

Towards a holographic model of D -wave superconductorsJiunn-Wei Chen,^{1,*} Ying-Jer Kao,^{1,†} Debaprasad Maity,^{1,2,‡} Wen-Yu Wen,^{1,2,§} and Chen-Pin Yeh^{1,||}¹*Department of Physics and Center for Theoretical Sciences, National Taiwan University, Taipei 10617, Taiwan*²*Leung Center for Cosmology and Particle Astrophysics, National Taiwan University, Taipei 106, Taiwan*

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The holographic model for S-wave high T_c superconductors developed by Hartnoll, Herzog, and Horowitz is generalized to describe D-wave superconductors. The $3 + 1$ dimensional gravitational theory consists of a symmetric, traceless second-rank tensor field and a $U(1)$ gauge field in the background of the anti-de Sitter black hole. Below T_c the tensor field, which carries the $U(1)$ charge, undergoes the Higgs mechanism and breaks the $U(1)$ symmetry of the boundary theory spontaneously. The phase transition characterized by the D-wave condensate is second order with the mean field critical exponent $\beta = 1/2$. As expected, the AC conductivity is isotropic below T_c , and the system becomes superconducting in the DC limit but has no hard gap.

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I. INTRODUCTION

One of the unsolved mysteries in modern condensed matter physics is the mechanism of the high temperature superconducting (HTSC) cuprates [1]. These materials are layered compounds with copper-oxygen planes and are doped Mott insulators with strong electronic correlations. The pairing symmetry is unconventional, and there is a strong experimental evidence showing that it is the D-wave [2]. It is speculated that the pairing between electrons is mediated via strong antiferromagnetic spin fluctuations in the system. A prominent strong-coupling theory is proposed by Anderson, called the resonant valence bond theory, which describes liquid state with spin singlets. Upon hole doping, the Néel order is destroyed and gives rise to superconductivity [3]. Several gauge theories have been proposed to formulate the resonant valence bond physics, by enforcing the double occupation constraint in the strong-coupling limit [4]. The problem is difficult due to the strong-coupling nature of the theory. Although significant progress has been made in the past few years, alternative approaches may be valuable to tackle the problem.

One alternative approach is the holographic correspondence between a gravitational theory and a quantum field theory, which first emerged under AdS/CFT correspondence [5–7]. This method has provided a useful and complementary framework to describe strong interaction systems without a sign problem (see, e.g., [8–15]). In the original top-down approach, both the gravity side and the field theory side of the theories are precisely known. Later applications assume that the correspondence exists among different pair of theories and try to make predictions from

one side of the correspondence. More specifically, in this bottom-up approach, usually the gravity side of the theory is explicitly constructed with the desired symmetries, then physical observables (matrix elements) of the field theory side are predicted through the above mentioned correspondence.

Recently, a gravitational model of hairy black holes [16,17] have been used to model S-wave HTSC [18–21]. In those class of models the Abelian symmetry of a complex scalar field is spontaneously broken (i.e., the Higgs mechanism) below some critical temperature. The Meissner effect was soon observed by including magnetic field in the background [22,23]. The effect of superconducting condensate on the holographic fermi surface has been studied by calculating fermionic spectral function [24–26]. Interestingly, the properties of spectral function appeared to have similar behavior to that found in the angle resolved photo-emission experiment. Motivated by all of these S-wave studies, holographic dual to the P-wave superconductor has been proposed by coupling a $SU(2)$ Yang-Mills field to the black hole, where a vector hair develops in the superconducting phase [27–30]. The behavior of fermionic spectral function has also been studied in those P-wave superconducting backgrounds [31]. So far, the bottom-up construction of holographic superconductor assuming the existence of gauge/gravity duality has been discussed. However, in the string theory framework people also have studied the top-down approach considering various D-brane configurations in the anti-de Sitter (AdS) black hole background [32].

In this work, we try to construct a minimal gravitational model that models D-wave HTSC. We replace the complex scalar field in [18] by a tensor field whose condensate breaks the symmetry spontaneously below T_c and the condensate becomes zero and the symmetry is restored above T_c . The critical exponent β gives the mean field value $1/2$. The real part of the conductivity computed from the linear response has a delta function at zero frequency,

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which corresponds to static superconductivity below T_c . Above T_c , the delta function disappears as expected, and the conductivity becomes constant in frequency. It is expected that there is no “hard gap” in the real part of the conductivity, and the conductivity should be isotropic even though the condensate is not (for a model calculation, see [33]). Both features are seen in our results.

