# Analyzing the effect on CMB in a parity and charge-parity violating varying alpha theory

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In this paper we study in detail the effect of our recently proposed model of parity and charge-parity (PCP) violating varying alpha on the Cosmic Microwave Background (CMB) photon passing through the intragalaxy-cluster medium (ICM). The ICM is well known to be composed of magnetized plasma. According to our model, the polarization and intensity of the CMB would be affected when traversing through the ICM due to nontrivial scalar photon interactions. We have calculated the evolution of such polarization and intensity collectively, known as the Stokes parameters of the CMB photon during its journey through the ICM and tested our results against the Sunyaev-Zel'dovich (SZ) measurement on the Coma galaxy cluster. Our model contains a PCP-violating parameter, $\beta$, and a scale of alpha variation $\omega$. Using the derived constrained on the photon-to-scalar conversion probability, $\bar{P}_{\gamma\to\phi}$, for the Coma cluster in [34,35]we found a contour plot in the $(\omega, \beta)$ parameter plane. The $\beta = 0$ line in this parameter space corresponds to well-studied Maxwell-dilaton type models which has lower bound on $\omega \gtrsim 6.4 \times 10^9$ GeV. In general, as the absolute value of $\beta$ increases, the lower bound on $\omega$ also increases. Our model in general predicts the modification of the CMB polarization with a nontrivial dependence on the parity-violating coupling parameter $\beta$. However, it is unconstrained in this particular study. We show that this effect can in principle be detected in the future measurements on CMB polarization such that $\beta$ can also be constrained.

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## I. INTRODUCTION

There has been growing interests in the recent past to extend all the standard model of particle physics and test against the present day high-precession measurements. Parity violation has already been proved to be one of its simplest and straightforward extension. It is already a well-established fact that there exists parity (P) and charge-parity (CP) violation in the electroweak sector. This particular observation drives people for the last several years to study various different possible sources of parity and charge-parity (PCP) violation beyond the standard model [1–5]. The basic idea of all these models is to add an explicit parity-violating term in the Lagrangian. Interestingly all those different PCP-violating models predict different potentially observable phenomena such as cosmic birefringence [1,2] and left-right asymmetry in the gravitational wave dynamics [3,4] which could be detectable in the future experiments. Recently we have also constructed a PCP-violating model [6] in the framework of "varying alpha theory" with the advantage over that of other scalar field model such as Carroll's in that the origin of the parity violation may be better physically motivated.

String theory gives us ample evidence to consider theories of varying fundamental constants in nature. Since string theory is actually a higher-dimensional theory, all the fundamental constants are emergent because of dimensional reduction. So our hope is that future high-precession cosmological as well as laboratory experiments may provide some signatures of new physics, which also includes the variation of fundamental constants.

After the proposal of a consistent framework of variation of fine-structure constant $\alpha$ by Bekenstein [7], an extensive effort have been made for the last several years on the theoretical [8–10] as well as the observational side [11–15] of this $\alpha$ variation. The important point to mention that it is not the other observable effect but the effect of direct fine-structure constant variation on the cosmology which has been considered extensively in the literature. What we want to emphasize is that our recently proposed PCP violating extension to this model opens up the possibility to test it against various other observable effect apart from just the variation of $\alpha$ in the direct laboratory measurement [16] as well as indirect cosmological measurement. This is the indirect cosmological measurement leading to the stringent constraint on a varying alpha model which is the main subject of study of our present papr. We have already put constraints on our model parameters space against various laboratory experiments like BFRT [17], PVLAS [18], and Q&A [19]. The main goal of all these experiments is to measure the change of states of a polarized laser beam propagating through the region of an externally applied magnetic field. An external magnetic

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field-induced polarization in a model of scalar(pseudoscalar) coupled with an electromagnetic field has been the subject of study for a long time [20–35]. The model that we recently introduced also exhibits this effect induced from the PCP violating term in our varying fine-structure constant theory [6]. This motivates us to use a different class of experiments to constrain the parameters of a given varying fine-structure constant theory. Such approach has not been explored before.

In terms of simple well-known dilaton or axion electrodynamic models our model can be thought of as a natural generalization of all those where we have both parity-even and parity-odd coupling with a photon. But more importantly it is not just an arbitrary addition but a basic well-known underlying assumption of the varying fine-structure constant which dictates to us the form of the scalar field coupling function with the electromagnetic field up to some unknown constant which will be determined from the observation. So, from our current study not only can we constrain those constant parameters but also can shed some light on the possible variation of the fine-structure constant over a cosmological time. Our current study will be particularly focused on CMB observation and how it constrains our model parameter in the same spirit as of all the previous studies separately on the scalar or axion electrodynamics models. In this regard our study can, therefore, be thought of as a coherent study of all those scalar and axion electromagnetic models studied so far. Our model has two independent parameters, namely, $\omega$ and $\beta$. It is the parameter $\beta$, the ratio between axion and scalar type coupling with a photon field, which parametrizes the PCP violating coupling strength. In the present study, we will see how this PCP violating parameter $\beta$ affects various observable quantities.

In our previous work [16] we put bounds on our model parameters based on the birefringence and the dichroism of the vacuum induced from the nontrivial coupling of a photon in the laboratory-based experiment. We would like to point out that the bounds we derived in our previous study are completely excluded compared with the bound we found in the present study. This essentially tells us that with the current experimental parameter values it is impossible to see any signal of birefringence and the dichroism in those laboratory experiments.

In this paper, we will be exploring another class of cosmological observations to constrain our model parameters. We will analyze the effect of our PCP violating varying the fine-structure constant model on the CMB photon when passing through the ICM. From various cosmological observations it has already been verified that the ICM consists of strong magnetized plasma with the magnetic field up to $30\mu$G. In the presence of this ICM plasma, the CMB photons encounter an inverse Compton scattering with the electron. This effect is known as SZ effect. This scattering process does not affect the number density but changes the energy distribution of the incoming CMB photons. As is well known, CMB photons coming from the last scattering surface encode a wealth of information related to the properties of structure formation and more importantly the information about the inflationary dynamics in the very early Universe. All the important effects on CMB photon after the last scattering, therefore, should be carefully investigated. As we just mentioned, the SZ effect is one of those which has already been studied quite intensively. If there exists some light scalar field that couples to a photon, then we should be able to see the modification of the CMB spectrum due to the nonzero photon-to-scalar conversion probability amplitude in the presence of background magnetized plasma. There exist many different models where this phenomena can occur. In this regard, standard axion-photon and dilaton-photon systems have been studied quite extensively from theoretical as well as phenomenological points of view [28–33]. Another model called the chameleon model [36] has also a nontrivial effect on CMB [34,35]. In our model which is the generalization of the Bekenstein-Sandvik-Barrow-Magueijo (BSBM) theory of the varying fine-structure constant, has also a natural coupling between scalar and photon with parity violation. In this paper we will explore in detail the effect of our model on the CMB photon passing through the ICM magnetized plasma, with emphasis on the effect of PCP violation.

We organize this paper as follows: in Sec. II, after briefly reviewing our PCP violating ''varying alpha theory'' [6], we will analyze in detail the optical properties and calculate the evolution of the Stokes parameter of the electromagnetic wave when it is passing through the magnetized plasma. In the subsequent Sec. III, we will analyze the effects of our model on a CMB photon. We calculate the evolution of the Stokes parameters of a CMB photon when passing though the ICM magnetized plasma. As we have mentioned before, we will test our result against the SZ measurement of a particular galaxy cluster, the Coma Galaxy cluster. The model of the ICM magnetized plasma we will be using is the well-known power spectrum model. We will analytically calculate approximate expressions for the Stokes parameters of the incoming CMB photon after passing through the ICM magnetic field and plasma of a galaxy cluster. Then in Sec. IV, after briefly reviewing the general properties the galaxy cluster magnetic field, we will use our approximate expression of the photon-to-scalar conversion probability, $\bar{P}_{\gamma\to\phi}$, which is responsible for the additional modification of the CMB temperature over the standard SZ effect, to constrain our model parameter. We will use the derived upper bound on $\bar{P}_{\gamma\to\phi}$ from the Coma cluster in the Ref. [34] to constrain the scale of variation of the fine-structure constant $\omega$. Subsequently in Sec. V, we discuss the modification to the polarization Stokes parameter of the CMB photon and its observational aspects. Until now we do not have any observation on the change of polarization of the CMB photon due to the ICM mainly because of experimental difficulty. We suggest that several recent experiments on the polarization measurement such as STP-Pole, ALMA, POLAR, which are either ongoing or under development, with their high angular resolution can in principle help to constrain the parameter space of our model. Concluding remarks and future prospects are provided in Sec. VI.

## II. OPTICS IN A PCP VIOLATING VARYING ALPHA THEORY

A varying alpha theory [7–9] is usually referred to as a theory of spacetime variation of the electric charge of any matter field. The fine-structure constant in such a theory, therefore, conveniently parametrized by $\alpha = e_0^2 e^{2\phi(x)}$ in natural units. According to above definition this theory enjoys a shift symmetry in $\phi$ i.e. $\phi \to \phi + c$ and also the modified U(1) gauge transformation $e^{\phi}A_\mu \to e^{\phi}A_\mu + \chi_{,\mu}$. So, an unique gauge-invariant, shift-symmetric and PCP violating Lagrangian for the modified scalar-electromagnetic fields can be written as

$$\mathcal{L} = M_p^2 R - \frac{\omega^2}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4}e^{-2\phi}F_{\mu\nu}F^{\mu\nu} + \frac{\beta}{4}e^{-2\phi}F_{\mu\nu}\tilde{F}^{\mu\nu} + \mathcal{L}_m, \quad (1)$$

where electromagnetic field strength tensor can be expressed as

$$F_{\mu\nu} = (e^{\phi}\mathbf{a}_\nu)_{,\mu} - (e^{\phi}\mathbf{a}_\mu)_{,\nu} = \mathbf{A}_{\nu,\mu} - \mathbf{A}_{\mu,\nu}, \quad (2)$$

with $\mathbf{A}_\mu = e^{\phi}\mathbf{a}_\mu$ as a modified electromagnetic gauge potential. $R$ is the curvature scalar and $\beta$ is the PCP violating coupling parameter to be determined from the observation. we also set $e_0 = 1$ for convenience. As can be easily seen, the above action reduces to the usual form when $\phi$ is constant. The parameter $\omega$ sets a characteristic scale of the theory above which one expects Coulomb force law to be valid for a point charge. Shift symmetry protects the scalar field against any arbitrary potential function in our Lagrangian. Of course one can break this shift symmetry by introducing a potential term, which has recently been studied in [37]. We will leave this for our future study in the context of PCP violating varying alpha theory.

