeed violated, then there should have some interesting consequences. Motivated by this, we have introduced a parity-violating Lagrangian [6]

$$\mathcal{L} = \frac{\omega}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{4} e^{-2\phi} F_{\mu\nu} F^{\mu\nu} - \frac{\beta}{4} e^{-2\phi} F_{\mu\nu} \tilde{F}^{\mu\nu} + \mathcal{L}_m, \quad (3)$$

where  $\beta$  is a free coupling parameter in our model. As we can see, the scalar field  $\phi$  plays a similar role as that of the dilaton in the low-energy limit of string theory and M-theory, with the important difference that it induces a PCP violating electro-magnetic interaction in our case. For our purpose, we assume  $\beta$  as a free but small parameter. Here, we want to emphasize that the model can be thought of as a unified framework for dealing with different phenomena. At the present level of experimental accuracy, investigations of parity or charge-parity-violating beyond-standard model may shed some new light about the fundamental laws of physics. With the interest of phenomenological impacts on various experimental observations, subsequently we will discuss about some consequences of our model based on the laboratory experiments.

The equations of motion are

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} F^{\mu\nu}) + \partial_\mu \phi (-F^{\mu\nu} + \beta \tilde{F}^{\mu\nu}) = 0, \quad (4)$$

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} \phi) = \frac{e^{-2\phi}}{2\omega} [-F_{\mu\nu} F^{\mu\nu} + \beta F_{\mu\nu} \tilde{F}^{\mu\nu}]. \quad (5)$$

We now explore the theoretical predictions of such coupling on the rotation of plane of polarization as well

as ellipticity for an electro-magnetic wave propagating through a transverse magnetic field.

### III. CALCULATION OF OPTICAL ROTATION AND ELLIPTICITY

In this section, we explicitly estimate the rotation angle of the plane of polarization and the ellipticity due to the PCP violating scalar-photon coupling of our model. The equations of motion for Maxwell and scalar fields turn out to be,

$$\begin{aligned}\nabla \cdot \mathbf{E} &= 2\nabla\phi \cdot \mathbf{E} - 4\beta\nabla\phi \cdot \mathbf{B}, \\ \partial_\eta(\mathbf{E}) - \nabla \times \mathbf{B} &= 2(\dot{\phi}\mathbf{E} - \nabla\phi \times \mathbf{B}) - 4\beta(\dot{\phi}\mathbf{B} + \nabla\phi \times \mathbf{E}), \\ \nabla \cdot \mathbf{B} &= 0, \\ \partial_\eta \mathbf{B} + \nabla \times \mathbf{E} &= 0.\end{aligned}\quad (6)$$

As we have mentioned in the previous section, the definitions of electro-magnetic field strengths are  $F_{i0} = (\partial_i \mathbf{A}_0 - \partial_0 \mathbf{A}_i) = \mathbf{E}_i$  and  $F_{ij} = (\partial_i \mathbf{A}_j - \partial_j \mathbf{A}_i) = \epsilon_{ijk} \mathbf{B}_k$ , where  $i = 1, 2, 3$  and  $\epsilon$  is the three spatial dimensional Levi-Civita tensor density.

At this point, it is worthwhile to mention that different theoretical models based on the  $e^{-2\phi} F_{ab} F^{ab}$  type scalar-photon coupling or the standard QCD  $\phi F_{ab} \tilde{F}^{ab}$  type axion-photon coupling mediated by the background magnetic or electric field have been considered extensively [19,21–26,29]. In a PCP violating varying-alpha theory we have both coupling terms. This motivates us to study the effect of both terms under the externally applied magnetic field.

In terms of the vector potential  $\mathbf{A}_\mu$  and the scalar field  $\phi$ , the linear order fluctuation equation in the strong background magnetic field  $\mathbf{B}_0$  takes the form

$$(\nabla^2 + \varpi^2)\mathbf{A}_x = 4i\beta\mathbf{B}_0\varpi\phi, \quad (7)$$

$$(\nabla^2 + \varpi^2)\mathbf{A}_y = -2\mathbf{B}_0\partial_z\phi, \quad (8)$$

$$(\nabla^2 + \varpi^2)\mathbf{A}_z = 2\mathbf{B}_0\partial_y\phi, \quad (9)$$

$$(\nabla^2 + \varpi^2)\phi = \frac{2\mathbf{B}_0^2}{\omega}\phi - \frac{2\mathbf{B}_0}{\omega}(\partial_y A_z - \partial_z A_y) - \frac{4i\beta\mathbf{B}_0\varpi}{\omega}A_x. \quad (10)$$

