



Near threshold proton–proton fusion in effective field theory

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ABSTRACT

The astrophysical S -factor for proton–proton fusion, $S_{11}(E)$, is obtained with the nuclear matrix element analytically calculated in pionless effective field theory. To the third order, the zero-energy result $S_{11}(0)$ and the first energy derivative $S'_{11}(0)$ are found to be $(3.99 \pm 0.14) \times 10^{-25}$ MeVb and $S_{11}(0)(11.3 \pm 0.1)$ MeV⁻¹, respectively; both consistent with the current adopted values. The second energy derivative is also calculated for the first time, and the result $S''_{11}(0) = S_{11}(0)(170 \pm 2)$ MeV⁻² contributes at the level of 0.5% to the fusion rate at the solar center, which is smaller than 1% as previously estimated.

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1. Introduction

As the trigger of stellar hydrogen burning, the proton–proton (pp) fusion reaction, $p + p \rightarrow d + e^+ + \nu_e$, plays a fundamental role in astrophysics. Its reaction rate, conventionally encoded by the astrophysical S -factor $S_{11}(E)$, with E the energy in the center of mass frame, is thus an important input in studies such as stellar evolution and solar neutrinos. For such a pivotal reaction, the value of $S_{11}(E)$ at the center of the Sun (with temperature $\sim 1.55 \times 10^7$ K which yields $E \sim$ a few keV), unfortunately cannot be determined or reliably extrapolated by terrestrial experiments. This is because they have to be performed at much higher energies to overcome the Coulomb barrier and get sensible statistics. Therefore, one has to rely on theory for predictions.

Ever since the first proposal of the pp chains by Bethe and Critchfield [1], the nuclear transition amplitude of the pp fusion, together with other solar fusion cross sections have been extensively studied. Those results were extensively reviewed first in Ref. [2] (SFI) and then most recently in Ref. [3] (SFII). For solar fusion, the temperature is much lower than typical energy scales in nuclear physics, thus only the first few terms in the expansion of $S_{11}(E)$ around $E = 0$ is needed for solar models. Currently, the recommended value of $S_{11}(0)$ in SFII has an error of 1%, which results in a 1% error in the pp fusion rate. The recommended value of $S'_{11}(0)$ also has an error of 1%, which only results in a less than 0.1% error in the fusion rate. $S''_{11}(0)$ was not given in SFII. However, Bahcall and May [4] estimated its con-

tribution to the rate to be $\sim 1\%$, comparable to the overall error in S_{11} . Thus, SFII recommended a modern calculation of $S''_{11}(0)$ be undertaken.

In this work, we calculate $S_{11}(E)$ using the nuclear effective field theory (EFT) with pions integrated out. One of the major ingredients, the square of the orbital matrix element $\Lambda^2(E)$ which $S_{11}(E)$ is proportional to, is extracted from the cross section of an analogue process: $\nu_e + d \rightarrow p + p + e^-$, analytically computed in Ref. [5].

This pionless EFT is applicable for low energy processes with the characteristic momentum p much smaller than the pion mass m_π [6–8], which is the case for solar pp fusion. In this theory, pions are integrated out. All the nucleon–nucleon interactions and two-body currents are described by point-like contact interactions with a systematic expansion in powers of p/m_π . A close analogy of this theory is the Fermi theory of four fermion contact interactions. The one- and two-body contributions both depend on the momentum cut-off but the sum does not. In pionless EFT, there is only one two-body current (with coupling $L_{1,A}$) in every weak interaction deuteron breakup process to next-to-next-to-leading (NNLO) in the p/m_π expansion [5]. This two-body current is a Gamow–Teller operator. The other two-body currents are either missing due to vector current conservation or the matrix elements are suppressed because of the orthogonality of the initial and final state wave functions in the zero recoil limit. This means the universal number $L_{1,A}$ encodes the two-body contributions for all low energy weak deuteron breakup processes, and it takes just one measurement to calibrate all the processes. This feature is also seen in the other complimentary approaches to the solar fusion such as potential models [9], hybrid version of EFT [10], and pionless EFT with dibaryon [11] (see SFII for more details).