In this section we will do the general analysis in detail on the scalar-photon mixing phenomena in the background plasma with magnetic field. Our study would be relevant to the present day and also future various precision cosmological as well as astrophysical optical measurements. The Maxwell and scalar field equations turn out to be of standard type with the modifications coming from nontrivial scalar field $\phi$ coupling

$$\Box\phi = \frac{e^{-2\phi}}{2\omega^2}[-F_{\mu\nu}F^{\mu\nu} + \beta F_{\mu\nu}\tilde{F}^{\mu\nu}], \quad (3)$$

$$\begin{aligned} \nabla\cdot\mathbf{E} &= -(-2\nabla\phi\cdot\mathbf{E} + 4\beta\nabla\phi\cdot\mathbf{B}),\\ \partial_\eta(\mathbf{E}) - \nabla\times\mathbf{B} &= 2(\dot\phi\mathbf{E} - \nabla\phi\times\mathbf{B}) - 4\beta(\dot\phi\mathbf{B} + \nabla\phi\times\mathbf{E}),\\ \boldsymbol{\nabla}\cdot\mathbf{B} &= 0,\\ \partial_\eta\mathbf{B} + \boldsymbol{\nabla}\times\mathbf{E} &= 0, \end{aligned} \quad (4)$$

As is well known from various measurements, at cosmological as well as astrophysical scales, there exists a background magnetic field which may have a significant effect on the electromagnetic field coming from various sources. In this paper we are particularly interested in studying the effect on the CMB photon. Studying the effect of some other external field on the CMB photon is of particular interest because of its prime importance in cosmology. With this motivation in mind, we will try to calculate the effect of the magnetized plasma background on the electromagnetic wave. In terms of vector potential i.e. $\mathbf{B} = \nabla\times\mathbf{A}$, the above equations can be written in the following suitable form:

$$\begin{aligned} (\nabla^2 - \partial_t^2)\mathbf{A} &= -4\beta\mathbf{B}\partial_t\phi - 2(\nabla\phi\times\mathbf{B})\\ (\nabla^2 - \partial_t^2)\phi &= \frac{2\mathbf{B}^2}{\omega^2}\phi - \frac{2}{\omega^2}\mathbf{B}\cdot(\nabla\times\mathbf{A}) + + \frac{4\beta}{\omega^2}\mathbf{B}\cdot\partial_t\mathbf{A} \end{aligned} \quad (5)$$

In this case we assume the background magnetic field is $\mathbf{B}$. Because of smallness of the effect we consider linear-order equations for the scalar-photon system. In the above derivation we use the gauge condition $\nabla\cdot\mathbf{A} = 0$ and consider the scalar potential $\mathbf{A}_0 = 0$. Now, assuming the propagation direction of the electromagnetic wave to be in the z direction, we take the form of the field's ansatz to be

$$\mathbf{A}(z, t) = \mathbf{A}^0 e^{-i\varpi t}; \qquad \phi(z, t) = \phi^0 e^{-i\varpi t}, \quad (6)$$

where $\mathbf{A} = \{\mathbf{A}_x, \mathbf{A}_y, 0\}$. In order to solve them analytically, we will follow the same procedure as in [20]. We further assume that the background magnetic field variation is very small compared to the scalar and the photon frequency $\varpi$. With this assumption we can approximate the dispersion operator to be

$$\begin{aligned} \partial_z^2 + \varpi^2 &= (\varpi + i\partial_z)(\varpi - i\partial_z) = (\varpi + k)(\varpi + i\partial_z)\\ &\simeq 2\varpi(\varpi + i\partial_z), \end{aligned} \quad (7)$$

assuming the dispersion relation to be $k = n\varpi$ with $|n - 1| \ll 1$. Therefore, we can write down the above system of Eq. (5) as

$$(i\partial_z + \varpi)\mathbf{A}_x - i(\mathbf{B}_y + 2\beta\mathbf{B}_x)\phi = 0 \tag{8}$$

$$(i\partial_z + \varpi)\mathbf{A}_y + i(\mathbf{B}_x - 2\beta\mathbf{B}_y)\phi == 0 \tag{9}$$

$$(i\partial_z + \varpi)\phi - \frac{\mathbf{B}^2}{\omega^2\varpi}\phi - \frac{i}{\omega^2}(\mathbf{B}_x - 2\beta\mathbf{B}_y)\mathbf{A}_y + \frac{i}{\omega^2}(\mathbf{B}_y + 2\beta\mathbf{B}_x)\mathbf{A}_x = 0. \tag{10}$$

Now if we take into account the plasma effects, the above set of linear equations can be expressed as follows:

$$\left(i\frac{d}{dz} + \mathcal{M}\right)\begin{pmatrix}\mathbf{A}_x\\ \mathbf{A}_y\\ \Phi\end{pmatrix} = 0, \tag{11}$$

$$\text{where}\quad \mathcal{M} = \begin{bmatrix} \varpi + \Delta_x & 0 & -i(\mathbf{B}_y + 2\beta\mathbf{B}_x)\\ 0 & (\varpi + \Delta_y) & i(\mathbf{B}_x - 2\beta\mathbf{B}_y)\\ \frac{i}{\omega^2}(\mathbf{B}_y + 2\beta\mathbf{B}_x) & -\frac{i}{\omega^2}(\mathbf{B}_x - 2\beta\mathbf{B}_y) & \varpi - \frac{\mathbf{B}^2}{\omega^2\varpi}\end{bmatrix}.$$

Here $\mathcal{M}$ is called the scalar and photon mixing matrix. The new notation are

$$\Delta_{x,y} = \Delta_{\rm QED} + \Delta_{\rm CM} + \Delta_{\rm plasma}. \tag{12}$$

All the terms in the second expression of the above equation have been considered before in the axion-photon study. $\Delta_{\rm QED}$ comes from the effect of vacuum polarization giving rise to the refractive index of the photon. This term is also known to be associated with the lowest order nonlinear Maxwell Lagrangian (Euler-Heisenberg term). $\Delta_{\rm CM}$ is known as Cotton-Mouton term which is the effect of birefringence of gases and liquids in the presence of a magnetic field. The last term is due to the background plasma through which the photon traversed. The usual expression for those terms are as follows:

$$\Delta^x_{\rm QED} = \frac{7}{2}\varpi\zeta, \qquad \Delta^y_{\rm QED} = 2\varpi\zeta\Delta^x_{\rm CM} - \Delta^y_{\rm CM} = 2\pi C\mathbf{B}_0^2, \qquad \Delta_{\rm plasma} = -\frac{\varpi^2_{\rm plasma}}{2\varpi} = 4\alpha_0\frac{\rho_e}{m_e}\frac{1}{2\varpi}, \tag{13}$$

where $\zeta = (\alpha_0/(45\pi)(\mathbf{B}_0/\mathbf{B}_c)^2$, $\mathbf{B}_c \equiv m_e^2/e = 4.41 \times 10^{13}$ G the critical field strength, $m_e$ the electron mass, $e$ the electron charge, and $\alpha$ the fine-structure constant. Note that in the above expression for $\zeta$, we ignore the correction due to the fine-structure constant variation as it contributes to the higher order in fluctuation in Eq. (5).

Now in order to solve the above set of equations we define

$$\mathbf{A} = e^{-\varpi(t-z)+i\zeta(z)}\bar{\mathbf{A}}; \qquad \phi = e^{-\varpi(t-z)+i\zeta(z)}\bar{\phi}, \tag{14}$$

where $\zeta'(z) = -\varpi^2_{\rm plasma}/2\varpi$. With these new variables the above set of equations can be written as follows:

$$\left(\frac{d}{dz} + \bar{M}\right)\begin{pmatrix}\bar{\mathbf{A}}_x\\ \bar{\mathbf{A}}_y\\ e^{iS}\Phi\end{pmatrix} = 0, \tag{15}$$

$$\text{where}\quad \bar{M} = \begin{bmatrix} 0 & 0 & -(\mathbf{B}_y + 2\beta\mathbf{B}_x)e^{-iS}\\ 0 & 0 & (\mathbf{B}_x - 2\beta\mathbf{B}_y)e^{-iS}\\ \frac{1}{\omega^2}(\mathbf{B}_y + 2\beta\mathbf{B}_x)e^{iS} & -\frac{1}{\omega^2}(\mathbf{B}_x - 2\beta\mathbf{B}_y)e^{iS} & 0\end{bmatrix},$$

where we define

$$S(z) = -\int_0^z\left(\frac{\varpi^2_{\rm plasma}}{2\varpi} - \frac{\mathbf{B}^2}{\omega^2\varpi}\right)dx. \tag{16}$$

In order to solve the above equations of motion we will make an approximation following Ref. [34], where the amplitude of the mixing matrix is small, i.e., $Tr[\mathcal{M}\mathcal{M}^\dagger] < 1$. Let us consider the solution of the form

$$\begin{pmatrix} \bar{\mathbf{A}}_x(z) \\ \bar{\mathbf{A}}_y(z) \\ e^{iS(z)}\Phi(z) \end{pmatrix} \simeq (\mathbb{I} + \mathcal{J}_1 + \mathcal{J}_2 + \cdots) \begin{pmatrix} \bar{\mathbf{A}}_x(0) \\ \bar{\mathbf{A}}_y(0) \\ e^{iS(0)}\Phi(0) \end{pmatrix}, \tag{17}$$

where

$$\mathcal{J}_1 = \int_0^z \bar{M}(x)dx = \begin{bmatrix} 0 & 0 & -\mathcal{B}_y^* \\ 0 & 0 & \mathcal{B}_x^* \\ \frac{\mathcal{B}_y}{\omega^2} & -\frac{\mathcal{B}_x}{\omega^2} & 0 \end{bmatrix};$$

$$\mathcal{J}_2 = \int_0^z \bar{M}'(x)\bar{M}(x)dx, \tag{18}$$

where

$$\mathcal{B}_i = \int_0^z (\mathbf{B}_i - 2\beta\epsilon^{ij}\mathbf{B}_j)e^{iS}dx, \tag{19}$$

with $\epsilon^{xy} = 1$, $\epsilon^{yx} = -1$. Once we know the approximate solution, we can write down the polarization states of the electromagnetic field under study in terms of Stokes parameters

$$I(z) = |\mathbf{A}_x|^2 + |\mathbf{A}_y|^2 \tag{20}$$

$$Q(z) = |\mathbf{A}_x|^2 - |\mathbf{A}_y|^2 \tag{21}$$

$$U(z) = 2\,\mathrm{Re}(\mathbf{A}_x^*\mathbf{A}_y) \tag{22}$$

$$V(z) = 2\,\mathrm{Im}(\mathbf{A}_x^*\mathbf{A}_y), \tag{23}$$

where $I$ is intensity, $Q(z)$, $U(z)$ are linear polarization, and $V(z)$ is circular polarization of the electromagnetic field. After traversing the path length z, polarization states take the following explicit form:

$$I(z) = I(0)(1 - P_{\gamma\to\phi}) + Q(0)\mathcal{Q}(z) + U(0)\mathcal{U}(z) - V(0)\mathcal{V}(z), \tag{24}$$

$$\begin{aligned} Q(z) &= Q(0)(1 - P_{\gamma\to\phi}) + I(0)\mathcal{Q}(z) + U(0)(\mathcal{U}(z) - 2\mathcal{L}_1(z)) - V(0)(\mathcal{V}(z) - 2\mathcal{L}_2(z)), \\ U(z) &= U(0)(1 - P_{\gamma\to\phi}) + I(0)\mathcal{U}(z) - V(0)(P_{\gamma\to\phi} - 2\mathcal{L}_3(z)) - Q(0)(\mathcal{V}(z) - 2\mathcal{L}_1(z)), \\ V(z) &= V(0)(1 - P_{\gamma\to\phi}) + I(0)\mathcal{V}(z) - U(0)(P_{\gamma\to\phi} - 2\mathcal{L}_3(z)) - V(0)(\mathcal{V}(z) - 2\mathcal{L}_2(z)), \end{aligned}$$

where we have defined

$$\begin{aligned} P_{\gamma\to\phi} &= \frac{1}{2\omega^2}(|\mathcal{B}_x|^2 + |\mathcal{B}_y|^2); \\ \mathcal{Q}(z) &= \frac{1}{2\omega^2}(|\mathcal{B}_x|^2 - |\mathcal{B}_y|^2), \end{aligned} \tag{25}$$

$$\begin{aligned} \mathcal{U}(z) &= \frac{1}{2\omega^2}(\mathcal{B}_x^*\mathcal{B}_y + \mathcal{B}_y^*\mathcal{B}_x); \\ \mathcal{V}(z) &= \frac{1}{2\omega^2}(\mathcal{B}_x^*\mathcal{B}_y - \mathcal{B}_y^*\mathcal{B}_x), \\ \mathcal{L}_1(z) &= \frac{1}{2\omega^2}\int_0^z(\mathcal{B}_x^{*\prime}\mathcal{B}_y + \mathcal{B}_x'\mathcal{B}_y^*); \\ \mathcal{L}_2(z) &= \frac{1}{2\omega^2}\int_0^z(\mathcal{B}_x^{*\prime}\mathcal{B}_y - \mathcal{B}_x'\mathcal{B}_y^*), \\ \mathcal{L}_3(z) &= \frac{1}{2\omega^2}\int_0^z(\mathcal{B}_x^{*\prime}\mathcal{B}_x + \mathcal{B}_y'\mathcal{B}_y^*). \end{aligned}$$

