

Absorption cross section in warped AdS₃ black hole revisited

This content has been downloaded from IOPscience. Please scroll down to see the full text.

JHEP09(2009)102

(<http://iopscience.iop.org/1126-6708/2009/09/102>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 140.112.101.4

This content was downloaded on 25/11/2014 at 03:25

Please note that [terms and conditions apply](#).

Absorption cross section in warped AdS_3 black hole revisited

Hsien-Chung Kao^a and Wen-Yu Wen^b

^a*Department of Physics, National Taiwan Normal University,
Taipei 116, Taiwan*

^b*Department of Physics and Center for Theoretical Sciences &
Leung Center for Cosmology and Particle Astrophysics, National Taiwan University,
Taipei 106, Taiwan*

E-mail: hckao@ntnu.edu.tw, steve.wen@gmail.com

ABSTRACT: We investigate the absorption cross section for minimal-coupled scalars in the warped AdS_3 black hole. According to our calculation, the cross section reduces to the horizon area in the low energy limit as usually expected in contrast to what was previously found. We also calculate the greybody factor and find that the effective temperatures for the two chiral CFT's are consistent with that derived from the quasinormal modes. Observing the conjectured warped AdS/CFT correspondence, we suspect that a specific sector of the CFT operators with the desired conformal dimension could be responsible for the peculiar thermal behaviour of the warped AdS_3 black hole.

KEYWORDS: AdS-CFT Correspondence, Black Holes

ARXIV EPRINT: [0907.5555](https://arxiv.org/abs/0907.5555)

Contents

1	Introduction and summary	1
2	Scalar field in warped AdS₃ black hole	3
3	Tortoise coordinate and effective potential	4
4	Revisit absorption cross section	5
5	Greybody factor	7
6	Effective CFT description	8
7	Superradiant modes	10
A	Warped AdS/CFT correspondence	10
A.1	Warped AdS calculation	10
A.2	Group theory calculation	12
A.3	Correlator in Poincarè coordinate	13

1 Introduction and summary

It has been proved to be universal that for all spherically symmetric black holes the low energy cross section for massless minimally coupled scalars is always equal to the area of the horizon [1]. Furthermore, if the energy dependence is retained, the greybody factor or decay rate can also be obtained from the corresponding conformal field theory of the black hole [2–8]. Making use of the greybody factor, we can correctly reproduced the Bekenstein-Hawking entropy of the black hole. Recently, a black hole solution in the topological massive gravity (TMG) with a negative cosmological constant was constructed [9]. It was soon realized that the black hole can be viewed as discrete quotients of the warped AdS₃ spacetime [10] just like the BTZ black hole as discrete quotients of the AdS₃. Following the conventions in [10], the metric of the warped AdS₃ black hole is given by

$$ds^2 = -N^2(r)dt^2 + \ell^2 R^2(r)[d\phi + N_\phi(r)dt]^2 + \frac{\ell^4 dr^2}{4R^2(r)N^2(r)}, \tag{1.1}$$

with

$$R^2(r) \equiv \frac{r}{4}(3(\nu^2 - 1)r + (\nu^2 + 3)(r_+ + r_-) - 4\nu\sqrt{r_+r_-(\nu^2 + 3)}),$$

$$N^2(r) \equiv \frac{\ell^2(\nu^2 + 3)(r - r_+)(r - r_-)}{4R^2},$$

$$N_\phi(r) \equiv \frac{2\nu r - \sqrt{r_+r_-(\nu^2 + 3)}}{2R^2},$$

and the horizon area

$$\mathcal{A}_H = \pi \left\{ 2\nu r_+ - \sqrt{r_+ r_- (\nu^2 + 3)} \right\}. \quad (1.2)$$

It is known that when $\nu = 1$, it reduces to the BTZ black hole case. It was conjectured in [10] that this background has a holographic description in terms of two-dimensional conformal field theory with uneven temperatures in the left and right sectors, given by

$$\begin{aligned} T_H^{-1} &= \frac{4\pi\nu}{\nu^2 + 3} \frac{T_L + T_R}{T_R}, \\ T_L &= \frac{\nu^2 + 3}{8\pi} \left(r_+ + r_- - \frac{\sqrt{r_+ r_- (\nu^2 + 3)}}{\nu} \right), \\ T_R &= \frac{\nu^2 + 3}{8\pi} (r_+ - r_-). \end{aligned} \quad (1.3)$$