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2. $S_{11}(E)$ from pionless EFT

The astrophysical S-factor, $S(E)$, for a nuclear reaction at kinetic energy E is related to the reaction cross section by

$$\sigma(E) = \frac{S(E)}{E} e^{-2\pi\eta(E)}. \quad (1)$$

The rapid-varying energy dependence of $\sigma(E)$ due to the Coulomb barrier is mostly accounted for in the exponential term which is controlled by the Sommerfeld parameter

$$\eta(E) = \frac{\alpha}{2} \sqrt{\frac{m_p}{E}}, \quad (2)$$

where $\alpha = 1/137.036$ is the fine structure constant and $m_p = 938.272$ MeV is the proton mass.¹

Using the same convention and inputs as SFII, we write the S-factor for the pp fusion as

$$S_{11}(E) = 6\pi^2 m_p \alpha \ln 2 \frac{\Lambda^2(E)}{\gamma^3} \left(\frac{g_A}{g_V}\right)^2 \frac{f_{pp}^R(E)}{(ft)_{0^+ \rightarrow 0^+}}, \quad (3)$$

where $\gamma = \sqrt{2\mu_{np}E_d} = 45.70$ MeV ($\mu_{np} = 469.459$ MeV is the proton-neutron reduced mass and $E_d = 2.224573$ MeV is the deuteron binding energy) is the deuteron binding momentum; $g_V = 1$ and $g_A = 1.2695$ are the usual Fermi and axial-vector coupling constants, respectively; and $(ft)_{0^+ \rightarrow 0^+} = 3071$ s is the ft value for superallowed Fermi transitions. The energy dependence of $S_{11}(E)$ is determined by two terms: (i) $\Lambda(E)$, the orbital matrix element, which is proportional to the nuclear transition matrix element, and (ii) $f_{pp}^R(E)$, the phase space factor in this nuclear β^+ process. We note the conventional way of separating the radiative correction in $S_{11}(E)$: the long-distance (so-called ‘‘outer’’) part, which is process-dependent, is included in the phase factor $f_{pp}^R(E)$ (annotated by a superscript ‘‘R’’), while the short distance (so-called ‘‘inner’’) part, which is process-independent, is taken into account by the product of $(g_A/g_V)^2$ and $1/(ft)_{0^+ \rightarrow 0^+}$. For a detailed account, see Ref. [12].

2.1. Orbital matrix element $\Lambda(E)$

At very low energy, the pp fusion predominantly goes from the 1S_0 partial wave state to the deuteron state (3S_1 with some 3D_1 mixture) through the spatial axial current operator. In pionless EFT, it takes the form [13]

$$A_k^- = \frac{g_A}{2} N^\dagger \tau^- \sigma_k N + L_{1,A} [(N^T P_k N)^\dagger (N^T \bar{P}^- N) + \text{h.c.}] + \dots, \quad (4)$$

where N denotes the nucleon field; $\tau^- \equiv (\tau_1 - i\tau_2)$; $\bar{P}^- \equiv \bar{\psi} - \bar{\psi}$; $P_k \equiv \tau_2 \sigma_2 \sigma_k / \sqrt{8}$; $\bar{P}^- \equiv \tau_2 \tau^- \sigma_2 / \sqrt{8}$; all σ 's and τ 's are the Pauli matrices for spin and isospin, respectively. The coupling constant $L_{1,A}$ of the leading axial two-body current appears at next-to-leading order (NLO) and there is no new two-body current contributing until next-to-next-to-next-to-leading order (N^3 LO). The orbital matrix element $\Lambda(p)$, with $p = \sqrt{m_p E}$, is then related to the nuclear matrix elements of A_k^- by

$$|\langle d; j | A_k^- | pp \rangle| = g_A C_\eta(p) \sqrt{\frac{32\pi}{\gamma^3}} \Lambda(p) \delta_k^j, \quad (5)$$

where j denotes the deuteron $|d\rangle$ polarization and

$$C_\eta^2(p) = \frac{2\pi\eta(p)}{e^{2\pi\eta(p)} - 1}, \quad (6)$$

is the square of the Sommerfeld factor. The effect of the deuteron recoil, with momentum $q \lesssim 0.4$ MeV, is suppressed by a factor $q^2/\gamma^2 < 10^{-4}$ and hence can be neglected. In the zero recoil limit the vector matrix element between pp and d vanishes because the wave functions are orthogonal.