II. A HOLOGRAPHIC MODEL FOR D-WAVE HTSC

Our goal is to consider a minimal (3 + 1 dimensional) holographic model that gives rise to (2 + 1 dimensional) D-wave superconductivity. The construction will be similar to that of the S-wave case [18] with a spontaneous local $U(1)$ symmetry breaking in the bulk leading to a spontaneous breaking of global $U(1)$ symmetry at the boundary. Thus, strictly speaking, the boundary theory is a superfluid. One can still study the current-current correlator, which could be interpreted as the conductivity.

To have a D-wave condensate at the boundary, we introduce a charged tensor field in our dual gravity theory. Assuming the D-wave condensate originating from electron-electron pairing, the D-wave nature gives a symmetric wave function for the pair. So, the wave function for this electron pair has to be a spin singlet such that its total wave function is antisymmetric. A 3×3 symmetric traceless tensor has five components, which can be used to describe a D-wave state. We will promote this symmetric traceless tensor field to include time components and denote the field as $B_{\mu\nu}$ ($\mu, \nu = 0, 1, 2, 3$), i.e., $B_{\mu\nu} = B_{\nu\mu}$ and $B_{\mu}^{\mu} = 0$. However, it is important to note that the interacting higher spin fields, in general, require satisfying additional constraints in addition to the equations of motion to remove the unphysical degrees of freedom, see the discussion in [34–36], for example. Observing that there is no available consistent model for our purpose, we would like to propose a truncated model that has sufficient ingredients to catch some features of a D-wave superconductor. It would be an important but difficult task to construct a complete theory, which we would like to postpone for the future in order to attain our simple goal through the present exercise.

The desired complete action including gravity, $U(1)$ gauge field, tensor field, and other auxiliary fields, may take the following form[37]:

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left[\left(R + \frac{6}{L^2} \right) + \mathcal{L}_m + \mathcal{L}_a \right],$$

$$\mathcal{L}_m = -\frac{L^2}{q^2} \left[(D_{\mu} B_{\nu\gamma})^* D^{\mu} B^{\nu\gamma} + m^2 B_{\mu\nu}^* B^{\mu\nu} + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right], \quad (1)$$

where R is the Ricci scalar, the $6/L^2$ term gives a negative cosmological constant, and L is the AdS radius that will be set to unity in the units that we use. $\kappa^2 = 8\pi G_N$ is the

gravitational coupling. D_{μ} is the covariant derivative in the black hole background ($D_{\mu} = \partial_{\mu} + iA_{\mu}$ in flat space), and q and m^2 are the charge and mass squared of $B_{\mu\nu}$, respectively. The terms associated with auxiliary fields are included in \mathcal{L}_a . This Lagrangian is the same as that which appears in [34] in the flat space limit. However, in the subsequent studies after [34] it appeared that construction of the higher spin field Lagrangian coupled to gravity or $U(1)$ gauge in a gauge invariant way is nontrivial even in Minkowski space [38]. But later on in the context of gravitational interaction [39] it has been shown that this task may have a solution in AdS space. Also, there exists a recent attempt to construct gauge invariant $U(1)$ charged massive spin 2 particles in AdS space at linear approximation [40]. In spite of all these studies fully consistent formulation of interacting higher spin gauge fields is still lacking. So, naturally the action (1) without \mathcal{L}_a , which we are going to study, may be plagued with spurious degrees of freedom that make the system unstable. However we continue our study with the truncated action hoping that the fully consistent construction of our theory, which has no instability, will still allow D-wave condensation.

\mathcal{L}_m might look more familiar with the rescaling $B_{\mu\nu} \rightarrow qB_{\mu\nu}$ and $A_{\mu} \rightarrow qA_{\mu}$. Here, we also concentrate on the “probe limit” [18] where the backreaction to the background can be ignored. This limit is exact when $q \rightarrow \infty$. In the probe limit, \mathcal{L}_m can be treated as a perturbation on top of the 3 + 1 dimensional AdS black hole background:

$$ds^2 = -g(r)dt^2 + \frac{dr^2}{g(r)} + r^2(dx^2 + dy^2), \quad (2)$$

where $g(r) = r^2 - \frac{r_0^3}{r}$ and r_0 is the horizon size. The Hawking temperature for this black hole $T = \frac{3r_0}{4\pi}$.

