Cosmological behavior of a parity and charge-parity violating varying alpha theory

Debaprasad Maity[1,2,*] and Pisin Chen[1,2,3,†]

[1]*Department of Physics and Center for Theoretical Sciences, National Taiwan University, Taipei 10617, Taiwan*
[2]*Leung Center for Cosmology and Particle Astrophysics, National Taiwan University, Taipei 106, Taiwan*
[3]*Kavli Institute for Particle Astrophysics and Cosmology, SLAC National Accelerator Laboratory, Menlo Park, California 94025, USA*

(Received 7 June 2010; published 19 April 2011)

In this paper we construct a phenomenological model in which the time variation of the fine-structure constant, α, is induced by a parity and charge-parity (PCP) violating interaction. Such a PCP violation in the photon sector has a distinct physical origin from that in the conventional models of this kind. We calculate the cosmological birefringence so induced in our model and show that it in turn produces a new nonvanishing multipole moment correlation between the temperature and the polarization anisotropies in the CMB spectrum. We have also calculated the amount of optical rotation due to a strong background magnetic field and the effect of our new PCP violating term on the variation of α during the cosmic evolution. We found that only in the radiation dominated era can the contribution of the new PCP violating term to the variation of α be nonvanishing.

DOI: 10.1103/PhysRevD.83.083516 PACS numbers: 98.80.Cq

I. INTRODUCTION

Both inflation and late-time cosmic acceleration have puzzled physicists for a long time. It has become clear that the final solutions to these may require new physics beyond general relativity and the standard model of particle physics in order to explain those observations. *A priori*, however, we do not have any clear idea about how to proceed unless we can identify some new guiding principles. Although several new principles, such as the holographic principle, have been introduced to explain cosmological phenomena, these are nonetheless still at the preliminary stage. An alternative would be the more conservative path of drawing an analogy from known physics.

As is well-known, in parity (P) and charge-parity (*CP*) symmetries are violated in the electroweak sector of the standard model particle physics. Considering this as a guiding principle, construction of a P and *CP* violating extension has been considered in the new physics models that produce inflation as well as late-time acceleration. For the last several years, many different parity-violating models have been put forward [1–5]. The very basic idea of all those models is to add an explicit parity-violating term in the Lagrangian. Because of its nature, this parity-violating term leads to cosmic birefringence [1,2] and left-right asymmetry in the gravitational wave dynamics [3,4]. String inspired models with nonstandard parity-violating interactions have also been discussed [5]. Various observable effects of these new parity-violating models have been extensively investigated in order to put constrains on the corresponding parameters.

In this paper we construct a parity and charge-parity (PCP) violating model in the framework of "varying-alpha theory". Some aspects of our model are similar to that proposed by Carroll [1]. But as we will see, our model has the advantage over Carroll's in that the origin of the parity violation may be more physically motivated.

Cosmological variation of fundamental constants in nature has gained considerable interests in the recent past because of two fundamental reasons. First, triggered by the string theory there has been a resurgence of motivation to reconsider the variation of fundamental constants in cosmology as well as particle physics model building. As is well known, string theory gives us a consistent framework, where the effective four-dimensional fundamental constants depend on the compactifications of the extra dimensions. In principle, therefore, all the so-called fundamental constants in our four-dimensional world could actually be spacetime varying functions. The dynamics of such varying "constants" actually depends on the specific compactification that we make. Second, increasingly high precession cosmological, as well as laboratory, experiments give us hope that the signature of new physics, including those that give rise to the variation of fundamental constants, may emerge in the near future.

In spite of the long history of the speculation of the variation of fine-structure constant [6], the first consistent, gauge-invariant and Lorentz invariant, framework of α variability was proposed by Bekenstein [7]. Subsequently this subject has attracted much attention and was extensively studied in [8–10], mainly due to the first observational evidence from the quasar absorption spectra that the fine-structure "constant" might change with cosmological time [11–13]. This observation suggests that the value of α may be lower in the past in a cosmological time scale, with $\Delta\alpha/\alpha = -0.72 \pm 0.18 \times 10^{-5}$ for redshift $z \approx 0.5$–3.5.

*debu.imsc@gmail.com
†chen@slac.stanford.edu

 © 2011 American Physical Society

We organize this paper as follows: in Sec. II we construct the PCP violating model in the photon sector after briefly reviewing the basic concept of "varying-alpha theory". Then we discuss the theoretical implication and prediction of our model in different cosmological phenomena. In Sec. III we study the cosmic birefringence phenomena and calculate the rotation angle of the polarization of the electromagnetic wave in a leading-order approximation. We then discuss its effect on the parity-violating correlation function in the CMB polarization spectrum. In Sec. IV we discuss the effect of the background magnetic field on the rotation of the plane of polarization. As we know, there exists magnetic fields at cosmic scales that may affect the CMB polarization due to some scalar field coupling. There exists several laboratory-based experiments that aim at measuring the change of polarization of a electromagnetic wave induced by such a nontrivial (pseudo) scalar-photon coupling in a background magnetic field. Motivated by all these, we calculate the amount of optical rotation induced in a background cosmic magnetic field, which has a direct contact with experiments. In the subsequent Sec. V we first briefly review the varying-alpha cosmology and then calculate the alpha variation induced by the PCP violating term. In general, it is very difficult to solve the type of equation of motion that appeared in our model. This was done in our calculation by using the matched approximation adopted from [9]. Concluding remarks and future prospects are provided in Sec. VI.

II. PARITY-VIOLATING VARYING-ALPHA THEORY

In this section we will start with a general discussion on the varying fine-structure constant theory from the standard literatures [7–9]. In the framework of the varying-alpha theory, the simplest way to induce the variation of α is by requiring that the electric charge varies as $e = e_0 e^{\phi(x)}$, where e_0 denotes the coupling constant of a particle and $\phi(x)$ is a dimensionless scalar field. The fine-structure constant is therefore $\alpha = e_0^2 e^{2\phi(x)}$. There is an arbitrariness involved in the definition of $\phi(x)$ due to the shift invariance, i.e. $\phi \to \phi + c$. An important point to mention here is that the well-known charge conservation is violated. But in order to be consistent with the quantum field theory, the new modified electromagnetic theory should be gauge-invariant. Since e is the electromagnetic coupling, the $\phi(x)$ field couples to the gauge field as $e^{\phi}(x)A_\mu$ in the Lagrangian and in the gauge transformation, which leaves the action invariant as