We assume the background magnetic field  $\mathbf{B}_0$  is applied in the x-direction. In the above derivation, we use the gauge condition  $\nabla \cdot \mathbf{A} = 0$  and specify the scalar potential  $\mathbf{A}_0 = 0$ .  $\varpi$  is the frequency of the electro-magnetic field. Now, in general it is very difficult to solve the above equation. We will try to solve it by choosing an appropriate ansatz for the electro-magnetic field which is usually used the laboratory set up. Therefore, taking the propagation direction of the electro-magnetic wave to be orthogonal to the external magnetic field  $\mathbf{B}_0$  say z-direction, we take the ansatz to be

$$\mathbf{A}(z, t) = \mathbf{A}^0 e^{-i\varpi t + ikz}; \quad \phi(z, t) = \phi^0 e^{-i\varpi t + ikz}. \quad (11)$$

As is clear from the above ansatz that the equation for  $A_z$  is no longer coupled with  $\phi$ . So, evolution of this component of a vector potential does not get affected by the external magnetic field. The other three equations for  $A_x, A_y, \phi$  turn out to be

$$\begin{bmatrix} (\varpi^2 - k^2) & 0 & -4i\beta\mathbf{B}_0\varpi \\ 0 & (\varpi^2 - k^2) & 2i\mathbf{B}_0k \\ \frac{4i\beta\mathbf{B}_0\varpi}{\omega} & -\frac{2i\mathbf{B}_0k}{\omega} & \left(\varpi^2 - k^2 - \frac{2\mathbf{B}_0^2}{\omega}\right) \end{bmatrix} \begin{pmatrix} \mathbf{A}_x \\ \mathbf{A}_y \\ \phi \end{pmatrix} = 0, \quad (12)$$

In order to have a consistent solution of the above matrix Eq. (12), the determinant of this  $3 \times 3$  matrix part of this equation should be zero. This consistency condition leads to three possible roots for the frequency  $\varpi$  of a electro-magnetic field as follows:

$$\varpi^2 = k^2, \quad \varpi_\pm^2 = k^2 + \delta_\pm \quad (13)$$

$$\delta_\pm = \frac{\mathbf{B}_0^2}{\omega}(1 + 8\beta^2) \pm \sqrt{\frac{\mathbf{B}_0^4}{\omega^2}(1 + 8\beta^2)^2 + \frac{4\mathbf{B}_0^2 k^2}{\omega}(1 + 4\beta^2)}. \quad (14)$$

Now, to establish a connection with the experimental setup, we consider the initial ( $t = 0, x = 0$ ) electro-magnetic field to be linearly polarized and making an angle with the external magnetic field  $\mathbf{B}_0$ , so that

$$\begin{aligned}\mathbf{A}_x(z = 0, t = 0) &= \cos\alpha; \\ \mathbf{A}_y(z = 0, t = 0) &= \sin\alpha; \\ \phi(z = 0, t = 0) &= 0.\end{aligned}\quad (15)$$

With these boundary conditions, we have a unique solution for the above system of equation as

$$\begin{aligned}\mathbf{A}_x &= (a_x e^{-i\varpi t} + b_x e^{-i\varpi t} + c_x e^{-i\varpi t})e^{ikz}, \\ \mathbf{A}_y &= (a_y e^{-i\varpi t} + b_y e^{-i\varpi t} + c_y e^{-i\varpi t})e^{ikz}, \\ \phi &= \phi_0(e^{-i\varpi t} - e^{-i\varpi t})e^{ikz},\end{aligned}\quad (16)$$

where

$$\begin{aligned}b_x &= -\frac{2\beta\varpi_+}{k}b_y = \frac{2\beta\varpi_+}{k}\frac{\delta_-}{\delta_+}c_y \\ &= -\frac{\varpi_+}{\varpi_-}\frac{\delta_-}{\delta_+}c_x = \frac{4i\beta\mathbf{B}_0\varpi_+}{\delta_+}\phi_0, \\ a_y &= \frac{2\beta\varpi}{k}a_x = \sin\alpha + \frac{k}{2\beta\varpi_-}\left(\frac{\delta_+ - \delta_-}{\delta_+}\right)c_x, \\ c_x &= \frac{1}{\mathcal{F}}\left(\cos\alpha - \frac{k\sin(\alpha)}{2\beta\varpi}\right), \\ \mathcal{F} &= \frac{4\beta^2(\varpi\varpi_- \delta_+ - \varpi\varpi_+ \delta_-) + k^2(\delta_+ - \delta_-)}{4\beta^2\varpi\varpi_- \delta_+}.\end{aligned}\quad (17)$$

