Entropic force scenarios and eternal inflation

Taotao Qiu[1,2,3,*] and Emmanuel N. Saridakis[4,5,†]

[1]*Department of Physics, Chung-Yuan Christian University, Chung-li 320, Taiwan*
[2]*Department of Physics and Center for Theoretical Sciences, National Taiwan University, Taipei 10617, Taiwan*
[3]*Leung Center for Cosmology and Particle Astrophysics, National Taiwan University, Taipei 106, Taiwan*
[4]*CASPER, Physics Department, Baylor University, Waco, Texas 76798-7310, USA*
[5]*National Center for Theoretical Sciences, Hsinchu, Taiwan 300*

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We examine various entropic inflation scenarios, under the light of eternality. After describing the inflation realization and the normal condition for inflation to last at the background level, we investigate the conditions for eternal inflation with the effect of thermal fluctuations produced from standard radiation and from the holographic screen. Furthermore, we incorporate stochastic quantum fluctuations through a phenomenological, Langevin analysis, studying whether they can affect the inflation eternality. In one-holographic-screen scenarios eternality can be easily obtained, while in double-screen considerations inflation is eternal only in the high-energy regime. Thus, from the cosmological point of view, one should take these into account before he can consider entropic gravity as a candidate for the description of nature. However, from the string theory point of view, inflation eternality may form the background for the "landscape" of string or M theory vacua, leading to new perspectives in entropy gravity.

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I. INTRODUCTION

After almost three decades of extensive research, inflation is now considered to be a crucial part of the cosmological history of the Universe [1], having affected indelibly its observational features. The inflation paradigm, in conventional as well as in higher dimensional frameworks [2–4], can successfully solve theoretical problems such as the flatness, the horizon, and the monopole ones, and moreover it provides the right behavior of primordial fluctuations with a nearly scale-invariant power spectrum.

One important issue that has to be encountered in inflationary cosmology is that of eternality, that is, whether inflation has a beginning and/or an end [5,6]. In general there are two different points of view for its incorporation. On the one hand, from the cosmological point of view, a picture that indicates that there are always parts of the Universe that inflate and parts that have exit inflation, like our own observable universe part, in a procedure without beginning, offers a way out from the singularity [7,8] and trans-Planckian [9] problems. However, in order for such a procedure to be the case in nature, one still needs to find a mechanism that can lead areas of the Universe of nonzero measure to exit inflation, since in our "local" part inflation has obviously ended. We stress that these parts must be of nonzero measure since a minimal requirement, that inflation has to end at least in one part amongst infinite eternally inflating areas, leads to an intense fine-tuned and anthropic description of nature [10]. On the other hand, from the string theory point of view, eternal inflation may form the background for (or be formed by) the landscape of string or M theory vacua [11], induced by flux compactification [12] or other mechanisms [13,14], that is, for the existence of a huge number of possible (false) vacua. Thus, from this point of view, inflation eternality has an additional reason of being a desirable feature of a cosmological theory, and furthermore one does not need to worry so much about the inflation exit, since it is adequate for it to be realized in only one area of the Universe (which is equivalent to choosing a specific vacuum along a landscape of approximately 10^{500} ones [14]).

Recently, an extended holographic picture was suggested by Verlinde [15], in which gravity is no longer a fundamental theory, but emerges from a statistic effect of a holographic screen (a similar scenario was discussed by Padmanabhan in [16], based on the earlier considerations of [17]). Such an "entropic" origin of gravity was based on the holographic principle, conjectured as a significant property of quantum gravity, stating that the physics of a volume of space is encoded on its boundary, such as the gravitational horizon [18]. Although there is a controversy in the foundations of entropic gravity itself [19], the idea is very interesting and the cosmological implications of its various scenarios were extensively studied in the literature. Besides the original formulation (see [20] and references therein), people proposed various inflation models of which inflation was assumed to be driven by one or two holographic screen(s). In the one-screen inflation model, the holographic screen acts as a boundary term in the Einstein equation, which can force the Universe to accelerated expansion even if the Universe is radiation dominated [21]. In two-screen model, an "inner" Schwarzschild screen was added inside the holographic

*xsjqiu@gmail.com
†Emmanuel_Saridakis@baylor.edu

screen, which made the inflation realization easier [22,23]. Additionally, there are many other implications of entropic gravity, such as explaining late-time acceleration [24,25] and incorporating black holes [26].

In the present work, we focus on the eternality issue of inflation realization in the various entropic scenarios of the literature, both at background and fluctuation levels. In particular, we examine on what conditions inflation can be eternal, and whether its eternality can be prevented by stochastic quantum fluctuations. The plan of the work is as follows. In Sec. II we briefly review Verlinde's basic entropic gravity formulation. In Sec. III we examine the eternality for the single-screen inflation scenario, while in Sec. IV we study the case of the double-screen inflation model. Finally, Sec. V is devoted to the summary of the results.

Lastly, we mention that, in the whole manuscript, in order to simplify the symbolism we set the light speed c and the reduced Planck $\hbar$ and the Boltzmann k_B constants to 1, but we keep Newton's constant explicitly for clarity.

II. VERLINDE'S BASIC SCENARIO

The basic ingredient of Verlinde's idea is that the boundary physics can be described by thermodynamics satisfying a holographic distribution, where the number of degrees of freedom on this holographic screen is proportional to its area, that is,

$$dN = \frac{dA}{G}, \tag{1}$$

where G is the Newtonian gravitational constant. Thus, the classical holographic entropy on this screen is given by

$$S_b = \frac{A}{4G} = \frac{\pi r_b^2}{G}, \tag{2}$$

where r_b is the radius location of the boundary surface $\mathcal{S}$. The variation of energy with respect to the radius will provide the entropic force [15]:

$$F_e = -\left(\frac{dE}{dr}\right)_b = -\left(T\frac{dS}{dr}\right)_b = -2\pi\frac{T_b r_b}{G}, \tag{3}$$

in which T_b is the temperature of the boundary of the system. Finally, due to the Unruh effect (when a test particle with mass m is located nearby the holographic screen the variation of the entropy on this screen with respect to the radius takes the form $\frac{dS}{dr} = -2\pi m$) the above force yields an entropic acceleration a_e of the form [27]:

$$a_e \equiv \frac{F_e}{m} = 2\pi T_b. \tag{4}$$

Note that the corresponding entropic pressure is negative $P_e = \frac{F_e}{A_b} = -\frac{1}{2G}\frac{T_b}{r_b}$, and so it is expected to realize an accelerating process of the Universe.

The above simple analysis can be straightforwardly generalized to the full relativistic case. The natural generalization of Newton's potential is $\phi = \frac{1}{2}\log(\xi^a\xi_a)$, where ξ^a is a global timelike Killing vector,[1] and the exponential e^ϕ represents the redshift factor that relates the local time coordinate to that at a reference point with $\phi = 0$, which we will take to be at infinity. One considers a holographic screen on a closed surface of a constant redshift ϕ, enclosing a certain static mass configuration with total mass M [15]. Because of the equipartition $M = \frac{1}{2}\int_{\mathcal{S}} T dN$, the Unruh temperature now writes as

$$T = \frac{1}{2\pi}e^\phi N\cdot\nabla\phi. \tag{5}$$

Therefore, for the total mass we obtain $4\pi GM = \int_{\mathcal{S}} e^\phi\nabla\phi\cdot dA$ or, expressed in terms of the Killing vector,

$$M = \frac{1}{8\pi G}\int_{\mathcal{S}} dx^a\wedge dx^b\epsilon_{abcd}\nabla^c\xi^d. \tag{6}$$

Thus, expressing the total mass in terms of the energy-momentum tensor, it results in the Einstein equations, namely,

$$2\int_\Sigma\left(T_{ab}-\frac{1}{2}Tg_{ab}\right)n^a\xi^b dV = \frac{1}{4\pi G}\int_\Sigma R_{ab}n^a\xi^b dV, \tag{7}$$

where Σ is the three-dimensional volume bounded by the holographic screen $\mathcal{S}$ and n^a is its normal. Since this relation can hold for all appropriate Killing vectors and for arbitrary screens [15], it is sufficient in providing the full Einstein equations.