The corresponding central charges are

$$c_L = \frac{\ell}{G} \frac{4\nu}{\nu^2 + 3}, \quad c_R = \frac{\ell}{G} \frac{5\nu^2 + 3}{\nu(\nu^2 + 3)}. \quad (1.4)$$

There have been several discussions on the thermodynamical properties of this black hole. Among them, we remark some unusual features: Firstly, a computation of the low energy absorption cross section was carried out in [11] and to our surprise it did not reduce to the horizon area even in the BTZ limit.¹ This result seems to be in contradiction with an earlier statement in [1] since a U(1) isometry is still preserved in the warped geometry, though the area has been rescaled by deformation. Secondly, the quasinormal modes of this background were discussed in [13] and their dispersion relation was shown no longer linear in contrast to what is expected from the usual AdS/CFT correspondence. When $\nu > 1$, a ω^2 term is included in the definition of the conformal dimension of the operator which is coupled to the bulk scalar field thanks to the deformation. It was argued in the conclusion that the conventional AdS/CFT correspondence only made sense for very small deformation, in other words, $3(\nu^2 - 1)\omega^2 \ll m^2(\nu^2 + 3)$ for scalar mass m . In this paper, we would like to investigate again these peculiar features more carefully. We find a disagreement with ref. [11]. More specifically, our computation shows that the low energy absorption cross section does reduce to the horizon area, and therefore the statement in ref. [1] still holds. In addition, we obtain the desired greybody factor,

$$\sigma_{\text{abs}} \propto \frac{(2h_+^* - 1) \sinh\left(\frac{\omega}{2T_H}\right)}{\omega \Gamma^2(2h_+^*)} \left| \Gamma\left(h_+^* + i\frac{\omega}{4\pi\tilde{T}_R}\right) \Gamma\left(h_+^* + i\frac{\omega}{4\pi\tilde{T}_L}\right) \right|^2. \quad (1.5)$$

Here, the effective temperatures are given by [13]

$$\tilde{T}_L = \frac{T_L}{\delta} = \frac{\nu^2 + 3}{8\pi\nu}; \quad (1.6)$$

$$\tilde{T}_R = \frac{T_R}{\delta} = \frac{(\nu^2 + 3)(r_+ - r_-)}{8\pi(\nu(r_+ + r_-) - \sqrt{r_+ r_- (\nu^2 + 3)})} \quad (1.7)$$

$$\text{with } \delta \equiv \nu(r_+ + r_-) - \sqrt{r_+ r_- (\nu^2 + 3)},$$

¹Although it was later shown in a separate paper [12] that using a different coordinate system, absorption cross section did reduce to horizon area at this critical point.

and the conformal dimension

$$h_+^* = \frac{1}{2} \left(1 + \sqrt{1 - \frac{12(\nu^2 - 1)}{(\nu^2 + 3)^2} \omega^2 + \frac{4m^2 \ell^2}{\nu^2 + 3}} \right). \quad (1.8)$$

We also carry out the computation of massless scalar field in the (warped) AdS/CFT correspondence and conjecture that in the dual conformal theory living on the (t, u) -plane there exists a specific operator \mathcal{O}_* with the desired dimension which is responsible for the peculiar thermal behaviour found from the quasinormal modes.

One evidence supporting this statement is that the greybody factor can be correctly reproduced by using correlator built from \mathcal{O}_* , that is

$$\left\langle \mathcal{O}_*(t, u) \mathcal{O}_*(t', u') \right\rangle \sim \frac{e^{\pm i \frac{2\nu\omega}{\nu^2+3}(u-u')}}{|t-t'|^{2h_+^*}}. \quad (1.9)$$

The structure of this paper is outlined as follows: in section 2 we review the derivation of the differential equation for a massive scalar field probing the AdS₃ black hole background. In section 3, we discuss the subtlety arising from which one of the two limits is taken first, the massless or the low energy limit. In section 4, we revisit the computation of the absorption cross section and then obtain the greybody factor in section 5. In section 6, we attempt to derive the same greybody factor from the perspective of the effective conformal field theory. In section 7, we have discussion on the superradiant modes. For completeness, we also include some discussion on the warped AdS₃/CFT₂ correspondence in the appendix.