$$e^{\phi}A_\mu \to e^{\phi}A_\mu + \chi_{,\mu}. \tag{1}$$

So, from the above considerations, the unique gauge-invariant and shift symmetric Lagrangian for the modified electromagnetic field can be written as

$$S_{\rm em} = -\frac{1}{4}\int d^4x\sqrt{-g}e^{-2\phi}F_{\mu\nu}F^{\mu\nu}, \tag{2}$$

where the new electromagnetic field strength tensor is defined as

$$F_{\mu\nu} = (e^{\phi}A_\nu)_{,\mu} - (e^{\phi}A_\mu)_{,\nu}. \tag{3}$$

In the above action and for the rest of this paper we set $e_0 = 1$ for convenience. As one can see, the above action reduces to the usual form when ϕ is constant. The dynamics of the $\phi(x)$ field is controlled by the kinetic term

$$\mathcal{L}_\phi = -\frac{\omega}{2}\int d^4x\sqrt{-g}\phi_{,\mu}\phi^{,\mu}, \tag{4}$$

which is clearly invariant under the shift symmetry of ϕ. Here the coupling constant ω can be written as $\hbar c/l^2$, where l is the characteristic length scale of the theory above which the Coulomb force law is valid for a point charge. From the present experimental constraints the energy scale, $\hbar c/l$, has to be above a few tens of MeV to avoid conflict with experiments.

One of the natural assumptions in constructing the above Lagrangian is time-reversal invariance. But we will relax this assumption and try to analyze its implications. An obvious term that is consistent with the varying-alpha framework, yet violates PCP, is $\tilde{F}_{\mu\nu}F^{\mu\nu}$, where $\tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\sigma\rho}F_{\sigma\rho}$ is the Hodge dual of the electromagnetic field tensor. In conventional electromagnetism this does not contribute to the classical equation of motion. But in the present framework this is no longer true because of its coupling with the scalar field $\phi(x)$. As we have explained in the introduction, at the present level of experimental accuracy PCP violation in the electromagnetic sector may not be ruled out, and if the PCP in this EM sector is indeed violated, then there should be some interesting consequences. Motivated by this, we write down a parity-violating Lagrangian:

$$\begin{aligned}\mathcal{L} &= M_p^2 R - \frac{\omega}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4}e^{-2\phi}F_{\mu\nu}F^{\mu\nu}\\ &\quad + \frac{\beta}{4}e^{-2\phi}F_{\mu\nu}\tilde{F}^{\mu\nu} + \mathcal{L}_m,\end{aligned} \tag{5}$$

where R is the curvature scalar and β is a free coupling parameter in our model. As we can see, the scalar field ϕ plays a similar role as that of the dilaton in the low-energy limit of string and M theories, the important difference being that it induces a PCP violating electromagnetic interaction. For our purpose, we assume β as a free but small parameter. Here we want to emphasize that the model can be thought of as a unified framework for dealing with different cosmological phenomena. At the present level of experimental accuracy, investigations of parity or charge-parity violation, beyond standard model, may shed some new light on the fundamental laws of physics. With the interest of phenomenological impacts on the present cosmological observations, subsequently we will discuss some consequences of our model.

Before this let us write down the full set of equations of motion:

$$G_{\mu\nu} = \frac{1}{M_p^2}(T^{\rm mat}_{\mu\nu} + T^{\Phi}_{\mu\nu} + e^{-2\phi}T^{\rm em}_{\mu\nu}), \tag{6}$$

where the energy-momentum tensors are

$$(a)\ T^{\rm mat}_{\mu\nu} = \frac{1}{2}g_{\mu\nu}\mathcal{L}_m - \frac{\partial\mathcal{L}_m}{\partial g_{\mu\nu}}, \tag{7}$$

$$(b)\ T^{\rm em}_{\mu\nu} = \frac{1}{2}e^{-2\phi}\left\{F_{\mu\alpha}F^{\alpha}_{\nu} - \frac{1}{4}g_{\mu\nu}F_{\mu\nu}F^{\mu\nu}\right\}, \tag{8}$$

$$(c)\ T^{\Phi}_{\mu\nu} = \frac{\omega}{2}\left\{\partial_\mu\phi\partial^\nu\phi - \frac{1}{2}g_{\mu\nu}\partial_\alpha\phi\partial^\alpha\phi\right\}. \tag{9}$$

The electromagnetic field equation then becomes

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}F^{\mu\nu}) + \partial_\mu\phi(-F^{\mu\nu} + \beta\tilde{F}^{\mu\nu}) = 0. \tag{10}$$

Varying it with respect to ϕ, we get

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g\partial^\mu}\phi) = \frac{e^{-2\phi}}{2\omega}[-F_{\mu\nu}F^{\mu\nu} + \beta F_{\mu\nu}\tilde{F}^{\mu\nu}]. \tag{11}$$

In the subsequent sections we will study some cosmic phenomena which may be relevant to the future precision cosmological measurements.

III. COSMOLOGICAL BIREFRINGENCE

Cosmological birefringence (CB) is a wavelength-independent rotation of a photon polarization vector after traversing a long cosmic distance. It has long been the subject of interest in the context of cosmic microwave background (CMB) phenomena [1,2,14,15] where its polarization properties crucially depend on CB. The origin of this effect may come from either cosmic inhomogeneities or some nontrivial coupling of photon with other fields. In this section, we will study this effect and show that the main contribution to CB comes from our PCP violating term in the Lagrangian in Eq. (5). In order to calculate this effect, we assume the background spacetime as the spatially-flat FRW expanding background. On that background we will compute the cosmic optical rotation which is the measure of CB. For this it useful to take the background FRW metric in the conformal time that is

$$ds^2 = a(\eta)^2(-d\eta^2 + dx^2 + dy^2 + dz^2), \tag{12}$$

where η is the conformal time and $a(\eta)$ is the conformal scale factor. Since the electromagnetic theory is conformal invariance in four dimensions, the Maxwell equations turn out to be of the standard type with the modifications coming from the nontrivial scalar field ϕ coupling:

$$\begin{aligned}&\nabla\cdot\mathbf{E} = 2\nabla\phi\cdot\mathbf{E} - 4\beta\nabla\phi\cdot\mathbf{B},\\ &\partial_\eta(\mathbf{E}) - \nabla\times\mathbf{B} = 2(\dot\phi\mathbf{E} - \nabla\phi\times\mathbf{B}) - 4\beta(\dot\phi\mathbf{B} + \nabla\phi\times\mathbf{E}),\\ &\nabla\cdot\mathbf{B} = 0, \quad \partial_\eta\mathbf{B} + \nabla\times\mathbf{E} = 0.\end{aligned} \tag{13}$$

The wave equation for the $\mathbf{B}$ then becomes

$$\ddot{\mathbf{B}} - \nabla^2\mathbf{B} = 2\dot\phi(\dot{\mathbf{B}} + 2\beta\nabla\times\mathbf{B}). \tag{14}$$

Now, what is important point to keep in mind here is that the definition of physical electromagnetic field strength we use are $F_{i0} = \mathbf{E}_i$ and $F_{ij} = \epsilon_{ijk}\mathbf{B}_k$. Where, $i = 1, 2, 3$ and ϵ is the three spatial dimensional Levi-Civita tensor density.