The above formulation of gravity as an entropic force, which could therefore lead to descriptions of the inflationary as well as the dark-energy epoches, is quite general. Nonetheless, one still needs to construct more precise and detailed models in order to proceed to a quantitative analysis of various cosmological eras. Concerning inflation, which is the subject of the present work, we stress here that the entropic origin of gravity itself is consistent with both finite or eternal behavior, similar to the case of standard gravitational theories. Thus, the eternality subject has to be examined in each explicit inflationary model separately, since it depends on the details of each scenario. This is performed in the next two sections.

III. THE SINGLE-SCREEN MODEL

Although the qualitative features of inflation in the entropic context were discussed in [28], the first explicit quantitative inflation model was proposed in [21]. The basic feature of this model is that acceleration is driven by an

[1] Note that in the present manuscript we follow the $(+,-,-,-)$ metric signatures and thus, in the above definition, we have inverted the sign inside the logarithm comparing to the original expression in [15].

additional surface term in Einstein's equations, which comes from the holographic screen assumed by Verlinde. In this section, after briefly reviewing the model, we first examine the condition in which inflation lasts classically, then we study its eternality, incorporating thermal fluctuations, and finally we perform a detailed Langevin analysis, as a first approach to the stochastic quantum effects. In the following, we assume a flat Friedmann-Robertson-Walker background geometry with metric

$$ds^2 = dt^2 - a^2(t)\delta_{ij}dx^i dx^j, \tag{8}$$

where t is the cosmic time, x^i are the comoving spatial coordinates, and $a(t)$ is the scale factor. We also introduce the Hubble parameter $H = \dot{a}/a$, with a dot denoting differentiation with respect to t.

In the single-screen inflation model, one of the two Friedmann equations reads

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) + C_H H^2 + C_{\dot{H}}\dot{H}, \tag{9}$$

where C_H and $C_{\dot{H}}$ are dimensionless coefficients, which are expected to be bounded by $C_H < 1$ and $0 \leq C_{\dot{H}} \leq 3/4\pi$ according to the authors who first constructed the scenario [21].[2] Moreover, taking into account higher order corrections to the entropy, the above relation becomes

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) + C_H H^2 + C_{\dot{H}}\dot{H} + g\frac{G}{\pi}H^4, \tag{10}$$

where the dimensionless factor g incorporates the effective number of independent degrees of freedom which is dimensionless (note that since we have set $c = \hbar = k_B = 1$, GH^2 is dimensionless and thus all the formulas have are homogeneous in units). On the other hand, the second Friedmann equation writes as

$$H^2 = \frac{8\pi G}{3}\rho + C_H H^2 + C_{\dot{H}}\dot{H}. \tag{11}$$

Amazingly enough, the combination of (10) and (11) provides us the modified second Friedmann equation independent on C_H and $C_{\dot{H}}$, namely,

$$\dot{H} = -4\pi G(\rho + P) + g\frac{G}{\pi}H^4. \tag{12}$$

Moreover, differentiating (11) with respect to t and ignoring the higher order derivative of H under slow-roll approximation, we have the continuity equation:

$$\dot{\rho} + 3H(1 - C_H)(\rho + P) = 0 \tag{13}$$

[2]We mention here that the same authors, when they apply the same scenario in the late-time accelerated epoch instead of inflation, they use a different lower limit for C_H [24,29]. Although this behavior would require an explanation by those authors, for the purpose of the present work the lower limit of C_H is irrelevant, since only the sign of $C_H - 1$ plays a role, that is, its upper limit, which is always 1 in all works.

(note that this relation can be viewed as originating from the well-known generalized conservation equation in which matter exchanges energy with vacuum). Finally, the expression for the entropic pressure that drives inflation is

$$P_e = -\frac{2}{3}\rho_{c0}\left(\frac{H^2}{H_0^2} + \frac{gG}{\pi}\frac{H^4}{H_0^2}\right), \tag{14}$$

where we have introduced the critical energy density $\rho_c = 3H^2/(8\pi G)$, with the subscript 0 denoting the present-day value of a quantity. Clearly, when $gH^4/\pi = 4\pi(\rho + P)$, which, in the case of relativistically high energies, becomes $gH^4/\pi = 16\pi\rho/3$, we acquire $\dot{H} = 0$ and thus a de Sitter expanding phase can be obtained [21].

Before proceeding to the investigation of the eternality issues of the model at hand, let us make some comments. First, note that the last two terms in Friedmann equation (11) can be viewed as an "effective" inflation part, which can drive acceleration. Straightforwardly, the same equation can also be applied to the dark-energy epoch, that is, to explain the late-time acceleration, as it is done in the so-called "running cosmological constant" scenarios [30,31] (though from a different ideological point of view). In this approach, the time-dependent part of the effective cosmological constant $\Lambda(t)$ can be made to be $\propto H^2$, which is similar to the case at hand except for the $\dot{H}$ term. However, and more important, $\Lambda(t)$ should also contain a nonvanishing constant additive term, in order to fit the combined observational data [31]. This is a disadvantage of the present model, since one should provide an explanation for the modification of Friedmann equation (11) between early and late times, in order to acquire a realistic model. In the same lines, as it was mentioned above, one could put into question the bounds $C_H < 1$ and $0 \leq C_{\dot{H}} \leq 3/4\pi$ [21] that the authors give for their model. In summary, the one-screen model examined here seems to have ambiguities concerning the correct quantitative behavior. Clearly, one could study a generalization, including a constant term in (11), or abandon the aforementioned bounds in the model parameters. However, in the present work we prefer to remain in the original version of the scenario, and examine it under the eternality point of view, instead of trying to improve it first, which could be the subject of interest of a separate work.

A. Inflation eternality: Background analysis

Let us now discuss the inflation eternality realization in the one-screen entropic scenario. First, we have to determine the condition for a *global* inflation exit, that is, with no part of the Universe going on inflating. In the scalar-field driven inflation models, this is just the slow-roll condition. In the scenario at hand there is no field slow rolling, however, we can still use the definition of the "slow-roll" parameter $\epsilon \equiv -\dot{H}/H^2$, and thus the condition for inflation lasting remains $\epsilon \lesssim 1$ (this condition arises from the definition of inflation and thus it is model-independent), or equivalently

$$\dot{H} + H^2 \gtrsim 0. \tag{15}$$

Starting from Eqs. (10) and (11) and assuming that the Universe is filled with radiation ($P = \rho/3$), we can eliminate ρ and obtain an equation with $H(t)$ only:

$$\dot{H}(t)(1 - 2C_{\dot{H}}) = 2(C_H - 1)H^2 + g\frac{G}{\pi}H^4. \tag{16}$$

Using this equation, condition (15) imposes a constraint on $H(t)$, namely,

$$H \gtrsim \sqrt{\frac{\pi}{gG}[2C_{\dot{H}} - 1 + 2(1 - C_H)]}. \tag{17}$$

In summary, as long as $H(t)$ is larger than the value of the right-hand-side of the above condition, inflation will not globally exit, and thus we obtain its eternality realization at the background level. Taking into account the allowed intervals of the parameters, we deduce that this condition can be easily fulfilled. However, one must also investigate the role of fluctuations generated during inflation, on the eternality condition. This is performed in the next subsections.