After traversing the external magnetic field for a distance  $t = \ell$ , the electro-magnetic wave will be modified as

$$\begin{aligned} \mathbf{A}_x &= a_x e^{-i\varpi\ell} + b_x e^{-i\varpi_+\ell} + c_x e^{-i\varpi_-\ell}, \\ \mathbf{A}_y &= a_y e^{-i\varpi\ell} + b_y e^{-i\varpi_+\ell} + c_y e^{-i\varpi_-\ell}. \end{aligned} \quad (18)$$

From the above set of expressions, one can easily see that the vector potential describes an ellipse whose major axis deviates from the  $x$ -axis by an angle

$$\theta = \tan^{-1} \left( \sqrt{\frac{\sin^2 \alpha - \Gamma}{\cos^2 \alpha - \mathcal{L}}} \right), \quad (19)$$

where

$$\begin{aligned} \mathcal{L} &= 2a_x b_x \sin^2 \left( \frac{\Delta_+}{2} \right) + 2a_x c_x \sin^2 \left( \frac{\Delta_-}{2} \right) + 2c_x b_x \sin^2 \left( \frac{\Delta}{2} \right), \\ \Gamma &= 2a_y b_y \sin^2 \left( \frac{\Delta_+}{2} \right) + 2a_y c_y \sin^2 \left( \frac{\Delta_-}{2} \right) + 2c_y b_y \sin^2 \left( \frac{\Delta}{2} \right), \\ \Delta_+ &= (\varpi_+ - \varpi)\ell; \quad \Delta_- = (\varpi_- - \varpi)\ell; \quad \Delta = (\varpi_+ - \varpi_-)\ell \end{aligned} \quad (20)$$

Equation (19) yields the expression for the optical rotation of the plane of polarization as

$$\delta = \theta - \alpha \simeq \frac{\sin(2\alpha)}{4} \left( \frac{\mathcal{L}}{\cos^2(\alpha)} - \frac{\Gamma}{\sin^2(\alpha)} \right). \quad (21)$$

The ellipticity  $\epsilon$  is the measure of the phase difference between the two components of the vector potential after traversing a distance  $\ell$  through a magnetic region. From Eq. (18), the exact expression for the ellipticity is

$$\epsilon = \frac{1}{2} |\psi_x - \psi_y|,$$

where

$$\begin{aligned} \psi_x &= \tan^{-1} \left[ \frac{bx \sin(\Delta_+) + cx \sin(\Delta_-)}{ax + bx \cos(\Delta_+) + cx \cos(\Delta_-)} \right], \\ \psi_y &= \tan^{-1} \left[ \frac{by \sin(\Delta_+) + cy \sin(\Delta_-)}{ay + by \cos(\Delta_+) + cy \cos(\Delta_-)} \right]. \end{aligned} \quad (22)$$

These are the quantities that establish the direct connection with the experimental data. The similar analysis can be done for the background electric field as well. The analysis we have done so far is applicable to the laboratory experimental observations where a polarized monochromatic laser beam with a fixed momentum traverses a magnetic region. If we want instead to consider an unpolarized light, then we have to solve the above coupled system of Eqs. (6) in term of the spacetime coordinates. For example, in CMB polarization power spectrum, the initial state of the electro-

magnetic wave at the last scattering surface is completely unpolarized. So, our present analysis is not adequate to study CMB polarization. In our forthcoming paper we will consider a more detailed analysis of the background electro-magnetic field effect on the scalar-photon mixing and will study its cosmological connection. In the next section, we will consider various existing laboratory-based experimental results on the optical rotation  $\delta$  and ellipticity  $\epsilon$  to constrain our model parameter  $\beta$  and scale of varying fine-structure constant  $\omega$ .