2 Scalar field in warped AdS₃ black hole

To set up the notation for later computation, we will first repeat the derivation of the governing differential equation describing a massive scalar field probing the background described in eq. (1.1) [11, 13, 14]. The Klein-Gordon equation in a curved background is given by

$$\left(\frac{1}{\sqrt{-g}} \partial_\mu \sqrt{-g} \partial^\mu - m^2 \right) \Phi = 0. \quad (2.1)$$

Using separation of variables

$$\Phi(t, r, \theta) = e^{-i\omega t + i\mu\theta} \phi(r), \quad (2.2)$$

we obtain the radial equation

$$\frac{d^2 \phi(r)}{dr^2} + \frac{2r - r_+ - r_-}{(r - r_+)(r - r_-)} \frac{d\phi(r)}{dr} - \frac{(\alpha r^2 + \beta r + \gamma)}{(r - r_+)^2 (r - r_-)^2} \phi = 0, \quad (2.3)$$

with $\alpha = -\frac{3\omega^2(\nu^2 - 1)}{(\nu^2 + 3)^2} + \frac{m^2 \ell^2}{\nu^2 + 3}$,

$$\beta = -\frac{\omega^2(\nu^2 + 3)(r_+ + r_-) - 4\nu \left[\omega^2 \sqrt{r_+ r_- (\nu^2 + 3)} - 2\mu\omega \right]}{(\nu^2 + 3)^2} + \frac{m^2 \ell^2 (r_+ + r_-)}{\nu^2 + 3},$$

$$\gamma = -\frac{4\mu \left[\mu - \omega \sqrt{r_+ r_- (\nu^2 + 3)} \right]}{(\nu^2 + 3)^2} + \frac{m^2 \ell^2 r_+ r_-}{\nu^2 + 3}.$$

It is convenient to introduce the coordinate

$$z = \frac{r - r_+}{r - r_-} \tag{2.4}$$

such that the outer horizon is situated at $z = 0$ and spatial infinity at $z = 1$. The above equation then takes the form

$$z(1-z)\phi''(z) + (1-z)\phi'(z) + \left[\frac{A}{z} + \frac{B}{1-z} + C \right] \phi(z) = 0;$$

with

$$A = \frac{4(\omega\Omega_+^{-1} + \mu)^2}{(r_+ - r_-)^2(\nu^2 + 3)^2},$$

$$B = -\alpha,$$

$$C = -\frac{4(\omega\Omega_-^{-1} + \mu)^2}{(r_+ - r_-)^2(\nu^2 + 3)^2}, \tag{2.5}$$

with prime denoting differentiation with respect to z . Ω_+ and Ω_- are the angular velocity at the outer and inner horizon respectively, and

$$\Omega_+^{-1} = \nu r_+ - \frac{\sqrt{r_+ r_- (\nu^2 + 3)}}{2}, \quad \Omega_-^{-1} = \nu r_- - \frac{\sqrt{r_+ r_- (\nu^2 + 3)}}{2}. \tag{2.6}$$