B. Condition for eternal inflation: Thermal-fluctuation analysis

In the previous subsection we investigated the background evolution, extracting the corresponding condition for inflation lasting in the whole Universe. Here we examine the role of thermal fluctuations. Note that during inflation, the Universe will expand to nearly $e^3 \approx 20$ causally independent Hubble-sized regions in one Hubble time $\Delta t \approx H^{-1}$. In any of these regions, the energy density will be decreased by $\delta_c\rho = |\dot{\rho}|\Delta t \approx |\dot{\rho}|H^{-1}$, however, this may be corrected when we take into account the fluctuations. In particular, if there is nonzero probability that there exist regions of the Universe where the fluctuations are bigger than the corresponding classical change, then in these regions inflation will be eternal, despite the fact that in the other regions inflation will have an ending [6]. In a holographic system, the fluctuations are generated in a thermal way [21], so we define the condition for the inflation to be eternal as

$$\delta_t\rho > \delta_c\rho, \tag{18}$$

where $\delta_t\rho$ is the thermal fluctuation generated during inflation. We desire to emphasize that this condition looks similar to the well-known condition for eternal inflation driven by the scalar field [6]: $\delta_q\phi > \delta_c\phi$,[3] but the difference is that, in that case, fluctuations come from the quantum behavior of the scalar field. From this comparison one can see that our condition is very reasonable too.

[3]Note that the condition for eternal inflation in case of noncommutativity or nonminimal coupling has also been discussed; see [32,33], respectively.

Using (13) and the Friedmann equations (10) and (11), we can express the classical change of the energy density $\delta_c\rho$ as

$$\delta_c\rho = \frac{3(1 - C_H)H^2}{2\pi G(1 - 2C_{\dot{H}})}\left[(1 - C_H) - \frac{gC_{\dot{H}}}{\pi}GH^2\right], \tag{19}$$

where we have also made use of the fact that $\dot{\rho} = -4H\rho(1 - C_H)$ (a relation that holds for a radiation-dominated universe) is always negative. On the other hand, the thermal change $\delta_t\rho$ can be estimated through its correlation function in position space, namely, $\delta_t\rho = \sqrt{\langle\delta\rho^2\rangle}$, with the relation [34]:

$$\langle\delta\rho^2\rangle = C_V(R)\frac{T^2}{R^6}, \tag{20}$$

where $C_V(R) \equiv \partial\langle E\rangle/\partial T$ is the heat capacity in a sphere of radius R. Usually, two kinds of thermal fluctuations could be taken into account, namely, thermal particle fluctuations from normal radiation inside the bulk of the Universe, and holographic fluctuations from the boundary screen. According to different fluctuating mechanisms, the condition for eternal inflation will certainly be different. Thus, in the next two paragraphs we will investigate both of them separately.

1. Normal radiation

The energy density of normal radiation inside the bulk of universe can be written as [21]

$$\rho_{\rm tr} = 4\sigma\mathcal{G}(T)T^4, \tag{21}$$

where $\mathcal{G}(T) = 45g$ is the effective number of degrees of freedom at temperature T. The subscript "tr" stands for "thermal radiation." Thus, from the heat capacity definition we acquire

$$C_V = 960\pi g\sigma R_{\rm tr}^3 T^3, \tag{22}$$

where $R_{\rm tr}$ is the correlation length of radiation fluctuations, given as usual from $R_{\rm tr} = c_s/H$, with c_s the sound speed of the radiation. Therefore, we get the following thermal fluctuation of the system: $\delta_t\rho|_{\rm tr} = \sqrt{960\pi g\sigma T^5/R_{tr}^3}$. Setting $T = H/2\pi$ as the Gibbons-Hawking temperature, we finally obtain

$$\delta_t\rho|_{\rm tr} = \frac{1}{\pi^2}\sqrt{\frac{30g\sigma}{c_s^3}}H^4. \tag{23}$$

From (19) and (23) we can see that in the single-screen model, the condition for eternal inflation (18) with normal radiation fluctuation has the form,

$$\frac{1}{\pi^2}\sqrt{\frac{30g\sigma}{c_s^3}}H^4 > \frac{3(1-C_H)H^2}{2\pi G(1-2C_{\dot{H}})}\left[(1-C_H) - \frac{gC_{\dot{H}}}{\pi}GH^2\right],$$

which gives the constraint on H:

$$H > \frac{\sqrt{3}\pi(1-C_H)}{\sqrt{G}[2(1-2C_{\dot{H}})c_s^{-(3/2)}\sqrt{30g\sigma}+3(1-C_H)gC_{\dot{H}}]^{(1/2)}}. \quad (24)$$

Finally, we mention here that (24) has to be considered along with (17) in order for inflation to be eternal, since when inflation globally exits, there will be no eternal inflation. Thus, if the right-hand side of (24) is larger than that of (17), which depends on the parameter choice, we can take (24) as the eternality condition safely, while if it is not, (17) should be taken as the eternal condition. Again, we can see that the eternality conditions can be easily fulfilled.

2. Holographic screen

Now we focus on the case that the thermal-fluctuation is produced holographically. The number of (finite) degrees of freedom on the boundary screen is proportional to the surface area approximately $R_{\rm hs}^2$, where $R_{\rm hs}$ is the correlation length and "hs" stands for "holographic screen." The energy of each degree of freedom is roughly the temperature of the screen T in thermal equibrilium, thus the total energy of the holographic screen is $\langle E\rangle \sim R_{\rm hs}^2 T/G$, and the heat capacity $C_V \sim c_v R_{\rm hs}^2/G$ where c_v is a constant of the order of $\mathcal{O}(1)$ determined by the detailed microscopic quantities of quantum gravity. Therefore, (20) gives $\delta_t\rho|_{\rm hs} = T\sqrt{c_v/G}/R_{hs}^2$, and setting $R_{\rm hs} \simeq 1/H$ and $T = H/2\pi$ [23] we finally obtain

$$\delta_t\rho|_{\rm hs} = \sqrt{\frac{c_v}{G}}\frac{H^3}{2\pi}. \quad (25)$$

From (19) and (25) we deduce that in single-screen scenarios the condition for eternal inflation (18), under holographic fluctuations, becomes

$$\sqrt{\frac{c_v}{G}}\frac{H^3}{2\pi} > \frac{3(1-C_H)H^2}{2\pi G(1-2C_{\dot{H}})}\left[(1-C_H) - \frac{C_{\dot{H}}g}{\pi}GH^2\right],$$

which gives the constraint on H:

$$H > \frac{\pi(1-2C_{\dot{H}})}{6gC_{\dot{H}}(1-C_H)\sqrt{G}}\left[\sqrt{c_v + \frac{36gC_{\dot{H}}(1-C_H)^3}{\pi(1-2C_{\dot{H}})^2}} - \sqrt{c_v}\right], \quad (26)$$

and both (17) and (26) have to be taken into account for the same reason mentioned at the end of the above paragraph.

Before closing this subsection we have to make the following comment. In the above two paragraphs we calculated separately the thermal fluctuations from normal radiation and from the holographic screen, and we gave the corresponding conditions for eternal inflation, namely, relations (24) and (26), respectively. However, although these are two independent mechanisms, they can exist simultaneously and thus lead to a combined effect, namely, $\delta_t\rho|_{\rm tr} + \delta_t\rho|_{\rm hs} > \delta_c\rho$. This forces $H(t)$ to satisfy

$$H > \frac{-\pi(1-2C_{\dot{H}})\sqrt{c_v} + \sqrt{\pi^2 c_v(1-2C_{\dot{H}})^2 + 12\pi(1-C_H)^2[2(1-2C_{\dot{H}})c_s^{-3/2}\sqrt{30g\sigma}+3(1-C_H)gC_{\dot{H}}]}}{2\sqrt{G}[2(1-2C_{\dot{H}})c_s^{-3/2}\sqrt{30g\sigma}+3(1-C_H)gC_{\dot{H}}]} \quad (27)$$

for eternal inflation.