In the above expressions we assume the initial correlations as follows:

$$<\phi^*(0)\mathbf{A}_i(0)> = 0; \qquad <\mathbf{A}_i^*(0)\phi(0)> = 0; \qquad <\phi^*(0)\phi(0)> = 0. \tag{26}$$

The photon-to-scalar or scalar-to-photon transition amplitude is defined by $P_{\gamma\to\phi}(z)$. Variation of the fine-structure constant leads to an effective change in photon intensity, which in turn affects the CMB temperature. It can also induce polarization of the photon when traverses a long cosmological distance. We have already discussed in the introduction that we are interested in studying the effect of the ICM on the CMB photon traversing through it. Therefore, in order to make an estimate of the amount of effect due to the variation of the fine-structure constant, we have to consider a specific model of magnetic field **B** variation and also the electron density $\rho_e$ variation at the galaxy-cluster scale. As the CMB photon passes through the ICM, its frequency distribution changes due to the inverse Compton scattering SZ effect with the electrons in the plasma. This essentially means the nonvanishing photon-to-scalar conversion probability $P_{\gamma\to\phi}(z)$. In this paper we will estimate the effect due to this conversion probability considering the particular model of magnetic field and electron density variation in the galactic medium closely following the Ref. [34]. The particular model that we will be considering is the power spectrum model for the spatial variation of the galaxy cluster magnetic field $\mathbf{B}$ and the electron density $\rho_e$. We will also be discussing the effect on the polarization states of the CMB photon. It is well known that the initial states of the CMB photon are very lightly polarized compared to its intensity. According to the observation, fractional linear polarization compared to the intensity parametrized by $\langle Q(0)^2\rangle^{1/2}/I(0)$, $\langle U(0)^2\rangle^{1/2}/I(0) \sim \mathcal{O}(10^{-6})$ and the fractional circular polarization $\langle V(0)^2\rangle^{1/2}/I(0) \ll \mathcal{O}(10^{-6})$. So essentially the change of states of the CMB photon after traversing a long intergalactic distance $z$ is proportional to the initial intensity $I(0)$ and conversion probability $P_{\gamma\to\phi}(z)$.

$$\begin{aligned} I(z) &\simeq I(0)(1 - P_{\gamma\to\phi}); \qquad Q(z) \simeq I(0)\mathcal{Q}(z) \\ U(z) &\simeq I(0)\mathcal{U}(z); \qquad V(z) \simeq I(0)\mathcal{V}(z). \end{aligned} \tag{27}$$

Once we get the above approximate expression for the Stokes parameters for the electromagnetic wave, we can analyze the effect on the CMB photon, which is believed to be one of the important probes to understand the cosmology. As we have discussed before, the framework that we built up is readily applicable to analyze the evolution of states of the CMB photon passing through the galaxy clusters. In the following sections we will apply our framework and do the quantitative estimates of the temperature as well as polarization modulations on the CMB photon due to ICM magnetized plasma fields.

## III. THE POWER SPECTRUM MODEL AND THE EFFECT ON CMB

The most realistic model for the magnetic field $\mathbf{B}$ and the electron density $\rho_e$ in a galaxy cluster is described by the so-called power spectrum model [38]. The most relevant physical quantities are the two-point correlation functions of $\delta\mathbf{B}_i$ and $\delta\rho_e$, defined by

$$\begin{aligned} R_{\mathbf{B}ij}(x) &= <\delta\mathbf{B}_i(y)\delta\mathbf{B}_i(x+y)> \\ &= \frac{1}{4\pi}\int d^3k P_{\mathbf{B}ij}(k) e^{i\mathbf{k}\cdot\mathbf{x}}, \end{aligned} \tag{28}$$

$$R_{\mathbf{e}}(x) = <\delta\rho_e(y)\delta\rho_e(x+y)> = \frac{1}{4\pi}\int d^3k P_e(k) e^{i\mathbf{k}\cdot\mathbf{x}}. \tag{29}$$

In the power spectrum model the magnetic field fluctuation component is approximately assumed to be Gaussian random variable, i.e. $<\delta\mathbf{B}_i> = 0$, where the average is taken over full spatial length of a galaxy cluster through which the CMB photon is propagating. With this assumption one can show $P_{\mathbf{B}ij}(k) = \frac{1}{3}\delta_{ij}P_{\mathbf{B}}(k)$. In the above expression for the power spectrum we also assumed that the fluctuations are approximately position-independent. The corresponding correlation lengths for the fluctuation of electron density and the magnetic field are defined by [39]

$$L_{\mathbf{B}} = \frac{\int_0^\infty kdkP_{\mathbf{B}}(k)}{2\int_0^\infty k^2dkP_{\mathbf{B}}(k)}; \qquad L_e = \frac{\int_0^\infty kdkP_e(k)}{2\int_0^\infty k^2dkP_e(k)}. \tag{30}$$

Now in order to estimate the modified polarization of states and intensity of a CMB photon after traversing a distance $L$ through the galaxy cluster, the basic quantity we have to calculate is $G_{ij}(x) = <\mathcal{B}_i^*(y)\mathcal{B}_j(y+x)>$. The main goal is to express the above correlation functions in terms of two power spectra $P_{\mathbf{B}}(k)$ and $P_e(x)$. As we mentioned before, in order to calculate this we will closely follow the procedure of [34].

In order to avoid complications in our main text, we only quote our essential expressions which are directly related to the observable quantity. All the detailed calculations have been given in the Appendix. One can see that to the leading order in $1/\omega^2$ and $\beta$, the photon-to-scalar conversion probability can be written in the following compact form:

$$\bar{P}_{\gamma\to\phi} = \bar{P}^{\rm reg}_{\gamma\to\phi} + \bar{P}^{\rm ran}_{\gamma\to\phi} + \frac{8\beta^2}{3}\left(\int_{\bar{\Delta}}^\infty + \int_{\bar{\Delta}'}^\infty\right) kdk\mathcal{F}^k_{(1)}.$$

To simplify our further calculations in the above expression we consider the propagating photon with a single frequency $\varpi$ so that $\bar{\Delta} = \bar{\Delta}'$.. For convenience, we have separated the total scalar conversion probability amplitude into a term coming from the regular(''reg'') ICM magnetic field $\mathbf{B}_0$ and the other terms coming from the random (''ran'') magnetic field $\delta\mathbf{B}$. We will provide the expressions for $\bar{P}^{\rm reg}_{\gamma\to\phi}$ and $\bar{P}^{\rm ran}_{\gamma\to\phi}$ in our subsequent discussions. With these new definitions on the part of scalar-to-photon transition probability one can also check that the expression for the induced polarization of the CMB photon, after traveling through the ICM of length $L$, becomes

$$\bar{V}(L) \simeq -\beta I(0)\bar{P}^{\rm ran}_{\gamma\to\phi}, \tag{31}$$

$$\begin{aligned} \bar{Q}(L) &\simeq I(0)\bar{P}^{\rm reg}_{\gamma\to\phi}(\cos 2\theta - 4\beta\sin 2\theta), \\ \bar{U}(L) &\simeq I(0)\bar{P}^{\rm reg}_{\gamma\to\phi}(\sin 2\theta + 4\beta\cos 2\theta). \end{aligned} \tag{32}$$

One can immediately see that in addition to the standard scalar-photon coupling contribution, all the observable quantities depend nontrivially on the PCP-violating parameter $\beta$.

In order to calculate the amount of effect of the magnetized plasma on the incoming CMB photon, we need to consider observed power spectra $P_{\mathbf{B}}(k)$ and $P_e(k)$ of the magnetic field and electron density, respectively. On the small scales it is customary to parameterize the power spectrum by power law in momentum space as follows:

$$k^2 P_{\mathbf{B}}(k) = \mathcal{P}_{\mathbf{B}}\left(\frac{k}{k_0}\right)^{\gamma}; \qquad k^2 P_e(k) = \mathcal{P}_e^2 k^{\gamma}, \tag{33}$$

where $\mathcal{P}_{\mathbf{B}}$ and $P_e(k)$ are the normalization constants. $\gamma < -1$ and $k_0 = 1\ \mathrm{kpc}^{-1}$. A special universal value $\gamma = -5/3$ on small scale corresponds to the well-known spectral index for the three-dimensional Kolmogorov's theory of turbulence. We also assume that this power law form holds for a wide range of scales of the magnetic and electron density fluctuations in the ICM. Interestingly, observations on many different galaxy clusters suggest that on a wide range of spatial scales, the power spectrum is consistent with the Kolomogorov one. It is, therefore, straightforward to calculate

$$\int_{\bar{\Delta}}^{\infty} kdkP_{\mathbf{B}}(k) \propto \int_{\bar{\Delta}}^{\infty} kdkP_e(k) \propto \bar{\Delta}^{\gamma}, \tag{34}$$

where $\bar{\Delta} = (\frac{\varpi^2_{\mathrm{plasma}}}{2\varpi} - \frac{\bar{\mathbf{B}}_0^2}{\varpi\omega^2})$. The critical length scale $\bar{\Delta}$ is composed of two parts: the first part which is related to the well-known quantity called plasma frequency, is dependent on the average electron density, and the second part is dependent on the background magnetic field strength. The latter part also depends on the scale of the fine-structure constant variation $\omega^2$. In terms of the length scale, we can write

$$\frac{\varpi^2_{\mathrm{plasma}}}{2\varpi} \simeq 0.208 \times 10^2 \frac{1}{\mathrm{pc}}\left(\frac{2\pi 100\ \mathrm{GHz}}{\varpi}\right)\left(\frac{\bar{\rho}_e}{10^{-3}\ \mathrm{cm}^{-3}}\right),$$
$$\frac{\bar{\mathbf{B}}_0^2}{\varpi\omega^2} \simeq 4.29 \times 10^{-6} \frac{1}{pc}\left(\frac{2\pi 100\ \mathrm{GHz}}{\varpi}\right)\left(\frac{\bar{\mathbf{B}}_0}{30\mu\mathrm{G}}\right) \times \left(\frac{1\ \mathrm{GeV}}{\omega}\right)^2. \tag{35}$$