We assume general wave solutions of the form $\mathbf{B} = \mathbf{B}_0(\eta)e^{-i\mathbf{k}\cdot\mathbf{z}}$, and take the z direction as the propagation direction of the electromagnetic waves, i.e.,$\mathbf{k} = k\hat{\mathbf{e}}_z$, The equations for the polarization states, viz., $b_\pm(\eta) = \mathbf{B}_{0x}(\eta) \pm i\mathbf{B}_{0y}(\eta)$ turns out to be

$$\ddot{b}_\pm - 2\dot\phi\dot{b}_\pm + (\mathbf{k}^2 \mp 4\mathbf{k}\beta\dot\phi)b_\pm = 0, \tag{15}$$

while the equation of motion for the scalar field is

$$\ddot\phi - 2\frac{\dot a}{a}\dot\phi = \frac{1}{\omega a^2}[-(\mathbf{E}^2 - \mathbf{B}^2) + 4\beta\mathbf{B}\cdot\mathbf{E}]. \tag{16}$$

It is in general difficult to solve the above nonlinear coupled equations exactly. We therefore look for an approximate solution to the leading-order in the large ω limit. In this limit, the solution for the scalar field would be

$$\phi = B\int\frac{d\eta}{a(\eta)^2} + C + \mathcal{O}(\omega); \qquad \dot\phi = \frac{B}{a(\eta)^2} \tag{17}$$

where B and C are the integration constants. We also assume the coupling constant β and the value of the scalar field to be vary small based on the various observational constraints. From the above expressions, we see that the energy density of the scalar field is proportional to B. We therefore know that this constant must be very small in order for it not to have a backreaction to the background cosmological evolution.

Since the change of $b_\pm$ is expected to be small, we estimate the optical activity using the WKB method [16]. In the long wavelength limit and for small coupling constant β, we assume the solution of the above equation for $b_\pm$ to be

$$b_\pm = e^{ikS_\pm(\eta)}; \qquad S_\pm(\eta) = S^0_\pm + \frac{1}{k}S^1_\pm + \ldots \tag{18}$$

Therefore the solution based on the above ansatz is

$$S^0_\pm = \eta; \qquad S^1_\pm = -\frac{1}{2}(2i \pm 4\beta)\int\dot\phi d\eta. \tag{19}$$

It is clear from the above solution that the expression for the optical rotation of the plane of polarization is

$$\Delta = 4\beta \int_{\eta_i}^{\eta_f} \dot{\phi} d\eta = 4\beta |\phi(\eta_f) - \phi(\eta_i)|, \tag{20}$$

where η_i and η_f are the initial and final conformal time for the electromagnetic field to be detected. As expected, the leading contribution to the cosmic optical rotation comes from the PCP violating term. In order to connect with observations, we rewrite the above expression for the optical rotation to the leading-order in ω as

$$\Delta = 2\frac{h\beta}{H_0} \int_0^z \frac{(1+z)dz}{\sqrt{(\Omega_m + \Omega_{dm})(1+z)^3 + \Omega_{de}}}, \tag{21}$$

where h is the energy density of the scalar field, z is the redshift factor, and Ω's are the cosmological density parameters. In terms of the density of the scalar field we can write down the expression for the optical rotation as

$$\Delta \simeq \frac{M_p}{\omega\beta}\sqrt{\rho} \times 5.6 \times 10^{43}; \quad \rho = \frac{\omega^2 h^2 (1+z)^6}{2} \tag{22}$$

where M_p is the Planck constant. In the above expression we consider $z = 0.4$ just because observational data for radio galaxies and quasars have been analyzed in great detail for the redshift $z \geq 0.4$. As one can see from the above expression, the optical rotation is crucially dependent upon the scale of alpha variation, coupling constant β and the energy density of the new scalar field. In the next subsection we will investigate its impact on the CMB polarization and constrain the value of the parameter β in our model.

Effect of birefringence on CMB anisotropy

As we have already discussed, the CMB is one of the primary windows to peek into the early Universe. Recent CMB observations have reached remarkable precision and proved to be consistent with the so-called standard model of cosmology. With such high precision we can expect that the CMB may provide additional information to constrain new physics beyond the standard model. A positive answer is expected from the study of CMB polarization. In the context of parity-violating effects, there have already been many studies [17]. These violations might also have a measurable imprint on the observed CMBP pattern, whose statistical properties are constrained by the assumption of symmetry conservation.

It has been noted by several authors [2,18] that certain nonvanishing multipole moment correlations between the temperature anisotropy and polarization of the CMB could appear, if there exists parity-violating interaction in the photon sector. Such an interaction appears in our proposal in the framework of varying-alpha theory. As is well known, the angular distribution of the temperature anisotropy of the CMB can be expressed in terms of the expansion in spherical harmonics [19]:

$$\frac{\Delta T}{T}(\mathbf{n}) = \sum_{l,m} a^T_{lm} Y^T_{lm}(\mathbf{n}). \tag{23}$$

The polarization of the CMB is expressed in terms of a 2×2 traceless symmetric tensor $\mathcal{P}_{ab}(\mathbf{n})$ whose components are the Stokes parameters. This tensor can be decomposed into its irreducible "gradient" (or E) and "curl" (or B) parts that have opposite spatial parities. The angular distribution of this polarization tensor can thus be expressed in terms of the matrix spherical harmonics as [2,18]

$$\mathcal{P}^E_{ab}(\mathbf{n}) = \sum a^E_{lm} Y^E_{lm,ab}(\mathbf{n})$$
$$\mathcal{P}^B_{ab}(\mathbf{n}) = \sum a^E_{lm} Y^B_{lm,ab}(\mathbf{n}). \tag{24}$$