As in the S-wave case [18], an electric field can exist in the bulk by the appropriate choice of boundary conditions. The charged tensor field, which can be considered as charged particles, experiences a force under the electric field, with positive(negative) charges repelled(attracted) away from(toward) the black hole. On the other hand, the black hole tries to pull all the charged particles in it. At lower T , the black hole is smaller, and the gravitational pull is weaker. Thus, the positively charged particles have a greater chance to stay outside the horizon and form the condensate. At very large T , the gravitational force from the large black hole is strong enough to pull all the charged particles into the horizon such that there is no condensate. Thus, we have a phase transition.

We are interested in describing the D-wave superconductor in the continuum such that there is a condensate on the x - y plane on the boundary with translational invariance. Rotational symmetry is broken down to $Z(2)$ with the condensate changing its sign under a $\pi/2$ rotation on the x - y plane. To incorporate these features, we use an ansatz for the $B_{\mu\nu}$ and the gauge field A_{μ} , i.e.,

$$B_{\mu\nu} = \text{diagonal}(0, 0, f(r), -f(r)), \quad A = \phi(r)dt. \quad (3)$$

After plugging in this ansatz, we have the equation of motion for B

$$r^2 f''(r) + r \left[r \frac{g'(r)}{g(r)} - 2 \right] f'(r) + \left\{ \frac{r^2 \phi^2(r)}{g(r)^2} - \frac{[m^2 r + 2g'(r)]r}{g(r)} \right\} f(r) = 0, \quad (4)$$

and the corresponding Maxwell's equation is

$$r^2 \phi''(r) + 2r \phi'(r) - \frac{4f^2(r)\phi(r)}{r^2 g(r)} = 0, \quad (5)$$

where the $'$ is the derivative with respect to r .

We would like to choose the solution such that $\phi(r)$ has the asymptotic form

$$\phi(r) \rightarrow \mu + \frac{\rho}{r} \quad (6)$$

near the boundary ($r \rightarrow \infty$), where μ is interpreted as the chemical potential, and ρ as the charge density in the boundary theory. Here, we will first assume this and then show that indeed this can be satisfied later. Now, the asymptotic form of Eq. (4) has the asymptotic form

$$r^2 f''(r) - (m^2 + 4)f(r) \simeq 0 \quad (7)$$

near the boundary, which yields

$$f(r) \rightarrow f_0 r^{\Delta_+} + f_1 r^{\Delta_-}, \quad \Delta_{\pm} = \frac{1 \pm \sqrt{17 + 4m^2}}{2}. \quad (8)$$

If we interpret f_0 as the source and f_1 as the vacuum expectation value of the operator that couples to B at the boundary theory, we need $m^2 \geq -4$ (and $\Delta_- \leq 0$) such that the f_1 term is constant or vanishing at the boundary. After setting the source $f_0 = 0$ and using $\Delta_- \leq 0$, Eq. (5) indeed gives the asymptotic solution of Eq. (6). Note that the $f_0 r^{\Delta_+}$ term does not impose a constraint on m^2 by requiring that the third term on the left-hand side of Eq. (5) to be smaller than the other two terms since we have imposed $f_0 = 0$. One way to see this is to do the integration of the differential equations from the boundary, then $f(r) \rightarrow f_1 r^{\Delta_-}$, $\phi(r) \rightarrow \mu + \frac{\rho}{r}$ satisfy the asymptotic behaviors of Eqs. (4) and (5). The order parameter of the boundary theory can be read from the asymptotic behavior of B ,

$$\langle \mathcal{O}_{ij} \rangle = \begin{pmatrix} f_1 & 0 \\ 0 & -f_1 \end{pmatrix}, \quad (9)$$

where (i, j) are the indexes in the boundary coordinates (x, y) .

It is useful to note that the action and the equations of motion are invariant under the scaling

$$\begin{aligned} (t, r, x, y) &\rightarrow (t/c, cr, x/c, y/c), \\ (r_0, T, g(r)) &\rightarrow (cr_0, cT, c^2 g(r)), \\ (f(r), \phi(r)) &\rightarrow (c^2 f(r), c\phi(r)). \end{aligned} \quad (10)$$

Thus, we can always scale $\mu \rightarrow 1$. This also helps to keep track of the scaling dimension for observables, e.g., the scaling dimensions for μ , ρ , and f_1 are 1, 2, and $2 - \Delta_-$, respectively.