It can be easily observed from the above expressions that the value of magnetic field dependent part is in fact very small, even for $\omega \simeq \mathcal{O}(1)$ GeV, compared to the plasma frequency part. This is also in accord with our previous perturbative expansions of the various magnetic correlation functions in terms of the standard two-point correlation function. As is known for a typical galaxy cluster, if we consider the CMB photon frequency, $\omega \approx 30$–$300$ GHz then the inverse of the first line of the Eq. (35) takes the approximate value $\simeq 10^{-3} - 0.1$ pc. Therefore, all the observable quantities like $P_{\gamma\to\phi}, Q, U$ and $V$, which are sensitive to the critical length scale, is controlled by the plasma frequency of the intragalactic plasma[40]. It is important point to note that the measurement, so far, probes the power spectrum at the spatial scales larger than the few kiloparsecs. In order to proceed further we will assume, therefore, that the power spectrum Eq. (33), which holds for the momentum $k > k_*$, also includes the critical length scale $\bar{\Delta}$. On the other hand for $k < k_*$, the power spectrum remains almost constant and is consistent with the observation [41]. With these assumptions, following [34], the normalization constant for the assumed power spectrum $\mathcal{P}_{\mathbf{B}}$ and $\mathcal{P}_e^2$ can be approximately estimated as

$$k_0^{-\gamma}\mathcal{P}_{\mathbf{B}} \simeq \frac{2(\frac{\gamma}{\gamma+1} - \frac{1}{x})^{\gamma}}{(\log(x) - \frac{1}{\gamma})^{\gamma+1}} L_{\mathbf{B}}^{\gamma+1}\langle \delta\mathbf{B}\cdot\delta\mathbf{B}\rangle, \tag{36}$$

$$\mathcal{P}_e^2 \simeq \frac{2(\frac{\gamma}{\gamma+1} - \frac{1}{x})^{\gamma}}{(\log(x) - \frac{1}{\gamma})^{\gamma+1}} L_e^{\gamma+1}\langle \delta\rho_e\delta\rho_e\rangle, \tag{37}$$

where for the typical galaxies, the value of $x \sim 10$–$200$. From the assumed log-normal distribution of the electron density, it is straightforward to check that $I_e = 1 + \langle\delta\rho_e\delta\rho_e\rangle/\bar{\rho}_e^2$. For example, the electron density fluctuation measurement on our own galaxy suggests the approximate value of $I_e \sim 1$–$2$. In the subsequent analysis, we also assume $L_{\mathbf{B}} \approx L_e$ which is in accord with the various observations in different galaxy clusters.

Our analysis so far revealed that the fluctuating component of the ICM magnetic field $\delta\mathbf{B}$ and plasma density $\delta\rho_e$ fields play the crucial role in the modification of the CMB intensity and polarization tensor. On the other hand, the background regular components $(\mathbf{B}_0, \bar{\rho}_e)$ of those quantities set the inherent critical scale of the system. Using the above results, in the subsequent sections, we will estimate the amount of effects such as temperature variation (SZ-like effect) and also the induced polarization of the CMB making use of all the known parameters for the Coma cluster.

## IV. GALAXY CLUSTER'S MAGNETIC FIELD AND SZ-LIKE EFFECT

Because of the nontrivial scalar-photon interaction which depends on the nature of the magnetic field and plasma distribution, we have seen that the energy spectrum as well as the polarization states of the CMB photon change nontrivially when traversing through the ICM. Furthermore, in order to understand the early universe physics and also the physics of structure formation, it is essential to observe the correct initial state of the CMB photon at the last scattering surface (LSS). With that in mind, all the intermediate effects on the CMB photon, from the LSS to the observer on Earth, should be taken into consideration. We have already seen in our model that, because of the photon-to-scalar conversion probability, the modification of the intensity as well as polarization states of the CMB photon depends strongly on the strength and distribution of the magnetic field and plasma density in ICM.

It is an experimental fact that the ICM carries magnetic field with strength as high as $30\mu$G [42]. One of the common methods to determine the magnetic field profile of the ICM is to use the observation of the Faraday rotation of the plane of polarization of the electromagnetic wave coming from the extended polarized radio sources either behind or embedded within the galaxy cluster under study. Observations and numerical simulations suggests that the ICM magnetic field can be best described by the power spectrum model [39] that we have considered in our above analysis. The basic underlying assumption behind this power spectrum model is the statistical homogeneity and isotropy of the fluctuations of magnetic field and electron density on a large volume of the galaxy cluster under consideration. The linearly polarized radio emission experiences a rotation of the plane of polarization when it traverses through the ICM with a background magnetic field which has a component along the line of propagation. The observed polarization angle is proportional to the square of the wavelength and a quantity called RM, which is a proportionality constant. The mathematical expression for RM along the line-of-sight in the $\hat{\mathbf{z}}$ direction of a source located at $z_s$ is

$$\mathrm{RM}\,(z_s) = a_0 \int_0^{z_s} \rho_e(x\hat{\mathbf{z}})\mathbf{B}_z(x\hat{\mathbf{z}})dz, \tag{38}$$

where $a_0 = \alpha_0^3/\pi^{1/2}m_e^2$, $\mathbf{B}_z$ is the magnetic field along the line-of-sight, and the observer position is at $x = 0$.

In order to understand better about the magnetic field structure in the ICM, one needs to understand electron distribution as well. It is observed that the shape of the electron density distribution also has some correlation with the background magnetic field in the medium. Experimentally, for example, from the ROSAT full-sky survey, the electron density distribution has been determined from the X-ray surface brightness profile of the hot and diffused gas that fills the ICM. It is well -known that the radial profile of electron density from the galaxy core could be well fitted to a $\beta$ profile [43]:

$$\rho_e(r) = \rho_0\left(1 + \frac{r^2}{r_c^2}\right)^{-3\beta/2}, \tag{39}$$

where $\beta \sim \mathcal{O}(1)$ and positive. $\rho_0$ is the mean electron density and $r_c$ is the core radius of the galaxy cluster. For a typical galaxy cluster, the values of those parameters are $r_c \sim 100$–$200$ kpc, $\beta \sim 2/3$, and $\rho_0 \sim 0.0001$–$0.01$ cm$^{-3}$. In addition to the main component of the electron density there exists a fluctuating component that can be best characterized by using the power spectrum model. Standard magneto-hydrodynamic (MHD) simulation of the galaxy cluster formation suggests that the total magnetic energy should follow the power law behavior of electron density: $\langle \mathbf{B}^2\rangle \propto \langle\rho_e\rangle^\eta$. Various theoretical arguments and observations [44] predict that $\eta \simeq 1$.

As we have already mentioned before, the CMB photon encounters an inverse Compton scattering with the electrons in the ICM plasma. This effect is known as SZ effect. This scattering process changes the energy distribution of the incoming CMB photon. The CMB is believed to be one of the important cosmological probes. So, all the important effects on the CMB photon after the last scattering should be carefully investigated. We just stated that the SZ effect is one of those effects which have already been studied quite intensively. We have calculated in the previous section that because of nontrivial scalar-photon interaction, the frequency spectrum of the CMB changes because of the nonzero conversion probability amplitude. There exist many different kinds of scalar field models where this phenomena exists due to the nontrivial scalar-photon coupling. All those models have been studied quite extensively from the theoretical as well as phenomenological points of view [28–33]. All those fields are collectively known as axion-like particles (ALPs). One of the interesting examples, which has recently received much attention, is known as the chameleon field [36]. The effective mass of the chameleon field depends on the density of the surrounding matter distribution. Therefore, in the low-density region of space, this chameleon field also acts like an ALP. Extensive studies have been done on its effect on the cosmology, more specifically in the context of the present paper see [34,35]. As we have stated before, our model of PCP violating varying alpha predicts a nontrivial effect on the photon field traversing through the magnetized plasma. So, we will be able to see how our parity-violating coupling can lead to the various effects on the CMB photon. As we know the intensity of the CMB photon is related to its temperature. The variation of temperature, therefore, is related to the variation of the intensity as follows:

$$\frac{\delta T}{T_0} = \frac{(1 - e^{-\mu\varpi})}{\mu\varpi}\frac{\delta I}{I_0}, \tag{40}$$

where the Boltzmann factor $\mu = \frac{1}{k_B T_0}$ with average CMB temperature $T_0 \simeq 2.75$ K. We have seen before due to the scalar-photon coupling, the intensity of the CMB photon changes due to the ICM magnetic field and the electron density distribution. The point we would like to emphasize is that the variation of the fine-structure constant can lead to a new kind of nontrivial foreground effect on the CMB photon. The temperature variation of CMB due to the scalar-photon conversion probability, can then be expressed as

$$\frac{\delta T}{T_0} \approx \frac{(e^{-\mu\varpi} - 1)}{\mu\varpi}\bar{P}_{\gamma\to\phi}(L). \tag{41}$$

In terms of physical quantities, the expression for $\bar{P}_{\gamma\to\phi}(L)$ turns out to be

$$\begin{aligned}\bar{P}_{\gamma\to\phi}(L) = {}&\left(\frac{I_e^3\mathbf{B}_*^2}{\mathcal{N}^2\omega^2} - \frac{\mathbf{B}_0^2\cos(\bar{\Delta}L_{\mathrm{eff}})}{\mathcal{N}^2\omega^2}\right)\varpi^2\bar{\rho}_e^{-2}\\ &+ \frac{\pi L_{\mathrm{eff}} I_e^2(1+4\beta^2)\mathcal{N}^\gamma}{2\gamma\omega^2}\\ &\times\left(\frac{\mathcal{P}_e^2\mathbf{B}_*^2}{\bar{\rho}_e^2} + \frac{2I_e\mathcal{P}_{\mathbf{B}}}{3k_0^\gamma}\right)\varpi^{-\gamma}\bar{\rho}_e^\gamma,\end{aligned} \tag{42}$$

where we define

$$\mathbf{B}_*^2 = \mathbf{B}_0^2 + \frac{2}{3}\langle\delta\mathbf{B}\cdot\delta\mathbf{B}\rangle; \qquad \bar{\Delta} = \frac{\bar{\rho}_0\mathcal{N}}{\varpi}. \tag{43}$$

For the power spectrum model typically $-2 \le \gamma < -1$, which obviously includes the Kolmogorov model of three-dimensional turbulence, where the exponent $\gamma = -5/3$. Several observations suggest that the regular component, $\mathbf{B}_0$ of the ICM magnetic field, is much smaller than that of the random part $\delta\mathbf{B}$. For example the ICM magnetic field in the central region of the Coma cluster has been determined from the Faraday RMs measurement [45]. The strength of the regular part of the magnetic field is estimated to be $0.2 \pm 0.1\mu$G with the coherence length of the order of 200 kpc. Where as the strength of the random part is $8.5 \pm 1.5\mu$G with the coherence length on much shorter scales of $L_{\mathbf{B}} \sim 1$ kpc. If we assume that the coherence length, $L_e$, and the power spectrum of the electron-density fluctuation $\delta\rho_e/\bar{\rho}_e$ are proportional to that of the magnetic fluctuation, then the approximate expression for the dominant contribution to the scalar-to-photon conversion probability becomes

$$\bar{P}_{\gamma\to\phi}(L) \approx \frac{2I_e^3\langle\delta\mathbf{B}\cdot\delta\mathbf{B}\rangle}{3\mathcal{N}^2\omega^2}\varpi^2\bar{\rho}_e^{-2} + \frac{2\pi L_{\rm eff}I_e^2(1+4\beta^2)\mathcal{N}^\gamma}{6\gamma\omega^2} \times \frac{\mathcal{P}_e^2\langle\delta\mathbf{B}\cdot\delta\mathbf{B}\rangle}{\bar{\rho}_e^2}\varpi^{-\gamma}\bar{\rho}_e^\gamma. \tag{44}$$

At this point it is important to mention the behavior of the standard thermal SZ effect which changes the intensity of the CMB photon in the following way [46]:

$$\frac{\delta T}{T_0} = \frac{\kappa_B T_e}{m_e}\tau_0\left(\mu\varpi\coth\left(\frac{\mu\varpi}{2}\right) - 4\right), \tag{45}$$

with $T_e$ being the temperature of the electron in the ICM plasma. The quantity $\tau_0 = \int \sigma_T\rho_e(z)dz$ is known as optical depth. $\sigma_T$ is called Thompson cross section. One can see, therefore, that the thermal SZ effect is linear in electron density $\rho_e$ compared to the power law behavior of the SZ-like effect due to the varying alpha scalar field. Power law type frequency dependence in Eq. (44) compared to the nonpower law type frequency dependence in the standard thermal SZ effect could in principle be detectable from the observation in the future high-precession experiments.