One defines the correlation of the multipole moment coefficients, a^X_{lm}, $X = T, E, B$, as

$$C^{XX'}_l \equiv \langle a^X_{lm} a^{X'}_{lm} \rangle. \tag{25}$$

Clearly, correlations such as C^{XX}_l as well as C^{TE}_l all preserve P, while correlations such as C^{TB}_l and C^{EB}_l are obviously P violating, the appearance of which requires an explicitly P-violating interaction as mentioned earlier. The optical activity described earlier implies that if a correlation like C^{TE}_l does indeed arise due to reionization or otherwise, then the passage of the Thompson scattered photons through the scalar field ϕ background would produce the P-violating correlation term C^{TB}_l through the rotation [2,18]:

$$C'^{\mathrm{TB}}_l = C^{\mathrm{TE}}_l \sin 2\Delta \tag{26}$$

$$C'^{\mathrm{EB}}_l = \frac{1}{2}(C^{\mathrm{EE}}_l - C^{\mathrm{BB}}_l) \sin 4\Delta \tag{27}$$

where the primed quantities are rotated and Δ is the rotation of the plane of polarization of light. We clearly see that the effect of cosmic birefringence, which is parity-violating in nature, in our model can lead to some nonvanishing correlations.

The recent high precession cosmological observations put tight constraints on the possible amount of optical rotation compared to the previous studies [20–22]. The polarization data from radio galaxies and quasars for the redshift between $z = 0.425$ and $z = 2.012$ gives the average value of $\Delta = -.6^0 \pm 1.5^0$. On the other hand, the WMAP 7-years data [22] suggests that the rotation angle of the polarization plane would be $\Delta = -1.1^0 \pm 1.3^0$. That is, according to the WMAP polarization data there is no clear indication for the parity-violating interaction in the photon sector. However, as we have mentioned above, the most stringent constraint would come from the nonvanishing TB and EB correlations, whose values, as our model predicts, are different by a factor $\sin 2\Delta \sim 8\beta\delta\phi$. Since β is a free parameter to be fixed in our model, we need additional observational constraints to fix it. In the next section we will discuss the variation of α induced by our PCP violating term. In principle this will help us fix the β.

IV. EFFECT OF BACKGROUND ELECTROMAGNETIC FIELD

Apart from the cosmological or astrophysical observations, there exist various laboratory-based experiments such as BFRT [23], PVLAS [24], Q&A [25], BMV [26], etc., which make use of the photon-to-scalar-field conversion in the presence of a strong background magnetic or electric field for the indirect detection of new scalar fields. In this regard different theoretical models based on the dilaton-photon type coupling, $e^{-2\phi}F_{ab}F^{ab}$, or the standard QCD axion-photon type coupling, $\phi F_{ab}\tilde{F}^{ab}$, mediated by the background magnetic or electric field have been considered extensively. In our present model we have employed both these terms in a single varying-alpha framework. As a first step, in this section we will try to do a qualitative analysis of our model under the background magnetic field. We want to emphasize here that this study is important in the cosmological context as well. As we know, at cosmological scales there exist background magnetic fields. These cosmic magnetic fields may have a significant effect on the CMB polarization in addition to the scalar coupling effect that we describe in this paper. The polarization of CMB is known to have encoded the information of early Universe, specifically that of the inflationary epoch. The possibility of additional CMB polarizations induced by some other external field would undoubtedly complicate the issue and it must be clarified. With this motivation in mind, we calculate the effect of background electromagnetic field on the rotation plane of polarization. In terms of the vector potential, the main equations of our interests are

$$(\nabla^2 + \varpi^2)\mathbf{A}_x = 4i\beta\mathbf{B}_0\varpi\phi, \quad (28)$$

$$(\nabla^2 + \varpi^2)\mathbf{A}_y = -2\mathbf{B}_0\partial_z\phi, \quad (29)$$

$$(\nabla^2 + \varpi^2)\mathbf{A}_z = 2\mathbf{B}_0\partial_y\phi, \quad (30)$$

$$(\nabla^2 + \varpi^2)\phi = \frac{2\mathbf{B}_0^2}{\omega}\phi - \frac{2\mathbf{B}_0}{\omega}(\partial_y A_z - \partial_z A_y) - \frac{4i\beta\mathbf{B}_0\varpi}{\omega}A_x, \quad (31)$$

in the presence of background magnetic field $\mathbf{B}_0$ in the x direction. Because of the smallness of the effect, we consider only the linear order equations for the scalar-photon system. In the above derivation we used the gauge condition, $\nabla\cdot\mathbf{A} = 0$, and specified the scalar potential: $\mathbf{A}_0 = 0$. ϖ is the frequency of the fields. Let the propagation direction of the electromagnetic wave be orthogonal to the external magnetic field $\mathbf{B}_0$, say in z direction. We then write

$$\mathbf{A}(z,t) = \mathbf{A}^0 e^{-i\varpi t + ikz}; \qquad \phi(z,t) = \phi^0 e^{-i\varpi t + ikz}. \quad (32)$$

As is clear from the above ansatz, the equation for A_z is no longer coupled with ϕ. From the other three equations for A_x, A_y, ϕ, consistency condition leads to three roots for the frequency ϖ as follows:

$$\varpi^2 = k^2, \qquad \varpi_\pm^2 = k^2 + \delta_\pm \quad (33)$$

$$\delta_\pm = \frac{\mathbf{B}_0^2}{\omega}(1+8\beta^2) \pm \sqrt{\frac{\mathbf{B}_0^4}{\omega^2}(1+8\beta^2)^2 + \frac{4\mathbf{B}_0^2k^2}{\omega}(1+4\beta^2)}. \quad (34)$$

To establish the connection with the experimental setup, we can consider the initial ($t = 0, x = 0$) electromagnetic field to be linearly polarized and making an angle with the external magnetic field $\mathbf{B}_0$, so that

$$\begin{aligned} \mathbf{A}_x(z=0,t=0) &= \alpha_1 = \cos\alpha; \\ \mathbf{A}_y(z=0,t=0) &= \alpha_2 = \sin\alpha; \\ \phi(z=0,t=0) &= 0. \end{aligned} \quad (35)$$