In Fig. 1, we show the numerical result between the dimensionless quantities $f_1/\mu^{2-\Delta_-}$ and T/T_c for $m^2 = -1/4$. It is a second order phase transition. Numerically, the critical exponent β defined as $f_1 \rightarrow c(T_c - T)^\beta$ for $T_c - T \rightarrow 0^+$ is very close to the mean field value $\beta = 1/2$. Below we show that only the values $\beta = 1/2, 3/2, 5/2, \dots$ satisfy the equations of motion. Thus, without fine-tuning, one would get the mean field value $\beta = 1/2$.

Now we present the derivation. The metric $g(r)$ is a smooth function of $\epsilon = T_c - T$, while ϕ and f can be expanded as

$$\begin{aligned} g(r, T) &= a_g(r) + b_g(r)\epsilon + O(\epsilon^2), \\ \phi(r, T) &= \epsilon^k (a_\phi(r) + b_\phi(r)\epsilon + O(\epsilon^2)), \\ f(r, T) &= \epsilon^n (a_f(r) + b_f(r)\epsilon + O(\epsilon^2)). \end{aligned} \quad (11)$$

Since, Eq. (4) is a linear equation in f and the prefactors of $f''(r)$ and $f'(r)$ are polynomials of ϵ , the prefactor of $f(r)$ has to be a polynomial of ϵ as well in order to satisfy the equation. This implies k is an integer. At T_c and at the boundary, ϕ gives the value of chemical potential which is finite. This yields $k = 0$.

Analogously, the Maxwell equation, Eq. (5), is linear in ϕ . The prefactors of $\phi''(r)$ and $\phi'(r)$ are polynomials of ϵ and thus the prefactor of $\phi(r)$ is required to be a polynomial of ϵ . This yields $2n$ to be an integer. We also know that $n > 0$ for a second order phase transition, and hence only $\beta = 1/2, 3/2, \dots$ are allowed.

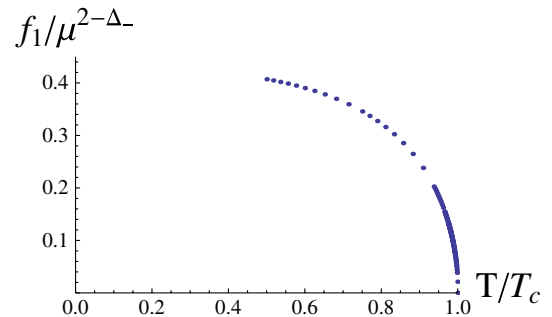


FIG. 1 (color online). The dimensionless D-wave condensate $f_1/\mu^{2-\Delta_-}$ shown as a function of T/T_c for $m^2 = -1/4$. The condensate goes to zero at $T = T_c \propto \mu$. The critical exponent $f_1 \rightarrow c(T_c - T)^\beta$ for $T_c - T \rightarrow 0^+$ is of the mean field value $\beta = 1/2$.

III. CONDUCTIVITY

In this section, we compute the conductivity of this D-wave HTSC by linear response. The conductivity tensor σ_{ij} can be defined through the linear response relation

$$J_i = \sigma_{ij} E_j, \quad (12)$$

where $i, j = 1, 2$. J , and E are the electric current and electric field, respectively. Following the approach of [18], we perturb the gauge field by $\delta A = e^{-i\omega t} A_x(r) dx$. To get a consistent set of equations, we also need to perturb $\delta B_{rx} = \delta B_{xr} = i b_{rx}(r) e^{-i\omega t}$ and $\delta B_{tx} = \delta B_{xt} = b_{tx}(r) e^{-i\omega t}$, respectively. The resulting equations of motion are

$$g A_x'' + g' A_x' + \left(\frac{\omega^2}{g} - \frac{4f^2}{r^4} \right) A_x = 0, \quad (13)$$

$$g b_{rx}'' + 2g' b_{rx}' + \left[\frac{g''}{2} - \frac{g'}{r} - 5 \frac{g}{r^2} + r_0^3 + \frac{(\omega - \phi)^2}{g} \right] b_{rx} = -\frac{2f}{r^3} A_x - \frac{(\omega - \phi)g'}{g^2} b_{tx}, \quad (14)$$

$$g b_{tx}'' + \left[-\frac{g''}{2} - \frac{g'}{r} - \frac{g}{r^2} + r_0^3 + \frac{(\omega - \phi)^2}{g} \right] b_{tx} = (\omega - \phi) g' b_{rx}. \quad (15)$$

In principle, we can also add other $\delta B_{\mu\nu}$ components in the perturbation. However, those components do not couple to δA_μ to the quadratic order in the action. So, if we set the initial condition of our system to be in the ground state before δA_μ perturbation being turned on then those extra B field perturbations will not be produced. However, in the full stability analysis, those δA_μ independent perturbations are important. We will defer this stability analysis to future study.