In this paper, to constrain our model parameters, we will use the bound on the photon-to-scalar conversion probability derived in the Ref. [34]. The specific result that we are going to use is for the nearby Coma galaxy cluster. There exist detailed measurements of the ICM magnetic field and also the SZ effect of this particular cluster by various experiments like OVRO, WMAP, and MITO. As has been derived in the Ref. [34], the upper bound on the photon-to-scalar conversion probability for the Coma cluster is

$$P^{\rm coma}_{\gamma\to\phi}(204\ \text{GHz}) < 6.2\times 10^{-5} \tag{46}$$

with the 95% confidence level. It is also argued that the above upper limit is not very sensitive to the value of the exponent of the power spectrum, $\gamma$. In this paper we are not going to discuss the derivation of the above constraint. Interested reader may consult the reference we mentioned. Now for the Coma cluster, the numerical value for the two parts of the scalar-to-photon conversion probability, Eq. (44), turns out to be

$$\begin{aligned}
&\frac{2\langle\delta\mathbf{B}\cdot\delta\mathbf{B}\rangle}{3\mathcal{N}^2\omega^2}\varpi^2\bar{\rho}_e^{-2} \\
&\approx \frac{2.7\times10^{15}}{\omega^2}\ \text{GeV}^2\left(\frac{\delta\mathbf{B}}{8.5\mu\text{G}}\right)^2 \\
&\quad\times\left(\frac{\varpi}{2\pi 204\ \text{GHz}}\right)\left(\frac{4\times10^{-3}\ \text{cm}^{-3}}{\rho_e}\right)^2, \\
&\frac{L_{\rm eff}\mathcal{N}^\gamma\mathcal{P}_e^2\langle\delta\mathbf{B}\cdot\delta\mathbf{B}\rangle}{6|\gamma|\omega^2\bar{\rho}_e^2}\varpi^{-\gamma}\bar{\rho}_e^\gamma \\
&\approx \frac{2.44\times4.07^\gamma\times10^{18+2\gamma}}{2\pi\omega^2}\ \text{GeV}^2\left(\frac{2\pi 204\ \text{GHz}}{\varpi}\right)^\gamma \\
&\quad\times\left(\frac{\rho_e}{4\times10^{-3}\ \text{cm}^{-3}}\right)^\gamma\left(\frac{\delta\mathbf{B}}{8.5\mu\text{G}}\right)^2\left(\frac{L_{\mathbf{B}}}{1\ \text{kpc}}\right)^{\gamma+1}\left(\frac{L}{200\ \text{kpc}}\right).
\end{aligned} \tag{47}$$

From the above expressions, the constraint, on the scale of fine-structure constant $\omega$, depends on several *a priori* unknown quantities like the ICM magnetic field $\mathbf{B}$, electron density $\rho_e$, Coherence length $L_{\mathbf{B}}$, exponent of the power spectrum of magnetic filed $\gamma$, etc. Accuracy of the constraints, therefore, depend severely on the observations of the properties of ICM. One can observe that depending upon the value of the power spectrum exponent $\gamma$, the frequency dependence of $\bar{P}_{\gamma\to\phi}$ changes, which in turn affects the upper bound of $\omega$. If we consider the aforementioned value of the power spectrum exponent $-2 < \gamma < -1$ and using the bound on $P^{\rm coma}_{\gamma\to\phi}(204\ \text{GeV})$ Eq. (46), one can get a contour plot (Fig. 1) for the parameter space $(\beta, y)$, where we denote $\omega = 10^y$. In the plot, we have considered the reasonable value of $I_e$ to be $\approx 1$ which is based on the electron density fluctuations in our own galaxy. The numerical value of all the other parameters are considered to be that of the aforementioned Coma galaxy cluster. The shaded region is excluded due to nonobservation of any SZ-like effect coming from the scalar field.

In order to elaborate more on the possible bounds on the parameter space and also compare with our previous results coming from the laboratory-based experiments [16], in what follows we consider a specific $\beta = 2$ line in the parameter space for two different values of $\gamma$. It is straightforward to check that for $\beta \approx 2$, one gets

$$\omega^2\bar{P}_{\gamma\to\phi} \approx (2.7\text{–}102.85)\times10^{15}. \tag{48}$$

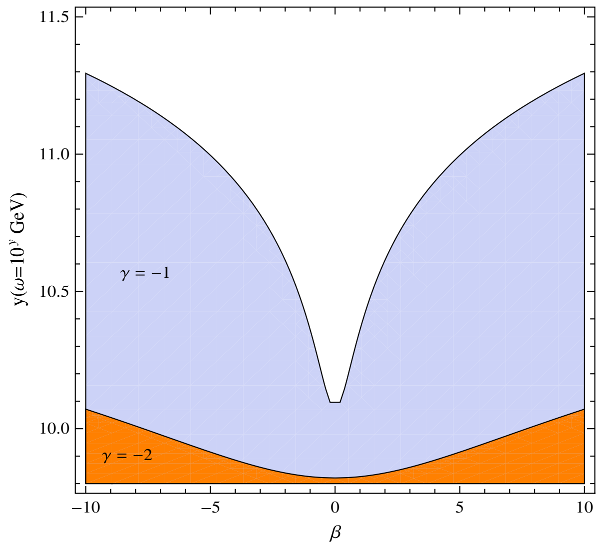


FIG. 1 (color online). Bounds on PCP-violating parameter $\beta$ and the scale of varying the fine-structure constant $\omega$ using the possible bound on scalar-to-photon conversion probability $P^{\rm coma}_{\gamma\to\phi}(204\ {\rm GeV}) < 6.2 \times 10^{-5}$ for the well-known Coma galaxy cluster. This bound has been derived in [34] from the SZ measurement on the Coma galaxy cluster.

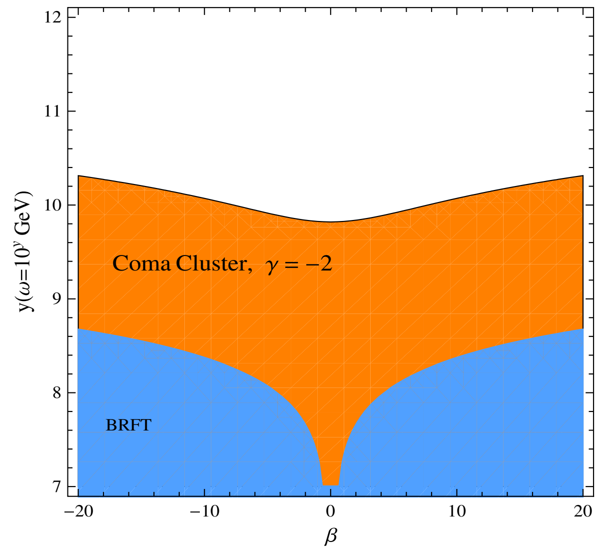


FIG. 2 (color online). Comparing bounds on $\beta$ and $\omega$ coming from CMB observation on the Coma galaxy cluster Fig. 1 and particular laboratory experiment called BFRT from our previous paper [16].

After using the above constraint coming from the Coma cluster Eq. (46), for $\beta = 2$ the above Eq. (48) gives us the lower bound on $\omega$ to be

$$\omega \geq (0.66\text{–}4.04) \times 10^{10}\ {\rm GeV}. \tag{49}$$

It is clear from the above contour plot that as absolute value of $\beta$ increases, the lower bound also increases for $\omega$.

At this point it is worth comparing our present analysis with our previous bound coming from the laboratory-based experiments we mentioned before [16]. In our previous study we had considered different laboratory-based experimental results to constrain the model parameters. Using the experimental constraint on the rotation and ellipticity of a polarized electromagnetic wave passing through a magnetized region, we derived the possible bounds to be $1 \leq \omega^2[{\rm GeV}^2] \leq 10^{13}$ and $-0.5 \leq \beta \leq 0.5$. The primary assumption behind those constraints was that $\beta$ should be less than unity. However, in our present study, it is clear that even if we consider $\beta = 0$, the parameter $\omega$ is always $\gtrsim$ few $\times\, 10^9$ GeV. It is also clear from the Fig. 2 that our previous bound on $\omega$ is completely excluded by the present CMB bound. Therefore, we can infer that if optical rotation and dichroism measured in the laboratory is sourced only by the possible variation of the fine-structure constant then it is almost impossible to see any positive signal with the current available values of the experimental parameters and accuracy of the experiment.

Our present analysis does not help us to constrain $\beta$. In order to constrain this we need to consider the polarization measurement of CMB. As we have mentioned before, with the present-day experimental precession, it is very difficult to measure the change of polarization in the galaxy clusters. In the subsequent section we will discuss the prediction of our model on the change of the polarization of the CMB due to the ICM magnetized plasma. The future experiments may shed some light on the existence of the parity violation through the CMB polarization measurement.

## V. PARITY-VIOLATING EFFECT ON CMB POLARIZATION

As we have already discussed before, in the presence of background ICM magnetic field and plasma density, the varying alpha scalar field alters the polarization states of the CMB photon. The leading-order contribution to this change of the polarization comes from the photon-to-scalar conversion probability $P_{\gamma\to\phi}$. The random part $P^{\rm ran}_{\gamma\to\phi}$ contributes to the Stokes parameter $\mathcal{V}$ which gives rise to the circular polarization of the CMB photon. On the other hand, the regular part, $P^{\rm reg}_{\gamma\to\phi}$, induces linear polarization of the CMB photon. The expressions for the induced polarization of the CMB along the line-of-sight are

$$\bar{\mathcal{V}}(L) \simeq -\beta \bar{P}^{\rm ran}_{\gamma\to\phi}, \tag{50}$$

$$\begin{aligned} \bar{\mathcal{Q}}(L) &\simeq \bar{P}^{\rm reg}_{\gamma\to\phi}(\cos 2\theta - 4\beta \sin 2\theta), \\ \bar{\mathcal{U}}(L) &\simeq \bar{P}^{\rm reg}_{\gamma\to\phi}(\sin 2\theta + 4\beta \cos 2\theta), \end{aligned} \tag{51}$$

where the expression for the regular part of the conversion probability is

$$\bar{P}^{\,\mathrm{reg}}_{\gamma\to\phi}(L) = \left(\frac{I_e^3\mathbf{B}_0^2}{\mathcal{N}^2\omega^2} - \frac{\mathbf{B}_0^2\cos(\bar{\Delta}L_{\mathrm{eff}})}{\mathcal{N}^2\omega^2}\right)\varpi^2\bar{\rho}_e^{-2} + \frac{\pi L_{\mathrm{eff}} I_e^2\mathcal{N}^\gamma\mathcal{P}_e^2\mathbf{B}_0^2}{2\gamma\omega^2\bar{\rho}_e^2}\varpi^{-\gamma}\bar{\rho}_e^{\gamma}, \tag{52}$$

and the expression for the random part is given in Eq. (44). We have already discussed that the contribution from the regular magnetic field part $\mathbf{B}_0$ is very small compared to the contribution from the random magnetic field $\delta\mathbf{B}$. To the leading order in $1/\omega^2$, therefore, the magnitude of the induced circular polarization $\bar{V}(L) \gg \bar{Q}(L), \bar{U}(L)$. It is easy to see that the dominant contribution in the Eq. (52) is coming from the electron density fluctuation. Using all the measured quantity for the Coma cluster, one gets

$$\frac{\pi L_{\mathrm{eff}}\mathcal{N}^\gamma\mathcal{P}_e^2\mathbf{B}_0^2}{2|\gamma|\omega^2\bar{\rho}_e^2}\varpi^{-\gamma}\bar{\rho}_e^{\gamma} \approx \frac{6.4\times 4.07^\gamma\times 10^{14+2\gamma}}{\omega^2}\ \mathrm{GeV}^2\left(\frac{2\pi 204\ \mathrm{GHz}}{\varpi}\right)^\gamma \times\left(\frac{\rho_e}{4\times 10^{-3}\ \mathrm{cm}^{-3}}\right)^\gamma\left(\frac{\delta\mathbf{B}}{0.2\mu\mathrm{G}}\right)^2\left(\frac{L_{\mathbf{B}}}{1\ \mathrm{kpc}}\right)^{\gamma+1}\left(\frac{L}{200\ \mathrm{kpc}}\right). \tag{53}$$