C. Effects from quantum corrections: Langevin analysis

In the previous subsection we investigated the conditions for eternal inflation involving thermal fluctuations generated both from normal radiation and holographic screen. Here, we examine the backreaction of the metric and the quantum fluctuations on the background spacetime, in order to see whether it can prevent inflation from eternality. In particular, we proceed to an indirect investigation of quantum fluctuations and incorporate them as a stochastic effect, without caring about their specific microscopic origin. Such an approach covers all possible effects of quantum fluctuations at the phenomenological level, as is used in many cosmological systems [33,35].

According to evolution equation (16) and following [35], we formulate the overall cosmological evolution, including the classical motion and the quantum fluctuations as a stochastic effect modeled through a random walk, which can then be described by a Langevin equation and analysis. In particular, considering the system being perturbed by microfluctuations described by a Gaussian white noise normalized as

$$\langle n(t)\rangle = 0, \qquad \langle n(t)n(t')\rangle = \delta(t-t'), \quad (28)$$

we can write the Langevin equation for Eq. (16) as

$$\dot{H}(t) = D_1H^2 + D_2H^4 + q_s n(t), \quad (29)$$

where we have defined the coefficients $D_1 \equiv -\frac{2(1-C_H)}{1-2C_{\dot{H}}}$, $D_2 \equiv \frac{gG}{\pi(1-2C_{\dot{H}})}$, $q_s \equiv \frac{\varepsilon G^{-3/4}}{1-2C_{\dot{H}}}$ (note that according to the bounds on $C_H, C_{\dot{H}}$, we deduce that $D_1 < 0$ and $D_2 > 0$). Note also that D_1 and D_2 have dimensions of 0 and -2, respectively. In the stochastic term the coefficient $G^{-3/4}$ is inserted as usual for dimensional reasons, and ε is a dimensionless coefficient with rather small value [35].

In Eq. (29), if the last term on the right-hand side is absent, we recover the usual equation of motion (16) and the system will follow a classical trajectory $H_c(t)$. Therefore, we expand $H(t)$ around its classical value $H_c(t)$ up to order $\mathcal{O}(q_s^2)$, namely,

$$H(t) = H_c(t) + q_s G^{(3/4)} H_1(t) + q_s^2 G^{(3/2)} H_2(t) + \mathcal{O}(q_s^3), \tag{30}$$

where $G^{(3/4)}$ and $G^{(3/2)}$ have been inserted for dimensional reasons. Substituting this expansion into (29) and setting the coefficients of the q_s powers to zero, we acquire the equations

$$\dot{H}_c = D_1 H_c^2 + D_2 H_c^4, \tag{31}$$

$$\dot{H}_1 = 2D_1 H_1 H_c + 4D_2 H_1 H_c^3 + G^{-(3/4)} n, \tag{32}$$

$$\dot{H}_2 = D_1(H_1^2 + 2H_c H_2) + D_2(6H_c^2 H_1^2 + 4H_c^3 H_2). \tag{33}$$

These equations cannot accept analytical solutions, however we can obtain approximate solutions in the limit $|t/\mathcal{T}| \ll 1$ where $\mathcal{T} \gg H^{-1}$, that is performing our calculation within one Hubble interval ($t \sim H^{-1}$) [35], with $t = 0$ the time where inflation starts, and then we impose the requirement that the stochastic fluctuations be able to stop the eternal inflation. The explicit calculations are presented in Appendix A. We find that stochastic fluctuations can hardly have any significant effect on preventing inflation from eternality caused at the background or thermal-fluctuation levels.

In summary, in this section we obtained the conditions for eternal inflation in one-screen entropic inflation scenarios, taking into account two types of thermal-fluctuation productions. As we saw, for general parameter choices the eternality conditions can be easily fulfilled. We also performed a Langevin analysis and we found that the quantum-stochastic effects from microfluctuations can hardly change the behavior, and thus in the model at hand, inflation has a large probability to be eternal. Actually this result had already been discussed in the original paper [21], and thus the authors resorted to quantum fluctuations in order to induce the inflation exit, but the incorporation of the quantum fluctuations remained at the qualitative level. However, in this section we performed a quantitative analysis and our conclusions seem trustworthy. Obviously, the Langevin analysis is not a full incorporation of quantum fluctuations, which is a relatively unknown subject, but it can provide phenomenologically trustworthy results. Definitely, one could try to incorporate quantum fluctuations differently, paying the price of being model dependent, but our above result seems difficult to to change.

IV. THE DOUBLE-SCREEN MODEL

Single-screen entropic inflation is the simplest model inspired from Verlinde's idea, and the scenario is interesting and totally different from other inflation models. However, this scenario presents difficulties in quantitatively describing the thermal history of the Universe due to the reason that the screen temperature is always much lower than that of the cosmological microwave background and thus the Universe would be unstable under Hawking radiation. In order to alleviate such a problem, a double-screen extension was proposed in [22,23]. In this model, apart from the usual outer holographic screen formed by the Hubble horizon (or a surface near it), an additional inner boundary has also been introduced, which was considered as the Schwarzschild horizon of the whole Universe. At early time, both screens give rise to mutually competing forces, which drive inflation, while at late times, when the inner screen evaporates, the remaining outer screen drives the Universe acceleration.

The Schwarzschild radius r_S is given by

$$r_S = 2GM_{\rm tot} = 2G\int_{M_b} \rho dV = \frac{8\pi G\rho}{3\beta^3 H^3}, \tag{34}$$

where we have used the fact that the volume of the Universe is $V = 4\pi r_H^3/3$, with $r_H = (\beta H)^{-1}$ the outer screen radius and β a dimensionless parameter quantifying the possible divergence of the outer screen from the Hubble horizon. Its corresponding temperature is given by

$$T_S = \frac{1}{8\pi GM_{\rm tot}} = \frac{3\beta^3 H^3}{32\pi^2 G\rho}, \tag{35}$$

and therefore its induced acceleration (with the simple entropy form) will be $a_S = 2\pi T_S$, but with the direction toward the inner screen, opposite to the outer one. Therefore, in double-screen entropic cosmology the total induced acceleration is

$$a_e = 2\pi(T_H - T_S) = \beta H\left(1 - \frac{3\beta^2 H^2}{16\pi G\rho}\right), \tag{36}$$

with $T_H = \beta H/(2\pi)$, namely, it incorporates a competition of entropic effects from the outer and the inner screens. Taking also the higher order corrections on the entropy expressions into account, one extracts the modified Friedmann acceleration equation in this scenario as [22,23]

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + f(\rho, H), \tag{37}$$

with the form of the surface function being

$$f(\rho, H) \simeq \beta^2 H^2\left(1 - \frac{3\beta^2 H^2}{16\pi G\rho}\right) + \frac{g_H G\beta^4 H^4}{4\pi} \times \left(1 - \frac{27 g_S \beta^6 H^6}{1024 g_H \pi^3 G^3 \rho^3}\right), \tag{38}$$

where g_H and g_S are the corresponding dimensionless correction coefficients for each boundary. Here we have neglected the higher order correction term that appeared in [22,23]. The cosmological system will close, as usual, by the consideration of the evolution equation of the total energy density ρ. In the case at hand, in which one may have flow through the boundaries, the corresponding equation is modified as [22,23]

$$\dot{\rho} + 3H(\rho + P) = \Gamma, \tag{39}$$

with the effective coupling term Γ being

$$\Gamma = \frac{27\beta^6 H^6}{1024\pi^3 G^3 \rho^3}\dot{\rho} + \frac{3\beta^2 H\dot{H}}{4\pi G}\left(1 - \frac{27\beta^4 H^4}{256\pi^2 G^2 \rho^2}\right), \tag{40}$$

at the classical level.