Equation (13) is very similar to the S-wave case and is decoupled from $\delta B_{\mu\nu}$. Near the boundary, we have

$$r^2 A_x'' + 2r A_x' \simeq 0, \quad (16)$$

which yields the asymptotic form

$$A_x \rightarrow A_{x,0} + \frac{A_{x,1}}{r}, \quad (17)$$

where $A_{x,0}$ is the x -component gauge field at the boundary whose time derivative gives E_x , and $A_{x,1}$ is the expectation value of the current operator J_x . The ratio of J_x and E_x is the frequency dependent conductivity

$$\sigma(\omega) \equiv \sigma_{xx}(\omega) = -\frac{iA_{x,1}}{\omega A_{x,0}}. \quad (18)$$

The fact that Eq. (13) depends only on A_x implies

$$\sigma_{yx}(\omega) = 0. \quad (19)$$

This is dictated by the reflection symmetry with respect to the $y = 0$ plane.

We are now focusing on the case $m^2 > -2$, where the asymptotic forms of Eqs. (14) and (15) are particularly simple:

$$r^2 b_{rx}'' + 4r b_{rx}' - (m^2 + 6) b_{rx} \simeq 0, \quad (20)$$

$$r^2 b_{tx}'' - (m^2 + 4) b_{tx} \simeq 0. \quad (21)$$

These two equations can be solved with

$$\begin{aligned} b_{rx} &\rightarrow b_{rx,0} r^{\tilde{\Delta}^+} + b_{rx,1} r^{\tilde{\Delta}^-}, \\ b_{tx} &\rightarrow b_{tx,0} r^{\Delta^+} + b_{tx,1} r^{\Delta^-}, \end{aligned} \quad (22)$$

where $\tilde{\Delta}_\pm = \frac{-3 \pm \sqrt{33 + 4m^2}}{2}$ and Δ_\pm is defined in Eq. (8). Here, we also identify $b_{rx,0}$ and $b_{tx,0}$ as the source terms and $b_{rx,1}$ and $b_{tx,1}$ are the normalizable fluctuations.

Near the horizon, $g(r) = 3r_0 dr + \mathcal{O}(dr^2)$ with $dr = r - r_0$ and $\phi(r) = \mathcal{O}(dr)$. The equations of motion become

$$9dr^2 A_x'' + 9dr A_x' + \frac{\omega^2}{r_0^2} A_x = 0, \quad (23)$$

$$\begin{aligned} 9dr^2 b_{rx}'' + 18dr b_{rx}' + \frac{\omega^2}{r_0^2} b_{rx} \\ = -\frac{\omega}{r_0^2 dr} b_{tx} - \frac{6f(r_0)}{r_0^4} dr A_x, \end{aligned} \quad (24)$$

$$9dr^2 b_{tx}'' + \frac{\omega^2}{r_0^2} b_{tx} = 9\omega dr b_{rx}. \quad (25)$$

The solutions near the horizon are

$$\begin{aligned} A_x &\rightarrow \bar{a}_{x,1} dr^{-i(\omega/(3r_0))} + \bar{a}_{x,2} dr^{i(\omega/(3r_0))}, \\ b_{tx} &\rightarrow 3ir_0 (\bar{b}_{rx,1} dr^{-i(\omega/(3r_0))+1} - \bar{b}_{rx,2} dr^{-i(\omega/(3r_0))} \\ &\quad - \bar{b}_{rx,3} dr^{i(\omega/(3r_0))+1} + \bar{b}_{rx,4} dr^{i(\omega/(3r_0))}), \\ b_{rx} &\rightarrow \bar{b}_{rx,1} dr^{-i(\omega/(3r_0))} + \bar{b}_{rx,2} dr^{-i(\omega/(3r_0))-1} \\ &\quad + \bar{b}_{rx,3} dr^{i(\omega/(3r_0))} + \bar{b}_{rx,4} dr^{i(\omega/(3r_0))-1}. \end{aligned} \quad (26)$$