This is clearly in magnitude of the order of $10^{-4}$ lower than that of $\bar{P}^{\mathrm{ran}}_{\gamma\to\phi}(L)$ mentioned before. If we use our previous bound on $\omega$ from the Eq. (49), the prediction of the linear polarization, which is coming from the regular part of the magnetic field in the ICM, comes out to be

$$\bar{P}^{\,\mathrm{reg}}_{\gamma\to\phi}(L) \le (0.89\text{–}1.51)\times 10^{-10}. \tag{54}$$

This is much less than that of the circular polarization which is proportional to

$$\bar{P}^{\,\mathrm{ran}}_{\gamma\to\phi}(L) \le 6.2\times 10^{-5}. \tag{55}$$

The interesting point we would like mention is about its nontrivial dependence on the parity-violating parameter $\beta$. We have mentioned before that $\beta$ could be greater than unity. Because of this fact, two linear polarizations $\bar{\mathcal{Q}}(L)$ and $\bar{\mathcal{Q}}(L)$ could be of opposite sign. In principle this effect can potentially be detectable from the next-generation experiments. There exist few earth-based experiments, such as SPT-Pol, ALMA, POLAR, which are either ongoing or under development, that have detectors to measure the polarization of CMB also on the low scale. All these experiments with high angular resolution could in principle shed some light on the parity-violating effect on the polarization of the CMB.

Before closing this section we would like to discuss the induced polarization coming from the random magnetic component of the ICM. We have seen that the circular polarization is induced by the random part of the magnetic field and the linear polarization can also get some contribution from the random part of the magnetic field. Since the magnitude and direction of the polarization Stokes parameters coming from the random part of the ICM magnetic field depend on its magnitude and direction, the average over the many lines-of-sight of those random contributions will vanish. The effective contribution from the random magnetic field, therefore, can be encoded in the variance $\sigma^2$ of the Stokes parameters $\mathcal{Q}$, $\mathcal{U}$ and $\mathcal{V}$. Now according to the standard definition, one can get the variance of the Stokes parameters $\mathcal{Q}$ to be [34],

$$\begin{aligned}\sigma^2_{\mathcal{Q}} &= \langle\mathcal{Q}(\Delta_1)\mathcal{Q}(\Delta_2)\rangle - \langle\mathcal{Q}(\Delta_1)\rangle\langle\mathcal{Q}(\Delta_2)\rangle,\\ &\approx \frac{1}{2}(P^2_{\gamma\to\phi}(\Delta_1,\Delta_2) - \beta^2 P^{\mathrm{ran}2}_{\gamma\to\phi}(\Delta_1,\Delta_2)\\ &\quad + P^{\mathrm{reg}2}_{\gamma\to\phi}(\Delta_1,\Delta_2)((1-16\beta^2)\cos 4\theta - 8\beta\sin 4\theta))\\ &\quad + (\Delta_2\to-\Delta_2).\end{aligned}$$

In the above derivation we have approximated $\mathcal{B}'s$ to be Gaussian and therefore all the expectation values can be written in terms of the two-point correlation function. With the similar definition, it can be easily shown that the variance of the other stokes parameters come out to be

$$\begin{aligned}\sigma^2_{\mathcal{U}} &\approx \frac{1}{2}(P^2_{\gamma\to\phi}(\Delta_1,\Delta_2) - \beta^2 P^{\mathrm{ran}2}_{\gamma\to\phi}(\Delta_1,\Delta_2)\\ &\quad - P^{\mathrm{reg}2}_{\gamma\to\phi}(\Delta_1,\Delta_2)((1-16\beta^2)\cos 4\theta - 8\beta\sin 4\theta))\\ &\quad + (\Delta_2\to-\Delta_2),\\ \sigma^2_{\mathcal{V}} &\approx \frac{1}{2}(P^2_{\gamma\to\phi}(\Delta_1,\Delta_2) + \beta^2 P^{\mathrm{ran}2}_{\gamma\to\phi}(\Delta_1,\Delta_2)\\ &\quad - P^{\mathrm{reg}2}_{\gamma\to\phi}(\Delta_1,\Delta_2)(1+4\beta^2)) + (\Delta_2\to-\Delta_2).\end{aligned}$$

As we have discussed in the previous section, the contribution from the random magnetic field is much greater than the regular contribution in the photon-to-scalar conversion probability. So to the leading order in magnitude we can clearly see from the above variance that all the polarization Stokes parameters are proportional to the $P^{\mathrm{ran}}_{\gamma\to\phi}$. The expression for the variance can be approximated as

$$\begin{aligned}\sigma^2_{\mathcal{Q}} &\approx \frac{1}{2}P^2_{\gamma\to\phi}(\Delta_1,\Delta_2) - \frac{\beta^2}{2}P^{\mathrm{ran}2}_{\gamma\to\phi}(\Delta_1,\Delta_2)\\ &\quad + (\Delta_2\to-\Delta_2),\\ \sigma^2_{\mathcal{U}} &\approx \frac{1}{2}P^2_{\gamma\to\phi}(\Delta_1,\Delta_2) - \frac{\beta^2}{2}P^{\mathrm{ran}2}_{\gamma\to\phi}(\Delta_1,\Delta_2)\\ &\quad + (\Delta_2\to-\Delta_2),\\ \sigma^2_{\mathcal{V}} &\approx \frac{1}{2}P^2_{\gamma\to\phi}(\Delta_1,\Delta_2) + \frac{\beta^2}{2}P^{\mathrm{ran}2}_{\gamma\to\phi}(\Delta_1,\Delta_2)\\ &\quad + (\Delta_2\to-\Delta_2).\end{aligned}$$

But as we have stated before, experimentally the random contribution is very difficult to measure with the present level of experimental accuracy. Regarding the problems of measurement, an elaborate discussion has been provided in Ref. [34]. We are not going to discuss it further. The essential point, that we would like to infer, is that for the contribution coming from the regular magnetic field part $\mathbf{B}_0$ of the ICM, we do not have such measurement problems. Although the magnitude of that contribution ($\sim 10^{-10}$) is very small compared to the intrinsic polarization of the CMB photon ($\sim 10^{-7}$), the recent experiment like ALMA, with the order of a few arc second angular resolution, could help to put stringent bounds on our model parameters or in principle could detect some positive signal regarding the parity violation in the photon sector.

## VI. CONCLUSIONS

The theory of varying the fine-structure constant has been the subject of intense study in the last several years. The cosmological impact of this variation has been studied quite extensively. Various cosmological as well as laboratory-based observations on this variation of the fine-structure constant have been considered to constrain the varying alpha parameter $\omega$. Recently, we have constructed a particular model based on this varying alpha theory which includes explicit PCP violation in the photon sector [6]. In this paper we have studied our aforementioned PCP violating varying alpha model in the light of a new class of cosmological observations which have not been considered before. We considered the SZ measurement of the Coma galaxy cluster to constrain our model parameters. In this particular measurement the temperature variation of CMB is being measured. The basic underlying mechanism behind this measurement is the existence of a nontrivial interaction between the photon and high-temperature plasma field in the ICM. As stated before, if there exists a light scalar field which has nontrivial coupling with the photon field then one would expect an additional SZ-like effect on CMB. This is what we have studied in detail in this paper. In our model [6] we have introduced a nontrivial PCP violating scalar-photon interaction within the varying alpha-theory framework. Although the experiments under consideration are insensitive to the properties of the background field due to the weakness of its coupling with matter, they nevertheless can help to constrain our varying alpha-model parameters $\omega$ and $\beta$ through the possible frequency-dependent upper limit on the temperature variation within the error bar of the usual thermal SZ measurement. We have calculated the approximate analytic expression of our model for those measurable quantities such as the Stokes parameters of a CMB photon passing through the ICM. The model is characterized by two independent parameters $\beta$ and $\omega$ that measure the strength of PCP violation and the scale of the fine-structure constant variation, respectively.

As we mentioned before in our previous study [16] we had considered different laboratory-based experimental results to constrain our model parameters. In our present study we use the SZ measurement of a CMB photon passing through the galaxy cluster to constrain our parameters. Using the measurement on the Coma galaxy cluster we found from Fig. 1, the lower bound on $\omega$ depends on the value of PCP-violating parameter $\beta$. If we choose the $\beta = 0$ line which corresponds to the standard Maxwell-dilaton type model, we approximately reproduce the known bound $\omega \geq 10^9$ GeV. According to our study in this paper the polarization measurement of a CMB photon is essential to constrain the parity-violating parameter. If the fine-structure constant is varying then the variation can lead to a certain degree of linear and circular polarizations to the CMB photon when it is passing through the magnetized medium. We have a definite prediction on the amount of circular polarization and linear polarization. But the shortcoming is that even though the circular polarization is induced by the parity-violating parameter $\beta$, the contribution is coming from the random magnetic field part of the ICM. As has been mentioned, it is very difficult to detect this signal mainly because of its random nature over a very small angular scale. In other words, the lines-of-sight are typically separated by a distance of the order of the coherence scale ($L_{\mathbf{B}}$) of random magnetic field. In order to detect the signal, one, therefore, needs to increase the angular resolution of an experiment to a very high precession. There exist measurements of polarization of photons at the galactic scale such as that of the Milky Way. We could in principle use those measurements to constrain our model parameter and also check the consistency with the present bound.