#### IV. CONSTRAINING $\beta$ AND $\omega$ PARAMETERS THROUGH LABORATORY EXPERIMENTS

The polarization properties of an electro-magnetic wave propagating through an external magnetic field can change if there exists a nontrivial (pseudo)scalar-photon coupling [21]. Based on these particular physical effects, various laboratory-based experiments have been devised to look for ultralight (pseudo)scalar particles. In this class of experiments, it is possible to make accurate measurements on the modification of the polarization state of a light beam. In a typical experiment, a linearly polarized laser beam is used to reflect  $N$  times between two mirrors, in a constant strong background magnetic field of strength  $\mathbf{B}_0$ . The magnetic field is perpendicular to the beam direction. Let the distance between the two mirrors be  $\ell$ , then the total length travelled by the laser beam in the magnetic field is  $L = N\ell$ . After traversing a distance  $L$ , which is usually of the order of a few Kilometers, it is possible to measure a minute ellipticity and a change in the rotation of the polarization plane.

As is well known, the vacuum itself has the magnetic birefringence property as dictated by QED. This effect is due to the dispersive effect induced by the virtual electron-positron pair in vacuum, which was first investigated by Heisenberg and Euler [39]. The ellipticity so induced serves as the background in the experiment that looks for birefringence or dichroism induced by a (pseudo)scalar particle that violates PCP. The QED contribution to the ellipticity can be written as

$$\mathcal{E} = N \frac{B_0^2 \ell \alpha_0^2 \omega}{15m_e^4}, \quad (23)$$

where  $\alpha_0 = e_0^2 = 1/137$  is the conventional fine-structure constant,  $\omega$  is the photon energy and  $m_e$  the electron mass. Here, we have assumed that the polarization vector of the initially linearly polarized beam makes a  $45^\circ$  angle relative to the direction of the external magnetic field. Consider, for example, a laser beam with wavelength  $\lambda = 1550$  nm, magnetic field  $B_0 = 9.5T$ , and length of travel  $N\ell = 25$  km. Then the resulting ellipticity (*cf.* Eq. (22)) would be  $2 \times 10^{-11}$  rad [40].

It is important to note that any physical mirror is transparent to the scalar field so that only the photon component of the beam is reflected. As in varying-alpha theory, matter

TABLE I. Laboratory experiments and their bounds on rotation  $\delta$  and ellipticity  $\epsilon$  measurements.

Experiment	$\lambda(\text{nm})$	$\mathbf{B}_0(\text{T})$	$L(\text{m})$	$N$	Rotation $\delta$ (rad)	Ellipticity $\epsilon$ (rad)
BFRT	514	3.25	8.8	250	$3.5 \times 10^{-10}$	-
PVLAS	1064	2.3	1	45000	$1.0 \times 10^{-9}$	$1.4 \times 10^{-8}$
		5.5			$1.2 \times 10^{-8}$	-
Q&A	1064	2.3	0.6	18700	$(-0.375 + 5.236) \times 10^{-9}$	-

field  $\psi$  coupled with the scalar field  $\phi$  through its electromagnetic mass correction at the loop level:

$$\mathcal{L}_{int} \sim \frac{e_0^2}{\omega} \bar{\phi} m_\psi^2 \bar{\psi} \psi + e_0 A_\mu \gamma^\mu \bar{\psi} \psi, \quad (24)$$

where we consider  $\bar{\phi}$  as the dimension full scalar field. It is natural to expect and obvious from the above equation that compared with the photon coupling,  $\phi$  coupling is significantly suppressed. This essentially sets the scalar component of the beam back to zero after each reflection [19]. The net effect after  $N$  reflections is  $\mathcal{E}(L) = N\mathcal{E}(\ell)$  where, in general,  $N\mathcal{E}(\ell)$  does not equal to  $\mathcal{E}(N\ell)$ . Thus, in order to take into account the effect of  $N$  reflections appropriately for a multiple-beam-path experiment, one needs to multiply the right-hand side of Eq. (21) and (22) by  $N$  (keeping everything else the same) while on the left-hand side, the length  $\ell$  of a single-path is now replaced by the total length  $L = N\ell$ .