With these boundary conditions, we can have a unique solution like

$$\begin{aligned} \mathbf{A}_x &= (a_x e^{-i\varpi t} + b_x e^{-i\varpi_+ t} + c_x e^{-i\varpi_- t})e^{ikz} \\ \mathbf{A}_y &= (a_y e^{-i\varpi t} + b_y e^{-i\varpi_+ t} + c_y e^{-i\varpi_- t}) \\ e^{ikz}\phi &= \phi_0(e^{-i\varpi_+ t} - e^{-i\varpi_- t})e^{ikz}, \end{aligned} \quad (36)$$

where

$$\begin{aligned} b_x &= -\frac{2\beta\varpi_+}{k}b_y = \frac{2\beta\varpi_+}{k}\frac{(-k^2+\varpi_-^2)}{(-k^2+\varpi_+^2)}c_y = -\frac{\varpi_+}{\varpi_-}\frac{(-k^2+\varpi_-^2)}{(-k^2+\varpi_+^2)}c_x = \frac{4i\beta\mathbf{B}_0\varpi_+}{(-k^2+\varpi_+^2)}\phi_0 \\ a_y &= \frac{2\beta\varpi}{k}a_x = \alpha_2 + \frac{k}{2\beta\varpi_-}\left[\frac{\varpi_+^2 - \varpi_-^2}{-k^2+\varpi_+^2}\right]c_x \qquad c_x = \frac{1}{\mathcal{F}}\left(\alpha_1 - \frac{k\alpha_2}{2\beta\varpi}\right) \\ \mathcal{F} &= \frac{4\beta^2(\varpi\varpi_-(-k^2+\varpi_+^2) - \varpi\varpi_+(-k^2+\varpi_-^2)) + k^2(\varpi_+^2 - \varpi_-^2)}{4\beta^2\varpi\varpi_-(-k^2+\varpi_+^2)}. \end{aligned} \quad (37)$$

While traversing through the region of external magnetic field, after $t = L$ the resulting interaction causes the wave solution to have a modified amplitude of the form

$$\mathbf{A}_x = a_x e^{-i\varpi L} + b_x e^{-i\varpi_+ L} + c_x e^{-i\varpi_- L}, \quad (38)$$

$$\mathbf{A}_y = a_y e^{-i\varpi L} + b_y e^{-i\varpi_+ L} + c_y e^{-i\varpi_- L}. \quad (39)$$

From the above set of expressions, we see that the vector potential describes an ellipse with the major axis at an angle

$$\theta \simeq \tan^{-1}\left(\frac{\alpha_2}{\alpha_1}\right) + \frac{\sin(2\alpha)}{4}\left(\frac{\mathcal{L}}{\cos^2(\alpha)} - \frac{\Gamma}{\sin^2(\alpha)}\right) \quad (40)$$

where

$$\mathcal{L} = 2a_x b_x \sin^2\left(\frac{\Delta_+}{2}\right) + 2a_x c_x \sin^2\left(\frac{\Delta_-}{2}\right) + 2c_x b_x \sin^2\left(\frac{\Delta}{2}\right),$$
$$\Gamma = 2a_y b_y \sin^2\left(\frac{\Delta_+}{2}\right) + 2a_y c_y \sin^2\left(\frac{\Delta_-}{2}\right) + 2c_y b_y \sin^2\left(\frac{\Delta}{2}\right),$$
$$\Delta_+ = (\varpi_+ - \varpi)L; \qquad \Delta_- = (\varpi_- - \varpi)L;$$
$$\Delta = (\varpi_+ - \varpi_-)L. \quad (41)$$

Now, Eq. (40) yields the expression for the optical rotation of the plane of polarization as

$$\delta = \frac{\sin(2\alpha)}{4}\left(\frac{\mathcal{L}}{\cos^2(\alpha)} - \frac{\Gamma}{\sin^2(\alpha)}\right). \quad (42)$$

This is the quantity that establishes the direct connection with the experimental data. The similar analysis can also be made for the background electric field as well. In our forthcoming paper we will consider a more detailed analysis of the background electromagnetic field effect on the scalar-photon mixing and its effect on the various laboratory as well as cosmological experiments. So far we have studied the effect of the scalar field on the polarization of the electromagnetic wave under various conditions that may arise in a laboratory or cosmological settings. In the next section we will consider the change of the scalar field or fine-structure constant under the background cosmological evolutions.

V. VARYING α COSMOLOGY

The effect of cosmic evolution on the variation of the fine-structure constant in the framework of the variation of a scalar field $\phi(x)$ has been extensively studied [8–10]. This has been referred to as the Bekenstein-Sandvik-Barrow-Magueijo (BSBM) theory. Here we only analyze the variation of α induced by the PCP violating effect. As we have already mentioned , the effective time-varying fine-structure constant is

$$\alpha(t) = e^{2\phi(t)}. \quad (43)$$

In the subsequent analysis we will switch over to the usual cosmic time. The fractional variation of α then becomes

$$\frac{\Delta\alpha}{\alpha(t_0)} = \frac{\alpha(t_0) - \alpha(t)}{\alpha(t_0)} = 1 - e^{2[\phi(t)-\phi(t_0)]}$$
$$\approx 2[\phi(t_0) - \phi(t)] = 2\Delta\phi(t), \quad (44)$$

where t_0 refers to the present epoch. The observational upper limit of the time variation of the fine-structure constant [12] then puts a constraint on the variation of the scalar field:

$$\frac{|\Delta\alpha|}{\alpha(t_0)} \simeq 10^{-5}. \quad (45)$$

In order to further constrain our model parameters we need to know the nature of solution for the scalar field $\phi(t)$. We will do so in the subsequent subsections.

A. General analysis

In this section we study the cosmological evolution of the scalar field during the various phases of the Universe evolution history. In the cosmological setting the equation of motion is

$$G_{\mu\nu} = \frac{1}{M_p^2}(\langle T^{\rm mat}_{\mu\nu}\rangle + T^{\Phi_H}_{\mu\nu} + e^{-2\phi}\langle T^{\rm em}_{\mu\nu}\rangle). \quad (46)$$

The average $\langle\cdots\rangle$ denotes a statistical average over the current state of the Universe. The electromagnetic field equation becomes

$$\nabla_\mu[e^{-2\phi}(\langle F^{\mu\nu}\rangle + \beta\langle\tilde{F}^{\mu\nu}\rangle)] = 0, \quad (47)$$

while variation with respect to the ϕ field gives the cosmological evolution for the field:

$$\Box\phi = \frac{e^{-2\phi}}{2\omega}[-\langle F_{\mu\nu}F^{\mu\nu}\rangle + \beta\langle F_{\mu\nu}\tilde{F}^{\mu\nu}\rangle]. \quad (48)$$

For future convenience we use the notation $\mathcal{L}_{\rm em} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$.