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## APPENDIX

In this section we will provide all the details of the calculation for the Stokes parameters. In order to get the expression for the photon-to-scalar conversion probability amplitude and the polarization of state we consider the following procedure. We divide the total magnetic field as a regular and a random part like $\mathbf{B} = \bar{\mathbf{B}} + \delta\mathbf{B}$. Similarly, we can define the total electron density as $\rho_e = \bar{\rho}_e + \delta\rho_e$, where $\bar{\rho}_e$ is the constant average electron density over the galaxy cluster $L$. Let us define a new quantity

$$\overline{\mathcal{D}\mathbf{B}^2}(z) = \frac{1}{z}\int_0^z (2\bar{\mathbf{B}}\cdot\delta\mathbf{B}(x) + \delta\mathbf{B}(x)\cdot\delta\mathbf{B}(x))dx = \frac{1}{z}\int_0^z \mathcal{D}\mathbf{B}^2(x)dx \quad \text{(A1)}$$

$$\bar{\delta}_e(z) = \frac{1}{z}\int_0^z \delta_e(x)dx = \frac{1}{z}\int_0^z \frac{\delta\rho_e(x)}{\bar{\rho}_e}dx, \quad \text{(A2)}$$

such that due to randomness of the density fluctuation over the length L, $\bar{\delta}_e(L) = 0$. If we define a new variable $Z = (1+\delta_e)x$, the integral

$$\mathcal{B}_i = \int_0^L (\mathbf{B}_i - 2\beta\epsilon^{ij}\mathbf{B}_j)e^{iS}dx = \int_0^L \frac{(\mathbf{B}_i - 2\beta\epsilon^{ij}\mathbf{B}_j)}{1+\delta_e} e^{-i(\bar{\Delta}+(\overline{\mathcal{D}\mathbf{B}^2}(Z)/\varpi\omega^2)Z}dZ, \quad \text{(A3)}$$

where

$$\bar{\Delta} = \left(\frac{2\pi\alpha_0\bar{\rho}_e}{2m_e\varpi} - \frac{\bar{\mathbf{B}}^2}{\varpi\omega^2}\right). \quad \text{(A4)}$$

In the above expressions we assume $|\bar{\delta}_e(z)| \ll 1$ along the photon path. For further simplification it would be useful to do another change of variable like

$$T = \left(1 + \frac{\overline{\mathcal{D}\mathbf{B}^2}(Z)}{\varpi\omega^2\bar{\Delta}}\right)Z. \quad \text{(A5)}$$

Therefore, the final expression for $\mathcal{B}_i$ takes the form

$$\mathcal{B}_i(\bar{\Delta}) = \int_0^{L_{\rm eff}} \frac{(\mathbf{B}_i - 2\beta\epsilon^{ij}\mathbf{B}_j)}{(1+\frac{\mathcal{D}\mathbf{B}^2(T)}{\varpi\omega^2\bar{\Delta}})(1+\delta_e)} e^{-i\bar{\Delta}T}dT, \quad \text{(A6)}$$

where $L_{\rm eff} = (1+\frac{\overline{\mathcal{D}\mathbf{B}^2}(L)}{\varpi\omega^2\bar{\Delta}})L$. As we have mentioned before, to estimate the intensity $I(z)$ and polarization states $\mathcal{Q}(z)$, $\mathcal{U}(z)$ and $\mathcal{V}(z)$ of the CMB photon, our main interest is $G_{ij}$. In term of the new variable as explained above, $G_{ij}$ takes the following form at different frequencies:

$$G_{ij}(\bar{\Delta}, \bar{\Delta}') = \langle \mathcal{B}_i^*(T,\bar{\Delta})\mathcal{B}_j(T',\bar{\Delta}')\rangle = \int_0^{L_{\rm eff}}\int_0^{L_{\rm eff}} \mathcal{F}_{ij}(T,T')e^{i(\bar{\Delta}T - \bar{\Delta}'T')}dTdT'. \quad \text{(A7)}$$

We define the correlation function as

$$\mathcal{F}_{ij}(T,T') = \left\langle \frac{(\mathbf{B}_i - 2\beta\epsilon^{ik}\mathbf{B}_k)(\mathbf{B}'_i - 2\beta\epsilon^{il}\mathbf{B}'_l)}{(1+\frac{\mathcal{D}\mathbf{B}^2(T)}{\varpi\omega^2\bar{\Delta}})(1+\frac{\mathcal{D}\mathbf{B}^2(T')}{\varpi'\omega^2\bar{\Delta}'})}\right\rangle \times \left\langle\frac{1}{(1+\delta_e)(1+\delta'_e)}\right\rangle. \quad \text{(A8)}$$

In the above expression for the correlation function we assume that the magnetic field fluctuation $\delta\mathbf{B}$ and electron density fluctuation $\delta\rho_e$ are uncorrelated. Isotropy of the fluctuations can simplify the above Eq. (A8) for $\mathcal{F}_{ij}(T,T')$ to

$$\mathcal{F}_{ij}(T,T') = \left\langle \frac{\bar{\mathbf{B}}_i^{\rm eff}\bar{\mathbf{B}}_j^{\rm eff}}{(1+\frac{\mathcal{D}\mathbf{B}^2(T)}{\varpi\omega^2\bar{\Delta}})(1+\frac{\mathcal{D}\mathbf{B}^2(T')}{\varpi'\omega^2\bar{\Delta}'})} + \frac{1}{3}\frac{\mathcal{E}_{ij}\delta\mathbf{B}\cdot\delta\mathbf{B}'}{(1+\frac{\mathcal{D}\mathbf{B}^2(T)}{\varpi\omega^2\bar{\Delta}})(1+\frac{\mathcal{D}\mathbf{B}^2(T')}{\varpi'\omega^2\bar{\Delta}'})}\right\rangle R_\delta(T,T'), \quad \text{(A9)}$$

where we have defined

$$\mathcal{E}_{ij} = (\delta_{ij} - 4\beta\epsilon_{ij} + 4\beta^2\epsilon_{ik}\epsilon_{jk}); \quad \bar{\mathbf{B}}_i^{\rm eff} = (\bar{\mathbf{B}}_i - 2\beta\epsilon^{ik}\bar{\mathbf{B}}_k) \quad R_\delta(T,T') = \left\langle\frac{1}{(1+\delta_e)(1+\delta'_e)}\right\rangle. \quad \text{(A10)}$$

We have assumed the isotropy and the approximate position-independent fluctuations of the cluster magnetic field and electron density. With this assumption the power spectrum of physical interests can then be defined by a simple Fourier transformation

$$G_{ij}(\bar{\Delta},\bar{\Delta}') == \frac{1}{4\pi}\int_0^{L_{\rm eff}} dT\int_0^{L_{\rm eff}} dT' \times \int d^3k\mathcal{F}_{ij}^k e^{ik(T-T')}e^{i(\bar{\Delta}T-\bar{\Delta}'T')}. \quad \text{(A11)}$$

In order to proceed further we will do some approximation adopting from [34]. We have mentioned before that we will be interested in dealing with the CMB photon passing through the ICM. The typical frequency of the CMB photon is $\varpi \simeq 10^{-5}$–$10^{-3}$ eV propagating over the distance around 100 kpc through the galaxy clusters. One can easily check therefore that in general $|\bar{\Delta}|L_{\rm eff} \gg 1$ as long as $2\pi\alpha_0\bar{\rho}_e/m_e$ is not finely tuned to be $\simeq \bar{\mathbf{B}}^2/\omega^2$. In order to get an approximate analytic expression for the above correlation function $G_{ij}(T,T')$, we further assume that the fluctuation of $\mathbf{B}$ and $\rho_e$ are such that $\mathcal{F}_{ij}^k$ falls off faster that $k^{-3}$ for $k > k_*$. Where $k_*^{-1}$ should be related to the characteristic coherent lengths $L_{\mathbf{B}}$ and $L_e$ of $\mathbf{B}$ and $\rho_e$ fluctuations, respectively, in a galaxy cluster under consideration. This is also believed to be a reasonable assumption that $\max(\bar{\Delta},\bar{\Delta}') \ll L_{\mathbf{B}}^{-1}, L_e^{-1}$. With all these assumptions and considering $\max(|\bar{\Delta}|L_{\rm eff}, |\bar{\Delta}'|L_{\rm eff}) \gg 1$ one can get the following expression to the leading order:

$$e^{-i(\Delta_- L_{\rm eff})}G_{ij}(\bar{\Delta},\bar{\Delta}') \approx \frac{2\cos(\Delta_- L_{\rm eff})}{\bar{\Delta}\bar{\Delta}'}\mathcal{F}_{ij}(0) - \frac{2\cos(\Delta_+ L_{\rm eff})}{\bar{\Delta}\bar{\Delta}'}\mathcal{F}_{ij}(L_{\rm eff}\hat{\mathbf{z}}) + \frac{\pi\sin(\Delta_- L_{\rm eff})}{2\Delta_-}\left[\int_{\bar{\Delta}}^\infty kdk\mathcal{F}_{ij}^k + \int_{\bar{\Delta}'}^\infty kdk\mathcal{F}_{ij}^k\right], \quad \text{(A12)}$$

where $\Delta_\pm = (\bar{\Delta} \pm \bar{\Delta}')/2$. $\hat{\mathbf{z}}$ is the direction along the propagation of light. In the above expression for the two-point correlation function

$$\mathcal{F}_{ij}(0) = \left\langle \frac{\bar{\mathbf{B}}_i^{\text{eff}} \bar{\mathbf{B}}_j^{\text{eff}}}{(1 + \frac{\mathcal{D}\mathbf{B}^2}{\varpi\omega^2\bar{\Delta}})^2} + \frac{1}{3} \frac{\mathcal{E}_{ij}\delta\mathbf{B}\cdot\delta\mathbf{B}}{(1 + \frac{\mathcal{D}\mathbf{B}^2}{\varpi\omega^2\bar{\Delta}})^2} \right\rangle \left\langle \frac{\bar{\rho}^2}{\rho^2} \right\rangle, \quad \text{(A13)}$$

where we have used the relation $\varpi'\Delta' = \varpi\Delta$. The effective distance $L_{\text{eff}}$, through which the CMB photon is traveling, is much larger than the coherence length of the fluctuations. So the leading contribution to $\mathcal{F}_{ij}(L_{\text{eff}})$ should be coming from the regular magnetic field part of the ICM

$$\mathcal{F}_{ij}(L_{\text{eff}}) = \left\langle \frac{\bar{\mathbf{B}}_i^{\text{eff}} \bar{\mathbf{B}}_j^{\text{eff}}}{(1 + \frac{\mathcal{D}\mathbf{B}^2(x)}{\varpi\omega^2\bar{\Delta}})(1 + \frac{\mathcal{D}\mathbf{B}^2(x+L_{\text{eff}})}{\varpi\omega^2\bar{\Delta}})} \right\rangle. \quad \text{(A14)}$$

Now we need to express the last two terms of Eq. (A11) in terms of known correlation functions. If we assume the fluctuation of $\rho_e$ is log-normal then one can write down

$$\left\langle \frac{\bar{\rho}^2}{\rho^2} \right\rangle = \left\langle \frac{\rho^2}{\bar{\rho}^2} \right\rangle^3 = I_e^3. \quad \text{(A15)}$$

Based on this log-normal distribution, it is consistent to separate the fluctuation $\rho_e$ into approximately independent short and long wavelength fluctuation such that $\rho_e = \bar{\rho}_e(1 + \delta_s)(1 + \delta_l)$. We also assume that the short wavelength fluctuations are linear up to some cutoff scale $k_{lin}^{-1}$. The long wavelength fluctuations are assumed to be above this scale and not necessarily linear. With the above assumptions, one can show [34]

$$R_\delta \simeq I_e^2(x)[1 + \bar{\rho}_e^{-2} R_e(x)]. \quad \text{(A16)}$$

From the above expression for the momentum $k \gg k_{lin}$, we can approximately have

$$R_\delta^k = I_e^2 \bar{\rho}_e^{-2} P_e(k)]. \quad \text{(A17)}$$

Now, let us define

$$\int_{\bar{\Delta}}^\infty kdk\mathcal{F}_{ij}^k = \bar{\mathbf{B}}_i^{\text{eff}} \bar{\mathbf{B}}_j^{\text{eff}} \int_{\bar{\Delta}}^\infty kdk\mathcal{F}_{(0)}^k + \frac{\mathcal{E}_{ij}}{3} \int_{\bar{\Delta}}^\infty kdk\mathcal{F}_{(1)}^k. \quad \text{(A18)}$$