In Ref. [5], various (anti)neutrino deuteron breakup processes were computed in pionless EFT up to N^2 LO with analytic expressions. The needed hadronic matrix element for the pp fusion can be extracted from the result of the similar process $\nu_e + d \rightarrow p + p + e^-$. The square of the orbital matrix element can be simplified and expressed as

$$\Lambda^2(p) = \frac{\pi\gamma^3}{2m_p p C_\eta^2} \frac{1}{1 - \epsilon\gamma\rho_d} [F_1'(p) + F_4'(p) + G_2'(p)] + \mathcal{O}(\epsilon^3), \quad (7a)$$

where $\rho_d = 1.764$ fm is the effective range parameter for deuteron, and the corresponding energy-dependent structure functions are

$$F_1'(p) = \frac{2m_p\gamma p}{\pi(p^2 + \gamma^2)^2} C_\eta^2(p) e^{4\eta(p)\tan^{-1}(\frac{p}{\gamma})}, \quad (7b)$$

$$F_4'(p) = \frac{1}{\pi} \text{Im}[B_0'^2(p)(A_{-1}(p) + \epsilon A_0(p) + \epsilon^2 A_1(p))], \quad (7c)$$

$$G_2'(p) = -\epsilon \frac{m_p}{\pi} \text{Im}\left[\sqrt{\frac{2\gamma}{\pi}} B_0'(p) \tilde{L}'_{1,A}(A_{-1}(p) + \epsilon A_0(p))\right] + \epsilon^2 \frac{m_p^2 \gamma}{2\pi^2} \text{Im}[\tilde{L}'_{1,A} A_{-1}(p)]. \quad (7d)$$

The complex integral

$$B_0'(p) = m_p \int \frac{d^3k}{(2\pi)^3} \frac{\sqrt{8\pi\gamma}}{k^2 + \gamma^2} C_\eta^2(k) \frac{e^{2\eta(k)\tan^{-1}(\frac{k}{\gamma})}}{p^2 - k^2 + i\delta} \Big|_{\delta \rightarrow 0}. \quad (8)$$

The p^i components of the pp scattering amplitude in the 1S_0 channel, $A_i(p)$, are found to be

$$\begin{aligned} A_{-1}(p) &= -\frac{4\pi}{m_p} \frac{1}{[1/a + \alpha m_p H(\eta)]}, \\ A_0(p) &= -\frac{2\pi}{m_p} \frac{r_0 p^2}{[1/a + \alpha m_p H(\eta)]^2}, \\ A_1(p) &= -\frac{\pi}{m_p} \frac{r_0^2 p^4}{[1/a + \alpha m_p H(\eta)]^3}, \end{aligned} \quad (9)$$

where $a = -7.82$ fm and $r_0 = 2.79$ fm are the scattering length and effective range parameter for 1S_0 , respectively (for consistency, we adopted the values extracted in Refs. [5,14]); and the function

$$H(\eta) \equiv \frac{\partial \ln \Gamma(i\eta)}{\partial(i\eta)} + \frac{1}{2i\eta} - \ln(i\eta), \quad (10)$$

depends on momentum via the Sommerfeld parameter $\eta(p)$. The renormalization scale μ -independent coupling constants $\tilde{L}'_{1,A}$ is defined as

$$\tilde{L}'_{1,A} = -\frac{(\mu - \gamma)}{m_p C_{0,-1}} \left[\frac{L_{1,A}}{g_A} - \pi \left(\frac{m_p}{2\pi} C_{2,-2} + \frac{\rho_d}{(\mu - \gamma)^2} \right) \right], \quad (11)$$

and when being evaluated at the pion-mass scale, $\mu = m_\pi \approx 140$ MeV, the low energy constants $C_{0,-1} = -3.77$ fm² and $C_{2,-2} = 7.50$ fm⁴.²

² As we take the $q^2/\gamma^2 \rightarrow 0$ limit in F_1 , F_4 , and B_0 , and redefine G_2 and $\tilde{L}'_{1,A}$ by some normalization factors, we add a prime to remind readers about these changes from Ref. [5].

¹ We use the units $\hbar = c = 1$.

Note that we keep the small expansion parameter ϵ explicit in Eqs. (7a)–(7d) to make the order of each term transparent. For evaluation, one has to make a series expansion in ϵ up to the maximum order at which the result is valid, and then set $\epsilon = 1$ in the end.

At zero energy, our result of $\Lambda^2(0)$ can be written in a compact form

$$\Lambda^2(0) = \frac{1}{1 - \epsilon\gamma\rho_d} [e^\chi - \gamma a(1 - \chi e^\chi E_1(\chi)) - \epsilon\gamma^2 a \tilde{L}'_{1,A}]^2 + \mathcal{O}(\epsilon^3), \quad (12)$$

where the function

$$E_1(\chi) = \int_\chi^\infty dt \frac{e^{-t}}{t} \quad (13)$$

yields a value of 1.4655 with $\chi \equiv \alpha m_p/\gamma = 0.1498$. Up to the order of ϵ^2 , this result is in agreement with Ref. [13] in which $\Lambda(0)$ is worked out to the order of ϵ^4 ; and the constant $\tilde{L}'_{1,A}$ defined therein is exactly the same as our $\tilde{L}'_{1,A}$. This result is also consistent with Ref. [15] by ignoring the $\tilde{L}'_{1,A}$ term, and Ref. [11] by a redefinition of $\tilde{L}'_{1,A}$.