Focusing on the early-time universe evolution, and, in particular, on the inflation realization, we assume that the Universe is radiation dominated and thus the equation of state of the total Universe content is $P = \rho/3$. Solving the equations of motion (37) and (39) up to leading order, one can obtain the following approximate solution for the Hubble parameter at early times [23]:

$$H^2 \simeq \frac{8\pi G}{3}\left[\rho + \frac{8(g_H - 4g_S)G^2}{69}\rho^2\right] = \frac{8\pi G}{3}\left(\rho + \frac{\rho^2}{\bar{\rho}}\right), \tag{41}$$

where $\bar{\rho} \equiv 69/(8\bar{g}G^2)$, with $\bar{g} = g_H - 4g_S$. An interesting property of this scenario is that when $\bar{g} > 0$, the Hubble parameter is proportional to the energy density at high-energy scales. Therefore, in this case the ρ^2 term could make the early-time inflation much easier to be realized, compared to single-screen models.

A. Inflation eternality: Background analysis

We now focus on the conditions for inflation eternality in double-screen scenarios. Similarly to the previous section, we consider that inflation will continue as long as condition (15) holds, since it is model independent. In order to simplify the expressions we restrict the analysis in the high-energy regime $\rho \gg \bar{\rho}$, where inflation is realized, since when $\rho < \bar{\rho}$ inflation will always end [23]. In the $\rho \gg \bar{\rho}$ case, the second term in (41) dominates, leading to a linear approximate relation between ρ and H:

$$\rho \simeq \sqrt{\frac{207}{64\pi\bar{g}}}\frac{H}{G^{3/2}}. \tag{42}$$

Substituting this into the second Friedmann equation (37) and considering also the radiation equation of state $P = \rho/3$, we can eliminate ρ, resulting in an equation depending only on H:

$$\dot{H} = -\sqrt{\frac{23\pi}{\bar{g}G}}H + (\beta^2 - 1)H^2 - \frac{\beta^4}{2}\sqrt{\frac{\bar{g}G}{23\pi}}H^3 + \frac{g_H\beta^4}{4\pi}GH^4 - \frac{g_S\beta^{10}}{\pi^3}\frac{\bar{g}}{184}\sqrt{\frac{\pi\bar{g}}{23}}G^{(5/2)}H^7. \tag{43}$$

Thus, condition (15) becomes

$$-\sqrt{\frac{23\pi}{\bar{g}G}}H + \beta^2H^2 - \frac{\beta^4}{2}\sqrt{\frac{\bar{g}G}{23\pi}}H^3 + \frac{g_H\beta^4}{4\pi}GH^4 - \frac{g_S\beta^{10}}{\pi^3}\frac{\bar{g}}{184}\sqrt{\frac{\pi\bar{g}}{23}}G^{(5/2)}H^7 \gtrsim 0. \tag{44}$$

In order to be more specific, as an example we consider the parameter choice of [22], that is $\beta = \sqrt{2}$,[4] $g_S = 0$, and $\bar{g} = g_H = 10^{16}$. Moreover, in the regime where $\rho \gg \bar{\rho}$, the relation (42) leads to $H \gg \sqrt{23\pi/(\bar{g}G)}$. Setting $s = H\sqrt{\bar{g}G/(23\pi)} \gg 1$, condition (44) can be simplified as

$$H\left\{-1 + 23s\left[\left(s - \frac{1}{23}\right)^2 + \frac{45}{529}\right]\right\} \gtrsim 0. \tag{45}$$

Since $s \gg 1$, this condition is always satisfied, that is, under the approximation $\rho \gg \bar{\rho}$, the Universe will not globally exit inflation. Note that this is consistent with the result of [23].

B. Condition for eternal inflation: Thermal-fluctuation analysis

In the previous subsection we extracted the requirement for inflation lasting at the background level, in the case of double-screen scenarios. Here we desire to incorporate the thermal fluctuations and, in particular, to examine under what conditions the inflation can be eternal. Similar to the single-screen case, we will estimate the change in energy density $\delta_t\rho$ due to thermal fluctuations arising from normal radiation and from the holographic screens, and we will compare it with the classical change $\delta_c\rho$. The condition for eternal inflation will again be $\delta_t\rho > \delta_c\rho$.

Concerning the classical change, things are different from the single-screen model due to the different background dynamics. In particular, using (39), (42), and (43), we obtain

$$\delta_c\rho \approx \frac{|\dot{\rho}|}{H} = \frac{3\left[\frac{\beta^2\dot{H}}{4\pi G}\left(1 - \frac{27\beta^4H^4}{256\pi^2G^2\rho^2}\right) - \frac{4\rho}{3}\right]}{1 - \frac{27\beta^6H^6}{1024\pi^3G^3\rho^3}}$$
$$\simeq \frac{3H^2}{8\pi Gs^4}\left\{69s^3\left[\left(s - \frac{1}{23}\right)^2 + \frac{22}{529}\right] - 3s^2 + 1\right\}. \tag{46}$$

However, concerning the thermal fluctuations, the calculations are similar to the single-screen scenario, since the mechanisms producing the fluctuations are independent of the background. Therefore, in the next subsection we study the conditions with both kinds of fluctuation productions.

1. Normal radiation

For standard radiation, the energy density writes as

$$\rho_{\rm tr} = \frac{3g_r}{16\pi}T^4, \tag{47}$$

[4] This β value ensures that at late times, when the Friedmann equations become those of general relativity, that is $H^2 = 8\pi G\rho/3$, the temperatures of the two screens will become equal, and thus according to (36) the acceleration of the Universe caused by entropic force will vanish and inflation will exit [22]. This provides an exiting mechanism of inflation that is absent in the single-screen model.

where g_r corresponds to the term $64\pi\sigma\mathcal{G}(T)/3$ of the single-screen model. Therefore, the heat capacity $C_V = g_r R_{\rm tr}^3 T^3$. Setting $R_{\rm tr} = c_s/H$ and $T = \beta H/2\pi$ we have

$$\delta_t\rho|_{\rm tr} = \sqrt{\frac{g_r\beta^5}{(2\pi)^5 c_s^3}}H^4. \quad (48)$$

Thus, comparing (46) and (48) we deduce that the condition for eternal inflation reads

$$\sqrt{\frac{g_r\beta^5}{(2\pi)^5 c_s^3}}H^4 > \frac{3H^2}{8\pi G s^4}\left\{69s^3\left[\left(s-\frac{1}{23}\right)^2+\frac{22}{529}\right]-3s^2+1\right\}.$$

This expression contains the higher order term of H and $s = H\sqrt{\bar{g}G/(23\pi)}$, and thus since $s \gg 1$, it can be simplified to give a constraint on $H(t)$ as

$$H > 9\pi\sqrt{\frac{23\bar{g}c_s^3}{2g_r\beta^5 G}}. \quad (49)$$

Since from this relation one can see that $H^2 > 9\pi\sqrt{(23\bar{g}c_s^3)/(2g_r\beta^5 G)}H \sim s/G \gg G^{-1}$ and thus $H \gg 1/\sqrt{G}$, eternal inflation can only be realized at a very high-energy regime.