In the standard electrodynamics both terms on the right-hand side of Eq. (48) vanish. The *PCP*-violating time variation of ϕ, and therefore that of α, causes the cosmic birefringence which in turn breaks the orthogonality properties of electromagnetic field, and as a result the term $\langle F_{\mu\nu}\tilde{F}^{\mu\nu}\rangle \simeq \langle\mathbf{E}\cdot\mathbf{B}\rangle$ can in principle be nonvanishing during the radiation epoch. We emphasize that this particular effect on the α variation was not present in the original BSBM theory. The other known contribution to the variation of α comes from nearly pure electrostatic or magnetostatic energy of the matter field. As has been extensively discussed in Refs. [7–9], the nonrelativistic matter contributes to the right-hand side of Eq. (48) through the spatial variation of the Coulombic mass. This contribution is parametrized by the ratio $\zeta_m = \mathcal{L}_{\rm em}/\rho$, where ρ is the energy density and $\mathcal{L}_{\rm em} \approx E^2/2$ for baryonic matter. BBN infers an approximate value for the baryon density of $\Omega_B \approx 0.03$ with a Hubble parameter $h_0 \approx 0.6$, implying $\Omega_{\rm CDM} \approx 0.3$. So, ζ_m depends strongly on the nature of the dark matter and can be either positive or negative with a modulus between 0 and 1.

Assuming a spatially-flat, homogeneous and isotropic Friedmann metric with expansion scale factor $a(t)$,

$$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2), \tag{49}$$

we obtain the Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_p^2}[\rho_m\{1 + e^{-2\phi}\zeta_m\} + e^{-2\phi}\rho_r + \rho_\phi] + \frac{\Lambda}{3} \tag{50}$$

where Λ is a constant cosmological vacuum energy density and $\rho_\phi = \frac{1}{2}[\dot{\phi}^2 + V(\phi)]$. For the scalar field we get

$$\ddot{\phi} + 3H\dot{\phi} = \frac{e^{-2\phi}}{\omega}\left[-2\zeta_m\rho_m + \frac{4}{a^3}\langle \mathbf{E}\cdot\mathbf{B}\rangle\right], \tag{51}$$

where $H \equiv \dot{a}/a$. The conservation equations for the non-interacting radiation and matter densities ρ_r and ρ_m, respectively, are

$$\dot{\tilde{\rho}}_m + 3H\tilde{\rho}_m = 0, \tag{52}$$

$$\partial_t(e^{-2\phi}\rho_r) + 4He^{-2\phi}\rho_r = 0, \tag{53}$$

where ρ_r is the radiation energy density. From the last equation one finds $\tilde{\rho}_r \equiv e^{-2\phi}\rho_r \propto a^{-4}$, while the solution for the matter density is $\tilde{\rho}_m = \{1 + e^{-2\phi}\}\rho_m \propto a^{-3}$. Equations (50)–(53) govern the Friedmann universe with a time-varying fine-structure constant $\alpha(t)$. They depend on the choice of the parameters ζ_m/ω and β/ω^2. In general it is difficult to solve the Eqs. (50) and (51). Since the effect of the new scalar field is expected to be very small on the background cosmological evaluation, we will try to solve the scalar field evolution equation in the leading-order approximation with the standard Hubble expansion included.

B. Evolution of scalar field in different cosmological era

In this section we analyze the evolution of the scalar field in the various cosmological eras. For simplicity, as well analytical purposes, we will ignore the potential term of the field.

The radiation dominated era

We here show that during the radiation era there exists a contribution to the variation of α through the PCP violating term as opposed to the usual Bekenstein theory. In this era the Friedmann equation is

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_p^2}\left[e^{-2\phi}\rho_r + \frac{1}{2}\dot{\phi}^2\right], \tag{54}$$

while the equation for the scalar field becomes

$$\frac{d}{dt}(\dot{\phi}a^3) = e^{-2\phi}\frac{4\beta}{\omega}\langle \mathbf{E}\cdot\mathbf{B}\rangle. \tag{55}$$

As we have discussed before, the average value of a radiation kinetic Lagrangian in pure radiation does not contribute to the α evolution. In order to solve the above equation for the scalar field, we need to know the average value of the PCP violating term in the action. However, we observe from Eq. (47) that in the plane wave limit the essential equation for our study is

$$\partial_0(a\mathbf{E}\cdot\mathbf{B}) = a\mathbf{E}\cdot(\nabla\cdot\mathbf{E}) + \frac{1}{a}\mathbf{B}\cdot(\nabla\cdot\mathbf{B}) + \dot{\phi}(2a\mathbf{E}\cdot\mathbf{B} - 4\beta\mathbf{B}\cdot\mathbf{B}). \tag{56}$$

It is clear from the above equation that $\mathbf{E}$ and $\mathbf{B}$ are not perpendicular to each other due to varying fine-structure constant. In the plane wave limit, we can ignore the first two terms because $\boldsymbol{\kappa}\cdot\mathbf{B} = \boldsymbol{\kappa}\cdot\mathbf{E} = 0$, where $\boldsymbol{\kappa}$ is the wave propagation direction. We then find

$$a\langle \mathbf{E}\cdot\mathbf{B}\rangle = \langle \mathbf{B}\cdot\mathbf{B}\rangle(2\beta + \theta e^{2\phi}), \tag{57}$$

where θ is the integration constant. Equation (56) is a first order differential equation in time. Therefore we can choose the initial condition to be orthogonal i.e. $\mathbf{E}\cdot\mathbf{B} = 0$ such that $\theta = -2\beta e^{-2\phi_0}$, where the initial value of ϕ is taken to be $\phi(t_i) = \phi_0$. The parameter β of our model therefore plays the main role in breaking the orthogonality of the electromagnetic field. The evolution equation for ϕ now becomes

$$\frac{d}{dt}(\dot{\phi}a^3) = \frac{8\beta^2\langle \mathbf{B}\cdot\mathbf{B}\rangle}{a\omega}(e^{-2\phi} - e^{-2\phi_0}). \tag{58}$$

We see that the variation of α depends quadratically in β. As we have mentioned before, in order to solve the above set of equations analytically, we invoke a self-consistent approximation which has been employed in [8]. The basic strategy of this approximation is that it invokes the background solution for the cosmological scale factor in the equation that governs the scalar field evolution. This is justified since at a late stage in the radiation era, the energy of the scalar field should fall faster than that of the radiation.