2.2. Phase space factor $f_{pp}^R(E)$

The phase space factor $f(E)$ in a nuclear β process is conventionally written as (see, e.g., Ref. [16])

$$f(E) = \frac{1}{m_e^5} \int_{m_e}^{Q+E} dW F(\pm Z, W) p_e W (Q + E - W)^2, \quad (14)$$

where $m_e = 0.511$ MeV is the electron (and positron) mass; W and $p_e = \sqrt{W^2 - m_e^2}$ are the relativistic energy and momentum of the emitted electron or positron; Z is the charge of the final nuclear state with the sign “ \pm ” being assigned to β^\mp emission, respectively; and Q is the difference of the total rest mass of the initial and final nuclear states. The Fermi function

$$F(\pm Z, W) = 2(1 + \gamma_0)(2p_e R)^{-2(1-\gamma_0)} e^{\pi v_e} \frac{|\Gamma(\gamma_0 + i v_e)|^2}{[\Gamma(2\gamma_0 + 1)]^2}, \quad (15)$$

where $\gamma_0 \equiv (1 - Z^2\alpha^2)^{1/2}$ and $v_e = \pm Z\alpha W/p_e$ take both the relativity of electron and the finite size of nucleus (through a spherical radius R) into account.

For pp fusion, $Z = 1$, $R = 2.1402$ fm, $Q = 0.420$ MeV, and the outer radiative correction evaluated to be $\delta_{pp}^{\text{out}} = 1.62\%$ [17,18], so the “corrected” phase space factor becomes

$$f_{pp}^R(E) = (1 + \delta_{pp}^{\text{out}}) f_{pp}(E) = 0.144 \left(1 + 9.04 \left(\frac{E}{\text{MeV}} \right) + 30.7 \left(\frac{E}{\text{MeV}} \right)^2 \right) + \mathcal{O}(E^3). \quad (16)$$

The linear term in E is in excellent agreement with [4]. A 0.1% error is assigned to $f_{pp}^R(0)$ in SFII. An error of the same size is assigned to $f_{pp}^R(E)$ in this work.

3. Results and discussion

To obtain a numerical result of $\Lambda^2(p)$, the last piece of information we need is the value of $L_{1,A}$. Using the reactor $\bar{\nu}_e d$ breakup data, $L_{1,A}$ is determined to be (3.6 ± 4.6) fm³ [19,20], and using the data of solar neutrino deuteron breakups through charged and

neutral currents, and $\nu_e e^-$ elastic scattering, $L_{1,A}$ is determined to be (4.0 ± 6.3) fm³ [20]. Both fits have taken into account the radiative corrections computed in Refs. [17,18]. Treating these two fits in two nucleon systems as independent and using the same average scheme as in Ref. [19], we obtain

$$L_{1,A} = (3.4 \pm 3.7) \text{ fm}^3, \quad (17)$$

where a -0.3 fm³ shift of the central value is introduced due to updating g_A from 1.267 to 1.2695. This range of $L_{1,A}$ is consistent with the naive dimensional analysis value $|L_{1,A}| \sim 6$ fm³ [21]. One can further constrain this two-body current by the tritium beta decay [3], which is a three nucleon system. This is carried out in potential model [9] and hybrid EFT [10], and both approaches yielded the determination of $S_{11}(0)$ with 1% accuracy.

Expanding Λ^2 in powers of E ,

$$\Lambda^2(E) = \Lambda^2(0) \left(1 + c_1 \left(\frac{E}{\text{MeV}} \right) + c_2 \left(\frac{E}{\text{MeV}} \right)^2 \right) + \mathcal{O}(E^3), \quad (18)$$

we find, by using the central value of $L_{1,A}$, the following expansions in ϵ for $\Lambda^2(0)$, c_1 and c_2 :

$$\begin{aligned} \Lambda^2(0) &\propto 1 + 0.057\epsilon + 0.054\epsilon^2 + 0.022\epsilon^3 + \dots, \\ c_1 &\propto 1 - 0.112\epsilon + 0.049\epsilon^2 + 0.020\epsilon^3 + \dots, \\ c_2 &\propto 1 - 0.000\epsilon + 0.057\epsilon^2 + 0.023\epsilon^3 + \dots. \end{aligned} \quad (19)$$

Although these results are only good up to $\mathcal{O}(\epsilon^2)$, the above $\mathcal{O}(\epsilon^3)$ terms indicate ~ 2 –3% corrections. This is what one typically expects for the pionless EFT, which has a small expansion parameter $\sim \gamma/m_\pi \simeq 1/3$. Therefore, we assign 3% higher-order corrections to $\Lambda^2(0)$, c_1 and c_2 .