2. Holographic screen

Concerning the holographic production of fluctuations, we can consider that it can mainly be produced on the outer screen [23]. Similar to the single-screen model, the total energy of the screen is $\langle E\rangle \sim R_{\rm hs}^2 T/G$, the heat capacity $C_V \sim c_v R_{\rm hs}^2/G$, and thus $\delta_t\rho|_{\rm hs} = T\sqrt{c_v/G}/R_{hs}^2$. Setting $R_{\rm hs} \simeq 1/H$ and $T = \beta H/2\pi$ (this β modification is the only difference between the two models) we finally obtain

$$\delta_t\rho|_{\rm hs} = \sqrt{\frac{c_v}{G}}\frac{\beta^3 H^3}{2\pi}. \quad (50)$$

From (46) and (50) we can see that the eternality condition (18) turns out to be

$$\sqrt{\frac{c_v}{G}}\frac{\beta^3 H^3}{2\pi} > \frac{3H^2}{8\pi G s^4}\left\{69s^3\left[\left(s-\frac{1}{23}\right)^2+\frac{22}{529}\right]-3s^2+1\right\}, \quad (51)$$

and in the regime $s \gg 1$, it can be simplified as

$$\left(\frac{4\beta^3\sqrt{c_v G}}{3} - 69\sqrt{\frac{\bar{g}}{23\pi}}\right)\sqrt{\frac{\bar{g}}{23\pi}}H^2 + 6\sqrt{\frac{\bar{g}}{23\pi}}H - 3 > 0. \quad (52)$$

For standard values of the parameters [$\bar{g} \gg G$, c_v, $\beta \approx \mathcal{O}(1)$] the expression on the left-hand side does not accept nontrivial real roots, and thus condition (52) is never satisfied. Therefore, we conclude that in the double-screen inflation model with the thermal fluctuations from the holographic screen, the background result is not changed.

Finally, if we desire to take into account simultaneously the thermal fluctuations from standard radiation and from the holographic screen, that is requiring $\delta_t\rho|_{\rm tr} + \delta_t\rho|_{\rm hs} > \delta_c\rho$, and under the approximation $s \gg 1$, we return again to (49) since the holographic fluctuation $\delta_t\rho|_{\rm hs}$ cannot have any contributions on eternal inflation. Thus, this is the total condition for eternal inflation, taking into account the thermal effect of fluctuations.

C. Effects from quantum corrections: Langevin analysis

In the previous subsections we extracted the requirement for eternal inflation in double-screen scenarios with both types of thermal-fluctuation productions. Similar to the single-screen case, in the present subsection we incorporate the microfluctuations, and we examine whether they can prevent eternal inflation. In particular, we perform a Langevin analysis in order to formulate the overall cosmological evolution, that is, the classical motion and the quantum fluctuations, as a stochastic effect modeled through a random walk.

According to Eq. (43) we can construct the corresponding Langevin equation as

$$\dot{H} = C_1 H + C_2 H^2 + C_3 H^3 + C_4 H^4 + C_7 H^7 + q_d n(t), \quad (53)$$

where $q_d \sim \varepsilon G^{-3/4}$, $n(t)$ satisfies (28), and the parameters $C_i (i = 1, 2, 3, 4, 7)$ are

$$C_1 = -\sqrt{\frac{23\pi}{\bar{g}G}}, \quad C_2 = \beta^2 - 1, \quad C_3 = -\frac{\beta^4}{2}\sqrt{\frac{\bar{g}G}{23\pi}},$$
$$C_4 = \frac{g_H\beta^4 G}{4\pi}, \quad C_7 = -\frac{g_S\beta^{10}}{\pi^3}\frac{\bar{g}}{184}\sqrt{\frac{\pi\bar{g}}{23}}G^{(5/2)}, \quad (54)$$

with dimensions of 1, 0, −1, −2, and −5, respectively.

Now, imposing the expansion solution (30) (with $q_s \to q_d$), we obtain the separate equations

$$\dot{H}_c = C_1 H_c + C_2 H_c^2 + C_3 H_c^3 + C_4 H_c^4 + C_7 H_c^7, \quad (55)$$

$$\dot{H}_1 = C_1 H_1 + 2C_2 H_1 H_c + 3C_3 H_1 H_c^2 + 4C_4 H_1 H_c^3 + 7C_7 H_1 H_c^6 + G^{-(3/4)}n, \quad (56)$$

$$\dot{H}_2 = C_1 H_2 + C_2(H_1^2 + 2H_2 H_c) + 3C_3 H_c(H_1^2 + H_2 H_c) + 2C_4 H_c^2(3H_1^2 + 2H_2 H_c) + 7C_7 H_c^5(3H_1^2 + H_2 H_c). \quad (57)$$

These equations cannot accept analytical solutions, however, we can extract approximate solutions in the limit $|t/\mathcal{T}| \ll 1$ with $\mathcal{T} \gg H^{-1}$, that is, performing our calculation within one Hubble interval ($t \sim H^{-1}$) [35], and then we impose the requirement stochastic fluctuations to be able to stop an eternal inflation. The explicit calculations are presented in Appendix B. We find that although it is very complicated to perform a general analysis due to high nonlinearity of the equations, in the specific parameter choice of [23] it is still hard for the microfluctuations to have a significant effect on preventing inflation from its eternality.

In summary, in this section we extracted the eternality conditions for inflation in double-screen entropic scenarios, with thermal fluctuations induced by normal radiation and holographic production. As we saw, inflation is eternal in the high-energy regime. Furthermore, we considered the quantum corrections from stochastic effects by performing the Langevin analysis, which were shown to hardly prevent the inflation eternality. On the other hand, in the low-energy regime, inflation exits globally [23].

V. CONCLUSIONS

In this work, we examined entropic cosmological scenarios under the light of inflation eternality. Going beyond Verlinde's basic idea, which cannot describe inflation quantitatively, we considered both the single-screen entropic inflation model [21], in which the holographic screen acts as an additional boundary term, as well as its double-screen extension [22,23], where one adds a second, inner holographic screen. In particular, after describing the inflation realization, we examined under what conditions the inflation is eternal, taking into account both the background evolution and mechanisms of thermal-fluctuation production. Furthermore, we incorporated quantum fluctuations through a phenomenological, stochastic, Langevin analysis, and we examined whether they can prevent the inflation from eternality. Although the Langevin analysis is not a full incorporation of quantum fluctuations, which is a relatively unknown subject, it can provide phenomenologically trustworthy results.

After describing the inflation realization and the normal condition for inflation to last at the background level, we discussed its eternality at the thermal fluctuations level. First, we defined the general condition for eternal inflation as $\delta_t\rho > \delta_c\rho$, which is very natural compared to the well-known relation of scalar-field-driven inflation. In the case of one-screen scenarios, we extracted the conditions for eternal inflation with fluctuations from normal radiation and the holographic screen, (24) and (26), respectively, which should be considered with (17). These eternality conditions can be easily fulfilled for general parameter choices. In the case of the double-screen model, we found that in the high-energy region when $\rho \gg \bar{\rho}$ and inflation is always going on, the condition for eternal inflation with radiation fluctuation remains (49), while for a fluctuation produced by the holographic screen, eternal inflation is not affected. Finally, after performing a Langevin analysis, we found that it might be hard in either of these two scenarios to prevent eternal inflation by stochastic effects, unless we severely fine-tune the parameters. As a side remark, however, we recognize that since the Langevin analysis is still a random walk simulation instead of a complete calculation of the quantum fluctuations, there could still be some ambiguity in the validity of this result, which deserves more study.

The above discussion indicates that from the cosmological point of view one must be careful with the entropic origin of gravity, since the induced inflationary dynamics must have an exit, at least at some parts of the Universe, in order to be consistent with observations. It seems that one-screen scenarios can easily lead to inflation eternality, while double-screen considerations present a better behavior. Finally, note that apart from these two models, in [36] the authors discussed the case where the holographic screen is open and the Brown-York surface stress tensor is introduced, while in [37] noncommutative geometry was used to describe the microstructure of the quantum spacetime of entropic gravity. It would be interesting to investigate the inflation realization in such scenarios, and then focusing on the eternality issues, examining whether the above behaviors can be improved.