Specifically, we assume that the scale factor is $a(t) = t^{1/2}$ for the radiation era. Changing the variable to $x = \frac{1}{2}\times\ln(t)$, we find that Eq. (58) becomes

$$\phi'' + \phi' = \mathcal{A}(e^{-2\phi} - e^{-2\phi_0}), \tag{59}$$

where $' \equiv d/dx$ and

$$\mathcal{A} = \frac{8\beta^2\langle \mathbf{B}\cdot\mathbf{B}\rangle}{\omega} \geq 0.$$

The above equation is very difficult to solve analytically. In order to get an analytic expression, let us assume that field variation is very small. Under this approximation we can write

$$\phi'' + \phi' + 2\mathcal{A}(\phi - \phi_0) = 0. \tag{60}$$

Equation (60) can be solved exactly for the varying fine-structure constant:

$$\phi = \phi_0 + \mathcal{C}_1 x^{-\alpha_+} + \mathcal{C}_2 x^{-\alpha_-}; \qquad \alpha_\pm = \frac{1}{4}\left(1 \pm \sqrt{1 - 8\mathcal{A}}\right). \tag{61}$$

In the above discussion for the orthogonality, we chose the initial value of the scalar field to be ϕ_0, which fixes $\mathcal{C}_2 = -\mathcal{C}_1 x_i^{\delta\alpha}$, where $\delta\alpha = \alpha_- - \alpha_+$. Other constants can be fixed by matching the value of a fine-structure constant at the matter-radiation equality epoch. With the above solution, the expression for the fine-structure constant during the radiation dominated era is

$$\alpha \sim \exp[\phi_0 + 2\mathcal{C}_1 t^{-\alpha_+} + 2\mathcal{C}_2 t^{-\alpha_-}]. \tag{62}$$

As we have already mentioned before, in the above solution the backreaction of the scalar field has not been considered in the background evolution. The standard radiation dominated cosmic expansion is therefore unperturbed. To check the validity of this approximation, we compare the leading-order behavior of the energy densities of the radiation and the scalar field:

$$e^{-2\phi}\rho_r \propto a^4 = \frac{1}{t^2}, \qquad \rho_\phi = \frac{\omega}{2}\dot{\phi}^2 \propto \frac{\mathcal{C}_1{}^2}{t^2}\frac{1}{\ln(t)^{2\alpha_+ + 2}}, \quad \frac{\mathcal{C}_2{}^2}{t^2}\frac{1}{\ln(t)^{2\alpha_- + 2}}. \tag{63}$$

As is clear from the above two expressions for the energy densities, the $\dot{\phi}^2$ term falls off faster than the radiation energy density as $t \to \infty$. From Eq. (62) we see that depending on the boundary conditions α can decrease or increase with time in the radiation dominated epoch. The change of α, on the other hand, is controlled by the average energy density of the radiation, $\mathcal{A}$, as well as the *PCP* violating coupling, β.

In the context of the subsequent cosmic expansion, the new PCP violating term in our Lagrangian does not contribute to the evolution of the scalar field ϕ. Therefore the corresponding variation of the alpha has the same evolution in the subsequent matter and dark energy dominated eras. This has been extensively discussed in Refs. [8–10].

VI. CONCLUSIONS

We have constructed a parity and charge-parity (PCP) violation model within the framework of the varying-alpha theory, popularly known as BSBM theory [7,9]. The origin of this violation in our model is the time variation of the charge, which is the basic assumption of this framework. One of the main motivations for this model is to search for new physics constrained by the presentday high precession data from cosmological observations. After constructing our model, we have calculated various relevant effects such as the cosmological birefringence, which has already become a standard observational parameter in CMB as well as in radio galaxy and quasar spectra observations. Although until now there has been no positive observational evidence of this parity-violating effect, future experiments with ever improved precession may hopefully help us identify this notion beyond the standard model. Our model also predicts that this new contribution to the fine-structure constant variation is effective mainly in the radiation-dominant era. In other eras, the variation is essentially the same as those extensively discussed in the literature [8–10]. Because of that, BBN (Big Bang Nucleosynthesis) becomes the main observational window to constrain the evolution of the PCP violating varying fine-structure constant. The electromagnetic coupling constant plays a very significant role in the nuclear abundance of our Universe. It happens that in our model the fine-structure constant has a power law time variation during the radiation-dominant era, whereas in the standard BSBM model it remains almost constant. So BBN should give us a strong constraint on the parity-violating parameter.

As is well known, BBN needs three essential input parameters which are the neutron-proton mass difference, Δm, the neutrino life time, τ_n, and the nuclear reaction rates. All of these parameters are directly or indirectly depending upon the fine-structure constant. There have been extensive studies on constraining the fine-structure constants through the light element abundance. The most updated bound on the total variation of the alpha is $-0.007 \leq \delta\alpha/\alpha_0 \leq 0.017$ at 95% C.L. [27]. In order to constrain the parity-violating parameter β, we need to know the amount of variation of the fine-structure constant after the radiation dominated epoch. To accomplish this, we need one more constraint deduced from a later time in cosmic evolution.

CMB anisotropy is another powerful tool to constrain the possible variation of a fine-structure constant from the matter dominated epoch to the present epoch. Variation of the alpha during the matter dominated epoch before CMB would change the time of recombination and the acoustic horizon associated with the photon-electron decoupling. The most updated bound on the variation of the fine-structure constant has been reported in [28] by using the latest WMAP 7-year data, and that is $-0.005 \leq \delta\alpha/\alpha_0 \leq 0.008$ at 95% C.L. By comparing the aforementioned two different bounds on the alpha variation deduced from two different cosmological time scales, it may be possible to constraint the PCP violating parameter β of our model. As a rough estimate, we take the difference between these two constraints and find the bound for the radiation-dominant era: $-0.002 \leq \delta\alpha_{\rm rad}/\alpha_0 \leq 0.009$, where $\delta\alpha_{\rm rad} \approx (2\mathcal{C}_1 t_{\rm eq}^{-\alpha_+} + 2\mathcal{C}_2 t_{\rm eq}^{-\alpha_-})$; $t_{\rm eq}$ is time of radiation-matter equality during the cosmic evolution.

Apart from the constraints deduced from cosmological and astrophysical observations, we have also done some qualitative analysis on the amount of the optical rotation due to background electromagnetic fields. Because of the existence of cosmic-scale magnetic fields in our Universe, polarization of the CMB photons may be sizable due to their coupling to the scalar field. With these considerations in mind, we believe that there exists experimental windows through which the validity of our model or the constraints of its parameters can be verified. As a first step, we have focused on establishing the qualitative behavior of our model in the present paper, but we did not investigate the observational constraints on its parameters. We hope to study this in more detail in the future.