Upon removing the poles in the last term of the above equation through the following ansatz

$$\phi(z) = z^p(1-z)^q u(z), \tag{2.7}$$

with

$$p = -i\sqrt{A}, \quad q = \frac{1}{2}(1 - \sqrt{1 + 4\alpha}),$$

we reach a standard form of the hypergeometric differential equation

$$z(1-z)u''(z) + \{c - (a+b+1)z\}u'(z) - abu(z) = 0, \tag{2.8}$$

with

$$a = p + q + \sqrt{C}, \quad b = p + q - \sqrt{C}, \quad c = 2p + 1.$$

It is well known that the two independent solutions are given by

$$u_1(z) = {}_2F_1(a, b; c; z), \tag{2.9}$$

$$u_2(z) = z^{1-c} {}_2F_1(a - c + 1, b - c + 1; 2 - c; z). \tag{2.10}$$

We comment that these solutions are standard in z coordinate, which agrees with those in refs. [13, 14]. In ref. [11], the computation was carried out in the r coordinate and a different set of independent solutions were obtained in equation (3.6) in their paper. Although their solutions are equivalent to ours up to linear combination, we suspect that their choice could cause some complication when comparing solutions in the asymptotic region and therefore makes it difficult to see how the cross section reduces to the horizon area at low energy.

3 Tortoise coordinate and effective potential

It is instructive to see equation (2.3) in the tortoise coordinate r^* , such that

$$\phi^*(r^*) \equiv z(r)\phi(r), \quad r^* = f(r). \tag{3.1}$$

$f(r)$ and $z(r)$ are uniquely determined by requiring that eq. (2.3) can be brought into the following form:

$$\left[-\frac{d^2}{dr^{*2}} - \omega^2 + U^*(r^*) \right] \phi(r^*) = 0. \quad (3.2)$$

We observe that in the spatial infinity, the effective potential

$$\lim_{r \rightarrow \infty} U^*(r) \rightarrow \frac{(\nu^2 + 3)(\nu^2 + 3 + 4m\ell^2)}{12(\nu^2 - 1)}. \quad (3.3)$$

In the case of the BTZ black hole where $\nu = 1$, we always expect an infinite potential wall in the spatial infinity. For a generic $\nu \neq 1$, however, it becomes a wall of finite height. Therefore for $\omega^2 < U^*(\infty)$ we may apply the Dirichlet boundary condition and the wavefunction vanishes in the spatial infinity. In contrast, for $\omega^2 \geq U^*(\infty)$ we expect the wavefunction to be nonvanishing there. As we will see later, this new feature may bring subtlety when both the massless and low energy limits are taken.

4 Revisit absorption cross section

To obtain the absorption cross section, we consider the scattering process that an in-going flux comes from the spatial infinity and interacts with the black hole. It is then partially reflected backwards as out-going flux to the spatial infinity and the rest absorbed into the black hole. One way to achieve this goal is to consider a wavefunction with pure in-going boundary condition at the horizon and carefully decompose it into the in-going and out-going parts in the asymptotic region. This determines the form of the wavefunction for the scalar field:

$$\phi = C_{\text{in}} z^p (1-z)^{(a+b-2p)/2} {}_2F_1(a, b; c; z), \quad (4.1)$$

where p is given in eq. (2.7), a, b, c in eq. (2.8) and C_{in} an arbitrary coefficient. Expanding this solution at small z , we obtain the asymptotic form near the outer horizon:

$$\phi_+ \simeq C_{\text{in}} \left(\frac{r - r_+}{r_+ - r_-} \right)^{-i \frac{2\mu + \omega \{ 2r_+ \nu - \sqrt{r_+ r_- (\nu^2 + 3)} \}}{(r_+ - r_-)(\nu^2 + 3)}} + \dots \quad (4.2)$$

Now let us consider the same wavefunction (4.1) near the spatial infinity:

$$\begin{aligned} \phi_\infty \simeq & C_{\text{in}} \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} \left(\frac{r}{r_+ - r_-} \right)^{-\frac{1}{2} + \frac{1}{2}\sqrt{1+4\alpha}} \left[1 + \frac{ab}{1-c+a+b} \left(\frac{r}{r_+ - r_-} \right)^{-1} \right] \\ & + C_{\text{in}} \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)} \left(\frac{r}{r_+ - r_-} \right)^{-\frac{1}{2} - \frac{1}{2}\sqrt{1+4\alpha}} \left[1 + \frac{(c-a)(c-b)}{1+c-a-b} \left(\frac{r}{r_+ - r_-} \right)^{-1} \right] + \dots \end{aligned} \quad (4.3)$$