However, the easy realization of eternal inflation in entropic gravity could still be very interesting from the string theory point of view, since it may form the background for the landscape of string or M theory vacua, that is, for the existence of a huge number of possible (false) vacua [11–14]. Additionally, it may also be worth investigating the probability of tunneling between different vacua, either Coleman-De Luccia–type [38] or Hawking-Moss–type [39]. In order to study these subjects, the full extension of our Langevin analysis will become important. Such considerations may lead to new perspectives in entropy gravity, and deserve further investigation.

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APPENDIX A: SOLUTION OF THE LANGEVIN EQUATIONS: THE SINGLE-SCREEN SCENARIO

Since we are dealing with stochastic variables, we perform the average of any physical quantity by defining the statistical measure. In particular, we use the Fokker-Planck approach and define the measure to be the physical volume of the Hubble patch, and thus the average is defined as

$$\langle H(t)\rangle_p = \frac{\langle H(t)e^{3N(t)}\rangle}{\langle e^{3N(t)}\rangle}, \qquad N(t) = \int_0^t H(t')dt'.$$

Since the Hubble patch that is eternally inflating will have an exponentially larger physical volume, taking the largest weight in the average at late times, the physical volume can be a good measure to characterize eternal inflation. Therefore, the average $\langle H(t)\rangle_p$ could be significantly changed by stochastic fluctuations if eternal inflation is realized. Furthermore, we shall use the functional technique developed in [35] and define a generating functional

$$W_t[\mu] = \ln\langle e^{M_t[\mu]}\rangle, \qquad M_t[\mu] = \int_0^t \mu(t')H(t')dt'.$$

Thus, $\langle H(t)\rangle_p$ can be evaluated by functionally differentiating $W_t[\mu]$ with respect to μ and setting $\mu = 3$, resulting to the following equations up to $\mathcal{O}(q_s^2)$:

$$\langle H(t)\rangle_p = \frac{\delta W_t[\mu]}{\delta\mu(t)}\bigg|_{\mu(t)=3} = \langle H(t)\rangle + 3\int_0^t \langle\langle H(t)H(t')\rangle\rangle dt', \tag{A1}$$

$$\langle\langle H(t)H(t')\rangle\rangle = \langle H(t)H(t')\rangle - \langle H(t)\rangle_p\langle H(t')\rangle_p. \tag{A2}$$

After these definitions we can proceed to the solution of the Langevin equations (31) and (33).

In general, these equations cannot accept analytical solutions, however we can obtain approximate solutions in the limit $|t/\mathcal{T}| \ll 1$ (with $\mathcal{T} \gg H^{-1}$), that is performing our calculations within one Hubble interval ($t \sim H^{-1}$), where $t = 0$ is the time where inflation starts. In this case we can assume an ansatz for the $H_c(t)$ solution, namely,

$$H_c(t) = H_{t0} + \tilde{\Lambda}_1\left(\frac{t}{\mathcal{T}}\right) + \tilde{\Lambda}_2\left(\frac{t}{\mathcal{T}}\right)^2 \ldots, \tag{A3}$$

where $H_{t0} \equiv H(t = 0)$. Inserting this ansatz into Eq. (31) and by rescaling $\Lambda_1 = \tilde{\Lambda}_1\mathcal{T}^{-1}$, $\Lambda_2 = \tilde{\Lambda}_2\mathcal{T}^{-2}$, we obtain

$$H_c(t) = H_{t0} + \Lambda_1 t + \Lambda_2 t^2 \ldots, \tag{A4}$$

with

$$\begin{aligned} H_{t0} &= \left(\frac{-15 \pm \sqrt{57}}{28}\frac{D_1}{D_2}\right)^{(1/2)}, \\ \Lambda_1 &= D_1 H_{t0}^2 + D_2 H_{t0}^4, \\ \Lambda_2 &= (D_1 H_{t0} + 2D_2 H_{t0}^3)\Lambda_1, \end{aligned} \tag{A5}$$

where we recall that $D_1 \equiv -\frac{2(1-C_H)}{1-2C_{\dot{H}}} < 0$ and $D_2 \equiv \frac{gG}{\pi(1-2C_{\dot{H}})} > 0$. Note that $\tilde{\Lambda}_{1,2}$ have dimensions of 1, while Λ_1 and Λ_2 have dimensions of 2 and 3, consistently with the dimensions of D_1 and D_2 as stressed before. One could also check that this leads to $\Lambda_1 < 0$, which is asymptotically quintessencelike inflation.

Thus, knowing the solution $H_c(t)$, we can acquire solutions of $H_1(t)$ and $H_2(t)$ as

$$H_1(t) = h_{1i}E(t)\left\{1 + \frac{1}{G^{3/4}h_{1i}}\int_0^t n(t')E^{-1}(t')dt'\right\}, \tag{A6}$$

$$H_2(t) = E(t)\int_0^t \Delta(t)H_1^2 E^{-1}(t')dt', \tag{A7}$$

where for convenience we have defined $E(t) \equiv e^{\int_0^t(2D_1H_c+4D_2H_c^3)dt'}$ and $\Delta(t) \equiv D_1 + 6D_2H_c^2$. Note that we have taken the initial condition for the first-order correction of the Hubble parameter to be h_{1i}, while initial conditions for higher order corrections have been neglected.

Now, from (30) we can straightforwardly write $\langle H(t)\rangle = H_c(t) + q_s G^{(3/4)}\langle H_1(t)\rangle + q_s^2 G^{(3/2)}\langle H_2(t)\rangle$, and therefore (A1) gives

$$\begin{aligned} \langle H(t)\rangle_p &= H_c(t) + q_s G^{(3/4)}\langle H_1(t)\rangle + q_s^2 G^{(3/2)}\langle H_2(t)\rangle \\ &\quad + 3q_s^2 G^{(3/2)}\int_0^t [\langle H_1(t)H_1(t')\rangle - \langle H_1(t)\rangle\langle H_1(t')\rangle]dt' \\ &= H_{t0} + \Lambda_1 t + q_s G^{(3/4)}h_{1i}E(t) + q_s^2 G^{(3/2)} \\ &\quad \times\left\{\langle H_2(t)\rangle + 3\int_0^t [\langle H_1(t)H_1(t')\rangle \right. \\ &\quad \left. - \langle H_1(t)\rangle\langle H_1(t')\rangle]dt'\right\}, \end{aligned} \tag{A8}$$

where we have used the fact that $\langle H_1(t)\rangle = h_{1i}E(t)$ and that

$$\begin{aligned} \langle H(t)\rangle\langle H(t')\rangle &= H_c(t)H_c(t') + q_s G^{(3/4)}[H_c(t)\langle H_1(t')\rangle \\ &\quad + H_c(t')\langle H_1(t)\rangle] + q_s^2 G^{(3/2)}[\langle H_1(t)\rangle \\ &\quad \times\langle H_1(t')\rangle + H_c(t)\langle H_2(t')\rangle + H_c(t') \\ &\quad \times\langle H_2(t)\rangle]. \end{aligned}$$

The last terms on the right-hand side of (A8) can be expressed using

$$\begin{aligned} \langle H(t)H(t')\rangle &= H_c(t)H_c(t') + q_s G^{(3/4)}[H_c(t)\langle H_1(t')\rangle \\ &\quad + H_c(t')\langle H_1(t)\rangle] + q_s^2 G^{(3/2)}[\langle H_1(t)H_1(t')\rangle \\ &\quad + H_c(t)\langle H_2(t')\rangle + H_c(t')\langle H_2(t)\rangle], \end{aligned}$$

and then, using relations (A6), (A7), and (28), we obtain

$$\begin{aligned} \langle H_1(t)H_1(t')\rangle &= h_{1i}^2 E(t)E(t')\left[1 + \frac{1}{G^{3/2}h_{1i}^2}\int_0^{\min(t,t')} E^{-2}(t_1)dt_1\right], \\ \langle H_2(t)\rangle &= E(t)\int_0^t \Delta(t_1)E(t_1) \\ &\quad \times\left[h_{1i}^2 + G^{-(3/2)}\int_0^{t_1} E^{-2}(t_2)dt_2\right]dt_1. \end{aligned} \tag{A9}$$