ACKNOWLEDGMENTS

This research is supported by Taiwan National Science Council under Project No. NSC 97-2112-M-002-026-MY3, by Taiwan's National Center for Theoretical Sciences (NCTS), and by US Department of Energy under Contract No. DE-AC03-76SF00515.

[1] S. M. Carroll, Phys. Rev. Lett. **81**, 3067 (1998).
[2] A. Lue, L. M. Wang, and M. Kamionkowski, Phys. Rev. Lett. **83**, 1506 (1999).
[3] C. R. Contaldi, J. Magueijo, and L. Smolin, Phys. Rev. Lett. **101**, 141101 (2008).
[4] T. Takahashi and J. Soda, Phys. Rev. Lett. **102**, 231301 (2009).
[5] D. Maity, P. Majumdar, and S. SenGupta, J. Cosmol. Astropart. Phys. 06 (2004) 005; P. Majumdar, Mod. Phys. Lett. A **19**, 1319 (2004); K. R. S. Balaji, R. H. Brandenberger, and D. A. Easson, J. Cosmol. Astropart. Phys. 12 (2003) 008.
[6] P. Jordan, Naturwissenschaften **25**, 513 (1937); Z. Phys. **113**, 660 (1939); E. Teller, Phys. Rev. **73**, 801 (1948); K. P. Stanyukovich, Dokl. Akad. Nauk SSSR **147**, 1348 (1962); Sov. Phys. Dokl. **7**, 1150 (1963); G. Gamow, Phys. Rev. Lett. **19**, 759 (1967); R. H. Dicke, *The Theoretical Significance of Experimental Relativity* (Gordon and Breach, New York 1965); R. H. Dicke, Science **129**, 621 (1959).
[7] J. D. Bekenstein, Phys. Rev. D **25**, 1527 (1982).
[8] H. B. Sandvik, J. D. Barrow, and J. Magueijo, Phys. Rev. Lett. **88**, 031302 (2002); Phys. Rev. D **65**, 123501 (2002); **66**, 043515 (2002); Phys. Lett. B **541**, 201 (2002); J. D. Barrow and D. Mota, Classical Quantum Gravity **19**, 6197 (2002).
[9] J. D. Barrow, H. B. Sandvik, and J. Magueijo, Phys. Rev. D **65**, 063504 (2002).
[10] K. A. Olive and Maxim Pospelov, Phys. Rev. D **65**, 085044 (2002); T. Chiba and K. Kohri, Prog. Theor. Phys. **107**, 631 (2002); J. P. Uzan, Rev. Mod. Phys. **75**, 403 (2003); D. S. Lee, W. Lee, and K. W. Ng, Int. J. Mod. Phys. D **14**, 335 (2005).
[11] M. T. Murphy *et al.*, Mon. Not. R. Astron. Soc. **327**, 1208 (2001).
[12] J. K. Webb, V. V. Flambaum, C. W. Churchill, M. J. Drinkwater, and J. D. Barrow, Phys. Rev. Lett. **82**, 884 (1999).
[13] J. K. Webb *et al.*, Phys. Rev. Lett. **87**, 091301 (2001).
[14] A. Dobado and A. Maroto, Mod. Phys. Lett. A **12**, 3003 (1997).
[15] D. Maity and S. SenGupta, Classical Quantum Gravity **21**, 3379 (2004).
[16] D. Maity, S. SenGupta, and S. Sur, Phys. Rev. D **72**, 066012 (2005); S. Kar, P. Majumdar, S. SenGupta, and S. Sur, Classical Quantum Gravity **19**, 677 (2002).
[17] G. C. Liu, S. Lee, and K. W. Ng, Phys. Rev. Lett. **97**, 161303 (2006); M. Li *et al.*, Phys. Lett. B **651**, 357 (2007); M. Shimon *et al.*, Phys. Rev. D **77**, 083003 (2008); J. Q. Xia *et al.*, Astron. Astrophys. **483**, 715 (2008); F. Finelli and M. Galaverni, Phys. Rev. D **79**, 063002 (2009); J. Q. Xia *et al.*, Astrophys. J. **679**, L61 (2008); M. Pospelov, A. Ritz, and C. Skordis, Phys. Rev. Lett. **103**, 051302 (2009); M. Li and X. Zhang, Phys. Rev. D **78**, 103516 (2008); J. Q. Xia *et al.*, Phys. Lett. B **687**, 129 (2010); L. Pagano *et al.*, Phys. Rev. D **80**, 043522 (2009).
[18] N. Lepora, arXiv:gr-qc/9812077.
[19] M. White and J. Cohn, Report No. TACMB-1, 2002.
[20] B. Feng *et al.*, Phys. Rev. Lett. **96**, 221302 (2006).
[21] P. Cabella, P. Natoli, and J. Silk, Phys. Rev. D **76**, 123014 (2007); T. Kahniashvili, R. Durrer, and Y. Maravin, Phys. Rev. D **78**, 123009 (2008); J. Q. Xia *et al.*, Astrophys. J. **679**, L61 (2008); J. Q. Xia *et al.*, Astron. Astrophys. **483**, 715 (2008); E. Y. Wu *et al.* (QUaD Collaboration), Phys. Rev. Lett. **102**, 161302 (2009); L. Pagano *et al.*, Phys. Rev. D **80**, 043522 (2009).
[22] E. Komatsu *et al.* (WMAP Collaboration), Astrophys. J. Suppl. Ser. **192**, 18 (2011).
[23] Y. Semertzidis *et al.* (BFRT Collaboration), Phys. Rev. Lett. **64**, 2988 (1990); R. Cameron *et al.* (BFRT Collaboration), Phys. Rev. D **47**, 3707 (1993).
[24] E. Zavattini *et al.* (PVLAS Collaboration), Phys. Rev. Lett. **96**, 110406 (2006).
[25] S. J. Chen, H. H. Mei, and W. T. Ni (Q & A Collaboration), Mod. Phys. Lett. A **22**, 2815 (2007).
[26] C. Rizzo (BMV Collaboration), 2nd ILIAS-CERN-CAST Axion Academic Training (2006), http://cast.mppmu.mpg.de/.
[27] T. Dent, S. Stern, and C. Wetterich, Phys. Rev. D **76**, 063513 (2007).
[28] S. J. Landau and C. G. Scoccola, arXiv:1002.1603.