Following the argument in [34], we can write down

$$\int_{\bar{\Delta}}^\infty kdk\mathcal{F}_{(0)}^k = \left\langle \frac{1}{(1 + \frac{\mathcal{D}\mathbf{B}^2}{\varpi\omega^2\bar{\Delta}})^2} \right\rangle \int_{\bar{\Delta}}^\infty kdkR_\delta^k + I_e^3 \int_{\bar{\Delta}}^\infty kdkR_{0\mathbf{B}}^k, \quad \text{(A19)}$$

$$\int_{\bar{\Delta}}^\infty kdk\mathcal{F}_{(1)}^k = \left\langle \frac{\delta\mathbf{B}\cdot\delta\mathbf{B}}{(1 + \frac{\mathcal{D}\mathbf{B}^2}{\varpi\omega^2\bar{\Delta}})^2} \right\rangle \int_{\bar{\Delta}}^\infty kdkR_\delta^k + I_e^3 \int_{\bar{\Delta}}^\infty kdkR_{1\mathbf{B}}^k, \quad \text{(A20)}$$

where new definitions are

$$\left\langle \frac{1}{(1 + \frac{\mathcal{D}\mathbf{B}^2(T)}{\varpi\omega^2\bar{\Delta}})(1 + \frac{\mathcal{D}\mathbf{B}^2(T')}{\varpi'\omega^2\bar{\Delta}'})} \right\rangle = \frac{1}{4\pi} \int d^3kR_{0\mathbf{B}}^k e^{i(T-T')\cdot k} \quad \text{(A21)}$$

$$\left\langle \frac{\delta\mathbf{B}\cdot\delta\mathbf{B}'}{(1 + \frac{\mathcal{D}\mathbf{B}^2(T)}{\varpi\omega^2\bar{\Delta}})(1 + \frac{\mathcal{D}\mathbf{B}^2(T')}{\varpi'\omega^2\bar{\Delta}'})} \right\rangle = \frac{1}{4\pi} \int d^3kR_{1\mathbf{B}}^k e^{i(T-T')\cdot k}. \quad \text{(A22)}$$

Therefore, all the physical quantities, Eq. (27) that we are interested in, can be expressed in terms of $G_{ij}$ as follows:

$$\begin{aligned} P_{\gamma\to\phi} &= \frac{1}{2\omega^2}\delta^{ij}G_{ij}; \\ \mathcal{Q}(z) &= -\frac{1}{2\omega^2}(\delta^{xi}\epsilon^{yj} + \delta^{yi}\epsilon^{xj})G_{ij}, \\ \mathcal{U}(z) &= \frac{1}{2\omega^2}(\delta^{xi}\epsilon^{xj} - \delta^{yi}\epsilon^{yj})G_{ij}; \\ \mathcal{V}(z) &= \frac{1}{2\omega^2}\epsilon^{ij}G_{ij}, \end{aligned} \quad \text{(A23)}$$

where index "i" is running for $x$ and $y$ coordinates. It is straightforward to check from the above expressions that, $V(z)$ is nonvanishing as expected from the PCP-violating term. Therefore, to the leading order, it should be proportional to $\beta/\omega^2$. By using the following identities, $\delta^{ij}\mathcal{E}_{ij} = \delta_i^i(1 + 4\beta^2)$ and $\epsilon^{ij}\mathcal{E}_{ij} = -4\beta\delta_i^i$, one gets

$$\begin{aligned} \bar{V}(L) \simeq -\frac{2\beta I(0)e^{-i(\Delta_- L_{\text{eff}})}}{3\omega^2} \Bigg[ &\frac{2\cos(\Delta_- L_{\text{eff}})I_e^3}{\bar{\Delta}\bar{\Delta}'} \\ &\times \left\langle \frac{\delta\mathbf{B}\cdot\delta\mathbf{B}}{(1 + \frac{\mathcal{D}\mathbf{B}^2}{\varpi\omega^2\bar{\Delta}})^2} \right\rangle + \frac{\pi\sin(\Delta_- L_{\text{eff}})}{\Delta_-} \\ &\times \left( \int_{\bar{\Delta}}^\infty + \int_{\bar{\Delta}'}^\infty \right) kdk\mathcal{F}_{(1)}^k \Bigg]. \end{aligned} \quad \text{(A24)}$$

The PCP-violating term in our Lagrangian, therefore, induces a certain degree of circular polarization to the incoming CMB photon propagating through the ICM magnetized plasma. The contribution strongly depends upon the fluctuating part of the ICM magnetic field. If we have only the regular part of the magnetic field, the induced circular polarization vanishes to the leading order in the PCP-violating parameter $\beta$.

If we consider the regular components of the ICM magnetic field $\bar{\mathbf{B}}_x = \mathbf{B}_0\cos\theta$ and $\bar{\mathbf{B}}_y = \mathbf{B}_0\sin\theta$, then one can easily show the following approximate expressions for the other Stokes parameters:

$$\begin{aligned} \bar{\mathcal{Q}}(L) &= -\frac{1}{2\omega^2}(\delta^{xi}\epsilon^{yj} + \delta^{yi}\epsilon^{xj})G_{ij} \\ &\simeq e^{-i(\Delta_- L_{\text{eff}})}\mathcal{A}(\bar{\Delta}, \bar{\Delta}')(\cos 2\theta - 4\beta\sin 2\theta), \\ \bar{\mathcal{U}}(L) &= \frac{1}{2\omega^2}(\delta^{xi}\epsilon^{xj} - \delta^{yi}\epsilon^{yj})G_{ij} \\ &\simeq e^{-i(\Delta_- L_{\text{eff}})}\mathcal{A}(\bar{\Delta}, \bar{\Delta}')(\sin 2\theta + 4\beta\cos 2\theta), \end{aligned} \quad \text{(A25)}$$

where the expression for $\mathcal{A}$ is

$$\omega^2\mathcal{A}(\bar\Delta,\bar\Delta') \approx +\frac{I_e^3\mathbf{B}_0^2\cos(\Delta_-L_{\rm eff})}{\bar\Delta\bar\Delta'}\left\langle\frac{1}{(1+\frac{\mathcal{D}\mathbf{B}^2}{\varpi\omega^2\bar\Delta})^2}\right\rangle - \frac{\mathbf{B}_0^2\cos(\Delta_+L_{\rm eff})}{\bar\Delta\bar\Delta'}\left\langle\frac{1}{(1+\frac{\mathcal{D}\mathbf{B}^2(x)}{\varpi\omega^2\bar\Delta})(1+\frac{\mathcal{D}\mathbf{B}^2(x+L_{\rm eff})}{\varpi\omega^2\bar\Delta})}\right\rangle + \frac{\pi\mathbf{B}_0^2\sin(\Delta_-L_{\rm eff})}{4\Delta_-}\left[\int_{\bar\Delta}^\infty kdk\mathcal{F}^k_{(0)} + \int_{\bar\Delta'}^\infty kdk\mathcal{F}^k_{(0)}\right].$$

Because of the PCP violation, two linear polarization states of the CMB are effected oppositely to the leading order in $\beta/\omega^2$. Finally, the expression for the photon-to-scalar conversion probability amplitude becomes

$$e^{i(\Delta_-L_{\rm eff})}\bar P_{\gamma\to\phi} = \frac{1}{2\omega^2}e^{i(\Delta_-L_{\rm eff})}\delta^{ij}G_{ij} \approx +\frac{I_e^3\cos(\Delta_-L_{\rm eff})}{\bar\Delta\bar\Delta'\omega^2}\left\langle\frac{\mathbf{B}_0^2+\frac{2}{3}\delta\mathbf{B}\cdot\delta\mathbf{B}}{(1+\frac{\mathcal{D}\mathbf{B}^2}{\varpi\omega^2\bar\Delta})^2}\right\rangle - \frac{\mathbf{B}_0^2\cos(\Delta_+L_{\rm eff})}{\bar\Delta\bar\Delta'\omega^2}\left\langle\frac{1}{(1+\frac{\mathcal{D}\mathbf{B}^2(x)}{\varpi\omega^2\bar\Delta})(1+\frac{\mathcal{D}\mathbf{B}^2(x+L_{\rm eff})}{\varpi\omega^2\bar\Delta})}\right\rangle + \frac{\pi\sin(\Delta_-L_{\rm eff})}{4\Delta_-\omega^2}\left[\mathbf{B}_0^2\left(\int_{\bar\Delta}^\infty+\int_{\bar\Delta'}^\infty\right)kdk\mathcal{F}^k_{(0)} + \frac{2(1+4\beta^2)}{3}\left(\int_{\bar\Delta}^\infty+\int_{\bar\Delta'}^\infty\right)kdk\mathcal{F}^k_{(1)}\right]. \quad \text{(A26)}$$

It is important to note that the photon-to-scalar conversion probability depends on the parity-violating parameter to the order $\beta^2$. The intuitive reason behind this is that the energy density of the electromagnetic field does not depend on $\beta$ linearly. In order to express all the above quantities in terms of magnetic and electron density power spectrum, we need to use perturbative expansion. To the leading order in $1/\omega^2$, all the correlation functions appeared in the above expressions for the Stokes parameters can be expressed as follows:

$$\left\langle\frac{1}{(1+\frac{\mathcal{D}\mathbf{B}^2}{\varpi\omega^2\bar\Delta})^2}\right\rangle \approx \left\langle 1-2\frac{\mathcal{D}\mathbf{B}^2}{\varpi\omega^2\bar\Delta}\right\rangle = 1-2\frac{\langle\delta\mathbf{B}\cdot\delta\mathbf{B}\rangle}{\varpi\omega^2\bar\Delta},$$
$$\left\langle\frac{1}{(1+\frac{\mathcal{D}\mathbf{B}^2(x)}{\varpi\omega^2\bar\Delta})(1+\frac{\mathcal{D}\mathbf{B}^2(x')}{\varpi\omega^2\bar\Delta})}\right\rangle \approx 1-2\frac{\langle\delta\mathbf{B}\cdot\delta\mathbf{B}\rangle}{\varpi\omega^2\bar\Delta},$$
$$\left\langle\frac{\delta\mathbf{B}(x)\cdot\delta\mathbf{B}(x')}{(1+\frac{\mathcal{D}\mathbf{B}^2(x)}{\varpi\omega^2\bar\Delta})(1+\frac{\mathcal{D}\mathbf{B}^2(x')}{\varpi\omega^2\bar\Delta})}\right\rangle \approx \langle\delta\mathbf{B}(x)\cdot\delta\mathbf{B}(x')\rangle \times\left(1-\frac{10}{3}\frac{\langle\delta\mathbf{B}\cdot\delta\mathbf{B}\rangle}{\varpi\omega^2\bar\Delta}\right). \quad \text{(A27)}$$

In the above expressions, we assume that the distribution of the fluctuating magnetic field $\delta\mathbf{B}$ in the ICM is approximately Gaussian. With this approximation, one can express all the unknown correlation functions in terms of ICM magnetic and electron density power spectrum, namely, $P_{\mathbf{B}}$ and $P_e$ as follows:

$$\int_{\bar\Delta}^\infty kdk\mathcal{F}^k_{(0)} = I_e^2\bar\rho_e^{-2}\left(1-2\frac{\langle\delta\mathbf{B}\cdot\delta\mathbf{B}\rangle}{\varpi\omega^2\bar\Delta}\right)\int_{\bar\Delta}^\infty kdkP_e(k),$$
$$\int_{\bar\Delta}^\infty kdk\mathcal{F}^k_{(1)} = \left(1-\frac{10}{3}\frac{\langle\delta\mathbf{B}\cdot\delta\mathbf{B}\rangle}{\varpi\omega^2\bar\Delta}\right)\left[I_e^2\bar\rho_e^{-2}\langle\delta\mathbf{B}\cdot\delta\mathbf{B}\rangle \times\int_{\bar\Delta}^\infty kdkP_e(k) + I_e^3\int_{\bar\Delta}^\infty kdkP_{\mathbf{B}}(k)\right]. \quad \text{(A28)}$$

It is important to note that all the relevant observable quantities depend on a critical length scale called $\bar\Delta^{-1}$.


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