Up to $\mathcal{O}(\epsilon^2)$, the full results are

$$\begin{aligned} \Lambda^2(0) &= 7.01(1 \pm 3\% \pm 3\%), \\ c_1 &= 2.24(1 \pm 0.7\% \pm 3\%), \\ c_2 &= 34.2(1 \pm 0.1\% \pm 3\%), \end{aligned} \quad (20)$$

where the first errors are from the uncertainty in $L_{1,A}$, and the second errors are from higher order ($\mathcal{O}(\epsilon^3)$) corrections.

The two sources of errors are correlated in the sense that when two near-threshold weak deuteron breakup processes have the same kinematics, the two processes will be governed by the same Gamow–Teller matrix element such that higher-order effects can be largely absorbed by shifting the value of $L_{1,A}$. Indeed, the two nucleon processes used to constrain $L_{1,A}$ and the pp fusion have similar kinematics – even though not exactly the same, so the combined error on $\Lambda^2(0)$ should be between 3% (100% correlated) to 4.2% (not correlated, added in quadrature). We will assign the combined error to be 3.6% which is slightly larger than the 3% error adopted in SFII with pionless EFT determination. Note that in SFII, a 0.9% error is assigned to $\Lambda^2(0)$ by using tritium beta decay rate to constrain the two body current. Here we just use the constraint of Eq. (17) available in the two nucleon sector and study its implication in $S_{11}(E)$.³

It is remarkable that c_1 and c_2 are quite insensitive to $L_{1,A}$. This is because for near threshold pp fusion, the energy dependence is dominated by the initial state pp scattering which alone

³ Using the SFII recommended value of $S_{11}(0)$, one can fix $L_{1,A} = (3.92 \pm 0.16)$ fm³, and reduce the corresponding error estimate in Eq. (20). Also one can match to recent determinations of two-body currents in chiral EFT (see, e.g., Refs. [10,22]) and obtain a better constraint than Eq. (17). However, this refinement has negligible impact on our main conclusions about $S'_{11}(0)$ and $S''_{11}(0)$, which will be discussed below.

gives $c_1 = 3.09$ and $c_2 = 36.2$ without $L_{1,A}$ dependence. c_1 and c_2 then receive -28% and -6% $L_{1,A}$ -dependent corrections, respectively, to reach the values of Eq. (20). This explains why the errors due to the uncertainty of $L_{1,A}$ in c_1 and c_2 are 23% and 4% of that in $\Lambda^2(0)$. This result along with the combined error assignment discussed above leads to error reduction in $S'_{11}(0)/S_{11}(0)$ and $S''_{11}(0)/S_{11}(0)$.

Combining our results of $\Lambda^2(E)$ and $f_{pp}(E)$, we found the S-factor $S_{11}(E)$ for the low-energy pp fusion yields

$$\begin{aligned} S_{11}(0) &= (3.99 \pm 0.14) \times 10^{-25} \text{ MeV b}, \\ S'_{11}(0) &= S_{11}(0)(11.3 \pm 0.1) \text{ MeV}^{-1}, \\ S''_{11}(0) &= S_{11}(0)(170 \pm 2) \text{ MeV}^{-2}. \end{aligned} \quad (21)$$

Our central value of $S_{11}(0)$ is the same as the pionless EFT value in SFII while the error is slightly larger as discussed above. As for $S'_{11}(0)$, our result is consistent with the adopted value in SFII: $S'_{11}(\text{SFII})(0) = (11.2 \pm 0.1)S_{11}(0) \text{ MeV}^{-1}$, which was obtained by Bahcall and May [4]. Finally, we also report a value of $S''_{11}(0)$, which was not computed before, with $\sim 1\%$ accuracy. At the center of the sun, this higher-order term has a contribution at the level of 0.5% to the fusion rate, which is smaller than $\sim 1\%$ that was previously estimated [3,4].

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