The ingoing wave boundary condition [41,42], which sets the wave falling into the horizon, demands $\bar{a}_{x,2} = \bar{b}_{rx,3} = \bar{b}_{rx,4} = 0$. We further set the divergent term $\bar{b}_{rx,2} = 0$ to keep the action finite. Now, $b_{rx,0(1)}$ and $b_{tx,0(1)}$ in Eq. (22) are linear combinations of $\bar{a}_{x,1}$ and $\bar{b}_{rx,1}$. So we have both normalizable and nonnormalizable solutions for b_{tx} and b_{rx} perturbations. The divergent source terms $b_{rx,0}$ and $b_{tx,0}$ near the boundary can be cancelled by counterterms [19].

In Fig. 2, we plot the real and imaginary part of $\sigma(\omega)$ for different T . The behaviors are similar to that of S-wave HTSC. $\text{Re}[\sigma(\omega)]$ has a delta function behavior at $\omega = 0$ corresponding to infinite DC conductivity when $T < T_c$. On the other hand, for $T \geq T_c$, the delta function and $\text{Im}[\sigma(\omega)]$ disappear, and $\text{Re}[\sigma(\omega)]$ becomes ω independent. There is no ‘‘hard gap’’ in our dual boundary superconducting system because $\text{Re}[\sigma(\omega)]$ does not vanish even

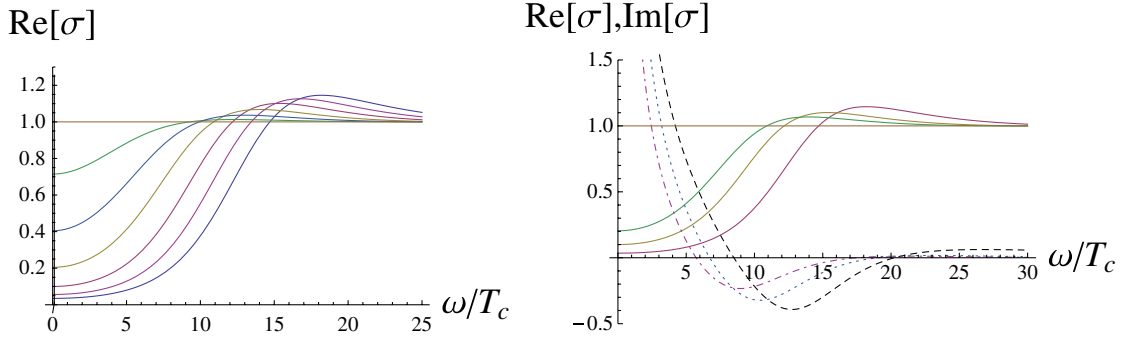


FIG. 2 (color online). The real (left plot) and imaginary (right plot) of conductivity shown as a function for frequency ω for different temperatures. Above T_c , $\text{Re}[\sigma(\omega)] = 1$ while $\text{Im}[\sigma(\omega)] = 0$. Below T_c , $\text{Re}[\sigma(\omega)]$ has a $\delta(\omega)$ delta function whose height decrease in T and vanishes at T_c . The right most curve has the lowest T , which implies the zero temperature gap $\omega_g/T_c \simeq 13$. (The construction in [18] for the S-wave gives the value 8 for this gap.)

for arbitrary small ω . One can also read off this soft gap from the plot, i.e., $\omega_g/T_c \simeq 13$. It is larger than the one obtained in the construction for the S-wave [18], where $\omega_g/T_c \simeq 8$. This may imply our D-wave pairing requires higher energy than the S-wave one.