In our analysis, we will consider various laboratory experiments as listed in the Table I. In all these experiments, a polarized laser beam with a particular wavelength has been used to measure the rotation and ellipticity of polarization. As we have discussed before, the basic underlying assumption of all these measurements is that there exists a nontrivial interaction of photon with the background field. The nature of this background field could be a Lorentz-invariant (pseudo) scalar or some Lorentz

violating vector field. The scalar field could be a QCD axion or some arbitrary dilaton field. As we saw, the scalar field that gives rise to the spacetime variation of fine-structure constant can also induce the polarization of an electro-magnetic wave through its PCP violating interactions. Rotation or ellipticity measurement therefore cannot distinguish different nature of the background interaction. Evidently, all the above experiments we mentioned are insensitive to the nature of the background field. Therefore, by analyzing the above experimental bounds on optical rotation  $\delta$  and ellipticity  $\epsilon$ , one can only constrain the parameters of a particular model without the explicit nature of the background field. It is therefore interesting to find out an observable that can distinguish different models.

All the above mentioned experiments have reported the upper limit on the optical rotation  $\delta$  except that PVLAS has reported the upper limit on ellipticity as well. It should be emphasized that in the case of the Q&A experiment, the error-bar in the optical rotation measurement is very large compared to its mean value. The amount of the optical rotation would therefore not be very conclusive, but we will still use its mean value for its upper bound to constrain our model parameters and compare with the other experimental constraints.

In Fig. 1, we have plotted the shaded exclusion regions in the two-dimensional plane spanned by the PCP violating

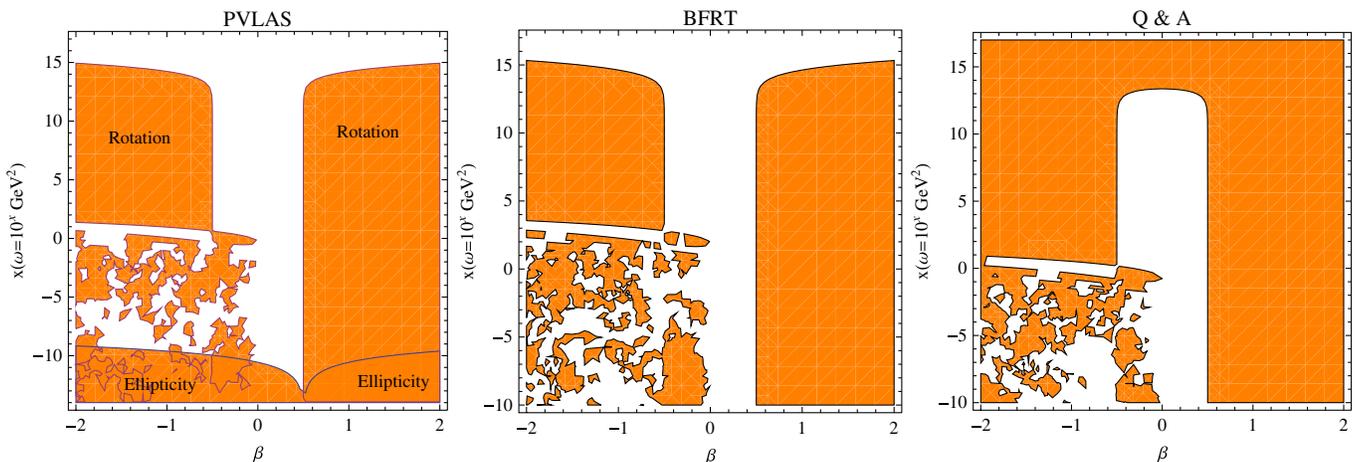


FIG. 1 (color online). Bounds on PCP violating parameter  $\beta$  and varying-alpha theory mass scale square  $\omega$  using different experimental results for the rotation and the ellipticity.

TABLE II. Laboratory constraints on PCP violating parameter  $\beta$ .

Range of $\omega$ in $\text{GeV}^2$	Bound on $\beta$	Experiment
$10^{-10} \leq \omega \leq 10^2$	$0 \leq \beta \leq 0.5$	PVLAS
$\omega \leq 10^2$		BERT, Q&A
$10^2 \leq \omega \leq 10^{13}$	$-0.5 \leq \beta \leq 0.5$	PVLAS, BFRT, Q&A
$\omega \geq 10^2$	$ \beta  \geq 1$	PVLAS, BFRT

parameter  $\beta$  and the scale of variation of the fine-structure constant  $\omega$ . Each plot corresponds to different experimental results. From these plots, we see that the bound on the scale of fine-structure constant variation based on the absence of ellipticity measured by PVLAS is  $\omega \geq 10^{-10} \text{ GeV}^2$ . On the other hand, the bound on the PCP violating parameter  $\beta$  based on the optical rotation measurement is almost the same for every experiment. As we can see from the contour plots of these experiments, the bound on PCP violating parameter falls into three different ranges as shown in Table II.