Thus, inserting (A9) into (A8) and keeping terms up to leading order in $E(t)$ and H_c we acquire

$$\begin{aligned} \langle H(t)\rangle_p &= H_{t0} + q_s G^{(3/4)}h_{1i} + \Lambda_1 t + 2q_s G^{(3/4)}h_{1i}H_{t0} \\ &\quad \times(D_1 + 2D_2H_{t0}^2)t + q_s^2 G^{(3/2)}h_{1i}^2(D_1 + 6D_2H_{t0}^2)t. \end{aligned}$$

This expression provides the stochastic-fluctuation corrected Hubble-parameter evolution, namely, the first two terms on the right-hand side give the classical result, while the last term provides the stochastic correction. Therefore, requiring the stochastic-fluctuations to be able to prevent eternal inflation, we need to impose

$$2q_sG^{(3/4)}h_{1i}H_{t0}(D_1+2D_2H_{t0}^2)+q_s^2G^{(3/2)}h_{1i}^2(D_1+6D_2H_{t0}^2) \lesssim \Lambda_1, \tag{A10}$$

where Λ_1 is given in (A5). This expression provides the constraints on the initial condition of stochastic fluctuations in leading order, namely, h_{1i}, as

$$r_1^s \lesssim h_{1i} \lesssim r_2^s, \tag{A11}$$

where

$$r_{1,2}^s = -\frac{H_{t0}}{q_sG^{(3/4)}} \times \frac{(D_1+2D_2H_{t0}^2) \pm \sqrt{2D_1^2+11D_1D_2H_{t0}^2+10D_2^2H_{t0}^4}}{D_1+6D_2H_{t0}^2}.$$

Inserting in these expressions the definitions of D_1, D_2, and q_s as functions of the parameters C_H and $C_{\dot{H}}$ [see Eq. (29)], we find that in the required intervals $C_H < 1$ and $0 \leq C_{\dot{H}} \leq \frac{3}{4\pi}$ [21] there are no real solutions. Thus, condition (A10) cannot be satisfied and therefore stochastic effects from quantum fluctuations cannot stop eternal inflation. We mention here that performing the calculations within one Hubble interval is a reasonable approximation, since the above result will still be valid if one considers successively many Hubble intervals.

APPENDIX B: SOLUTION OF THE LANGEVIN EQUATIONS: THE DOUBLE-SCREEN SCENARIO

In this appendix we solve Eqs. (55)–(57), using also the introductory relations of Appendix A. In general, these equations cannot accept analytical solutions; however, we can acquire approximate solutions in the limit $|t/\mathcal{T}| \ll 1$ (with $\mathcal{T} \gg H^{-1}$). Assuming an ansatz for the $H_c(t)$ solution as

$$\begin{aligned} H_c(t) &= H_{t0} + \tilde{\Sigma}_1\left(\frac{t}{\mathcal{T}}\right) + \tilde{\Sigma}_2\left(\frac{t}{\mathcal{T}}\right)^2 \ldots, \\ &= H_{t0} + \Sigma_1 t + \Sigma_2 t^2 \ldots, \end{aligned} \tag{B1}$$

where in the second step we use the similar rescaling as was done in Appendix A, with $\tilde{\Sigma}_{1,2}$ of dimension 1 while Σ_1 and Σ_2 of dimensions 2 and 3, respectively, consistently with the dimensions of C_i's stressed before. Substituting it into (55) we obtain

$$\begin{aligned} \Sigma_1 &= C_1H_{t0} + C_2H_{t0}^2 + C_3H_{t0}^3 + C_4H_{t0}^4 + C_7H_{t0}^7, \\ \Sigma_2 &= \tfrac{1}{2}(C_1 + 2C_2H_{t0} + 3C_3H_{t0}^2 + 4C_4H_{t0}^3 + 7C_7H_{t0}^6)\Sigma_1. \end{aligned} \tag{B2}$$

Thus, the solutions for H_1 and H_2 read

$$H_1(t) = h_{1i}e^{\int_0^t \Xi_1(t')dt'}\left\{1+\frac{1}{G^{3/4}h_{1i}}\int_0^t n(t')e^{-\int_0^{t'}\Xi_1(t'')dt''}dt'\right\}, \tag{B3}$$

$$H_2(t) = e^{\int_0^t \Xi_2(t')dt'}\int_0^t \Pi(t')H_1^2 e^{-\int_0^{t'}\Xi_2(t'')dt''}dt', \tag{B4}$$

where

$$\begin{aligned} \Xi_1(t) &= C_1 + 2C_2H_c + 3C_3H_c^2 + 4C_4H_c^3 + 7C_7H_c^6, \\ \Xi_2(t) &= C_1 + 2C_2H_c + 3C_3H_c^2 + 4C_4H_c^3 + 7C_7H_c^6, \\ \Pi(t) &= C_2 + 3C_3H_c + 6C_4H_c^2 + 21C_7H_c^5. \end{aligned}$$

Following the procedure of Appendix A we finally obtain

$$\begin{aligned} \langle H(t)\rangle_p &= H_{t0} + \Sigma_1 t + q_dG^{(3/4)}\langle H_1(t)\rangle \\ &\quad + q_d^2G^{(3/2)}\left\{\langle H_2(t)\rangle + 3\int[\langle H_1(t)H_1(t')\rangle - \langle H_1(t)\rangle\langle H_1(t')\rangle]dt'\right\} \\ &= H_{t0} + q_dG^{(3/4)}h_{2i} + \Sigma_1 t + q_dG^{(3/4)}h_{2i} \\ &\quad \times(C_1+2C_2H_{t0}+3C_3H_{t0}^2+4C_4H_{t0}^3+7C_7H_{t0}^6)t \\ &\quad + q_d^2G^{(3/2)}h_{2i}^2(C_2+3C_3H_{t0}+6C_4H_{t0}^2+21C_7H_{t0}^5)t. \end{aligned}$$

Therefore, requiring the stochastic fluctuations to be able to prevent the eternal inflation, we result to the constraint:

$$\begin{aligned} \Sigma_1 &> q_dG^{(3/4)}h_{2i}(C_1 + 2C_2H_{t0} + 3C_3H_{t0}^2 + 4C_4H_{t0}^3 \\ &\quad + 7C_7H_{t0}^6) + q_d^2G^{(3/2)}h_{2i}^2(C_2 + 3C_3H_{t0} + 6C_4H_{t0}^2 \\ &\quad + 21C_7H_{t0}^5). \end{aligned}$$

Inserting the definition of Σ's from (B2) and of C_i's from (54), we find that

$$r_1^d \lesssim h_{2i} \lesssim r_2^d, \tag{B5}$$

where

$$r_{1,2}^d = -\frac{X_0 \pm \sqrt{X_0^2 + 4P_0\Sigma_1}}{2q_dG^{3/4}P_0},$$

with

$$\begin{aligned} X_0 &= C_1 + 2C_2H_{t0} + 3C_3H_{t0}^2 + 4C_4H_{t0}^3 + 7C_7H_{t0}^6, \\ P_0 &= C_2 + 3C_3H_{t0} + 6C_4H_{t0}^2 + 21C_7H_{t0}^5. \end{aligned}$$

Relation (B5) imposes tight constraints on the model parameters. For example, in the particular parameter choice $\beta = \sqrt{2}$, $g_s = 0$, $g = g_H = 10^{16}$ of [22] it does not accept real solutions. Therefore, we conclude that on the double-screen model at hand, the stochastic effects cannot induce an exit from an eternal inflation caused by the background evolution and thermal fluctuations, unless one tunes the model parameters accordingly.

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