Unlike the case for the S-wave superconductor, the vanishing of the gap is actually expected in the D-wave case. In the BCS-type theory (see, e.g., [43]), the lowest dimensional D-wave operator for two fermion pairing is $O_{ij} = \psi^T(\vec{\partial}_i \vec{\partial}_j - \vec{\partial}^2 \delta_{ij}/2)\psi$, where $\vec{\partial}_i = \vec{\partial}_i - \vec{\partial}_i$ is the relative momentum between the two fermion which is invariant under Galilean transformation. The leading order Lagrangian in a weakly interaction theory is

$$\mathcal{L} = \mathcal{L}_0 - c(O_{ij}^\dagger + J_{ij}^\dagger)(O_{ij} + J_{ij}) + cJ_{ij}^2, \quad (27)$$

where \mathcal{L}_0 is the free Lagrangian, c is the coupling, and J_{ij} is an external source. Under a Hubbard Stratanovich transformation, the Lagrangian can be rewritten as

$$\begin{aligned} \mathcal{L}' = \mathcal{L}_0 + [B_{ij}^\dagger(O_{ij} + J_{ij}) + (O_{ij}^\dagger + J_{ij}^\dagger)B_{ij}] \\ + \frac{B_{ij}^\dagger B_{ij}}{c} + cJ_{ij}^2, \end{aligned} \quad (28)$$

where B and B^* are auxiliary fields. After integrating over the auxiliary fields, \mathcal{L} is recovered from \mathcal{L}' . It is clear that $\langle O_{ij} \rangle$ in \mathcal{L} is $\langle B_{ij} \rangle$ in \mathcal{L}' . The gap equation of \mathcal{L}' gives the dispersion relation

$$E_k = \sqrt{\left(\frac{k^2}{2m} - \mu\right)^2 + |B_{ij}k_i k_j|^2}. \quad (29)$$

The gap $|B_{ij}k_i k_j| \propto |k_x^2 - k_y^2|$ vanishes at four nodes $k_x^2 = k_y^2$. So, naturally gapless excitations can contribute to conductivity. This makes the conductivity for a D-wave superconductor gapless. In the S-wave case, however, the gap is isotropic and does not vanish in any direction leading to a hard gap in conductivity.

If we change the gauge field perturbation to $\delta A = e^{-i\omega t}(A_x(r)dx + A_y(r)dy)$, then there will be a response

from δB_{rx} , δB_{lx} , δB_{ry} , and δB_{ly} . $A(r) = A_x(r)\hat{x} + A_y(r)\hat{y}$ satisfies the same differential equation as Eq. (13):

$$g\mathbf{A}'' + g'\mathbf{A}' + \left(\frac{\omega^2}{g} - \frac{4f^2}{r^4}\right)\mathbf{A} = 0. \quad (30)$$

This shows that the conductivity is isotropic:

$$\sigma_{ij}(\omega) = \sigma(\omega)\delta_{ij}. \quad (31)$$

This might seem surprising at the first sight because the condensate is not isotropic. However, this is a consequence of the symmetries that σ_{ij} has in the D-wave case. In the linear response theory, σ_{ij} is a current-current correlator which can be schematically denoted as $\sigma_{ij} \sim \langle \Omega | [J_i, J_j] | \Omega \rangle$, where the matrix element denotes an ensemble average. Under a $\pi/2$ rotation along the z axis (R), $R^{-1}J_i R = \epsilon_{ij}J_j$, where ϵ_{ij} is an antisymmetric tensor, and assuming the ensemble average is governed by properties of the ground state which has the condensate structure of Eq. (9), so $R|\Omega\rangle = -|\Omega\rangle$. Then, $\sigma_{ij} \sim \langle \Omega | [J_i, J_j] | \Omega \rangle = \langle \Omega | R^{-1}[J_i, J_j]R | \Omega \rangle = \langle \Omega | [\epsilon_{ik}J_k, \epsilon_{jl}J_l] | \Omega \rangle$. This implies $\sigma_{xx} = \sigma_{yy}$ and $\sigma_{xy} = -\sigma_{yx}$. A similar analysis with parity operator with respect to the x axis gives $\sigma_{xy} = \sigma_{yx} = 0$. Thus, we have $\sigma_{ij} \propto \delta_{ij}$. An explicit microscopic model calculation [33] also yields an isotropic conductivity for a D-wave superconductor.

IV. CONCLUSION

We have constructed a minimal holographic model for high T_c D-wave superconductors. We follow closely the work of Hartnoll, Herzog, and Horowitz on the S-wave case. The 3 + 1 dimensional gravitational theory consists a symmetric, traceless second-rank tensor field and a $U(1)$ gauge field in the background of the AdS black hole. Below T_c , the tensor field is Higgsed to break the $U(1)$ symmetry at the boundary theory. The phase transition characterized by the D-wave condensate is second order with the mean field critical exponent $\beta = 1/2$. As expected, the AC conductivity is isotropic; below T_c , the

system becomes superconducting in the DC limit but has no hard gap.

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