In the above analysis, we have excluded the fluctuating region of  $\omega \leq 1 \text{ GeV}^2$  and  $\beta < 0$ . For this range of  $\omega$ , the bound on PCP violating parameter becomes  $0 < \beta < 0.5$  as shown in the Table II. Interestingly, if we consider the negative mean value of the optical rotation  $\delta = -0.375$  based on the Q&A experiment, the contour plot shows that  $\omega (\geq 10^{13} \text{ GeV}^2)$  is bounded from above. This in turn constrains the PCP violating parameter  $\beta$  to be always less than one. The main concern, however, is that the error-bar in this particular observational constraint on the optical rotation is much larger than its mean value. So, this upper bound on  $\omega$  may not be conclusive.

We have mentioned before that in order to be consistent with the observation of Coulomb force law, the scale of fine-structure constant variation  $\sqrt{\omega} = \hbar c/l$  should be greater than a few tens of MeV. From the ellipticity measurement of PVLAS, we found the lower bound on this scale to be  $\approx 10^{-2} \text{ MeV}$ , which is in direct conflict with Coulomb force law measurement. According to our model, if we assume a lower bound on  $\sqrt{\omega} \approx 10^3 \text{ MeV}$ , then with the present value of the experimental input parameters, PVLAS would not be able to measure the ellipticity down to the level of  $\epsilon \approx 1 \times 10^{-17}$ . This value is significantly lower than the present PVLAS bound of  $\epsilon \approx 1.4 \times 10^{-8}$ . One can also see from Table II that for  $\omega \geq 10^{14} \text{ GeV}^2$ , the bound is  $|\beta| \geq 1$  according to PVLAS and BFRT. This should be unacceptable in connection with the other physical parity-violating effects due to large PCP coupling. From all the above considerations, we conclude that the most reasonable bounds on both of our model parameters are  $1 \leq \omega \leq 10^{13} \text{ GeV}^2$  and  $-0.5 \leq \beta \leq 0.5$ .

## V. CONCLUSIONS

The theory of varying fine-structure constant has been the subject of intense study for the last several years. Cosmological impact of this variation has been studied quite extensively. Various cosmological as well as laboratory-based observations on this variation of fine-structure constant have been considered to constrain the varying-alpha parameter  $\omega$ . Recently we have constructed a particular model based on this varying-alpha theory which includes explicit PCP violation in the photon sector [6]. In this paper, we have studied our aforementioned PCP violating varying-alpha model in the light of a new class of laboratory observations which have not been considered before. All those experiments directly measure the change of the polarization state of a photon as it propagates through a background magnetic field. The basic underlying assumption behind all these measurements is the existence of a nontrivial interaction between photon and some unknown background field. As stated before in our model, we have introduced a nontrivial PCV violating scalar-photon interaction in the varying-alpha theory framework. Although the experiments under consideration are insensitive to the properties of the background field due to the weakness of its coupling with the matter, they nevertheless can help to constrain our varying-alpha model parameters  $\omega$  and  $\beta$  through the rotation of polarization and ellipticity measurement. We have calculated these two particular measurable quantities in our model. The model is characterized by two independent parameters  $\beta$  and  $\omega$  that measure the strength of PCP violation and the scale of fine-structure constant variation, respectively. We have considered three different laboratory-based experimental results to constrain our model parameters. All the experiments so far do not observe any positive signal for the rotation and ellipticity. So, the nonobservation give us the possible upper limit on those quantities. Using those upper limits, we found that the most suitable bound on our two model parameters are  $1 \leq \omega \leq 10^{13} \text{ GeV}^2$  and  $-0.5 \leq \beta \leq 0.5$ . An interesting point to note here is that with this lower bound on  $\omega$ , it is very hard to measure the ellipticity from laboratory experiments. As we have estimated, for  $\omega \approx 10^3 \text{ MeV}$ , the bound on ellipticity should be  $\epsilon \approx 1 \times 10^{-17}$ , which is far below the present experimental limit as well as sensitivity.

## ACKNOWLEDGMENTS

This research is supported by Taiwan National Science Council under Project No. NSC 97-2112-M-002-026-MY3, by Taiwan's National Center for Theoretical Sciences (NCTS), and by US Department of Energy under Contract No. DE-AC03-76SF00515.

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