On the other hand, we may also take the limit $r \rightarrow \infty$ in eq (2.3) first and it becomes

$$\phi''(r) + \frac{2}{r}\phi'(r) - \frac{\alpha r + \beta}{r^3}\phi(r) = 0. \quad (4.4)$$

It has two independent solutions

$$r^{-\frac{1}{2}} I_{-\sqrt{1+4\alpha}} \left(2\sqrt{\frac{\beta}{r}} \right), \quad r^{-\frac{1}{2}} I_{\sqrt{1+4\alpha}} \left(2\sqrt{\frac{\beta}{r}} \right), \quad (4.5)$$

where the modified Bessel function $I_n(x)$ has the following expansion at small x :

$$I_n(x) \simeq \frac{x^n}{\Gamma(1+n)} \left[1 + \frac{\Gamma(1+n)}{2^2 \Gamma(2+n)} x^2 + \dots \right]. \quad (4.6)$$

We then define the in-going and out-going waves as a linear combination of the leading terms, i.e.

$$\begin{aligned} \phi_\infty &\simeq \phi_\infty^{\text{in}}(r) + \phi_\infty^{\text{out}}(r), \\ \phi_\infty^{\text{in}} &= A_{\text{in}} \left(r^{-\frac{1}{2} + \frac{1}{2}\sqrt{1+4\alpha}} - \frac{i\eta}{\pi} r^{-\frac{1}{2} - \frac{1}{2}\sqrt{1+4\alpha}} \right), \\ \phi_\infty^{\text{out}} &= A_{\text{out}} \left(r^{-\frac{1}{2} + \frac{1}{2}\sqrt{1+4\alpha}} + \frac{i\eta}{\pi} r^{-\frac{1}{2} - \frac{1}{2}\sqrt{1+4\alpha}} \right). \end{aligned} \quad (4.7)$$

Here, η is some undetermined positive coefficient, which we take to be independent of ω . In comparison with eq. (4.3), we read

$$\begin{aligned} A_{\text{in}} &= \frac{C_{\text{in}}}{2} \left\{ \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} (r_+ - r_-)^{\frac{1}{2}(1-\sqrt{1+4\alpha})} \right. \\ &\quad \left. + i \frac{\pi}{\eta} \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)} (r_+ - r_-)^{\frac{1}{2}(1+\sqrt{1+4\alpha})} \right\}, \\ A_{\text{out}} &= \frac{C_{\text{in}}}{2} \left\{ \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} (r_+ - r_-)^{\frac{1}{2}(1-\sqrt{1+4\alpha})} \right. \\ &\quad \left. - i \frac{\pi}{\eta} \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)} (r_+ - r_-)^{\frac{1}{2}(1+\sqrt{1+4\alpha})} \right\}. \end{aligned} \quad (4.8)$$

We are now ready to compute the in-going flux via the following definition:

$$\mathcal{F} = \frac{2\pi}{i} (r - r_+) (r - r_-) [\phi^*(r) \phi'(r) - \phi(r) \phi'^*(r)]. \quad (4.9)$$

The absorption coefficient for the s-wave ($\mu = 0$) reads,

$$\mathcal{T} \equiv \frac{\mathcal{F}(\phi_+)}{\mathcal{F}(\phi_\infty^{\text{in}})} = \frac{\omega \mathcal{A}_H}{4\eta\sqrt{1+4\alpha}} \left| \frac{C_{\text{in}}}{A_{\text{in}}} \right|^2. \quad (4.10)$$

Note that A_{in} is generally quite complicated for arbitrary mass and angular frequency. Here, we would like to focus on the low energy limit for massless scalar. Under such conditions, we find that the coefficient of the first term in both curly brackets in eq. (4.8), $\frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}$, always reduces to 1 regardless of whether the low energy or the massless limit is taken first. On the other hand, the coefficient of the second term

$$\frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)} \rightarrow \begin{cases} 0, & \text{if } \omega \rightarrow 0 \text{ is taken first;} \\ \frac{2(\Omega_+^{-2} - \Omega_-^{-2})}{3(r_+ - r_-)^2(\nu^2 - 1)}, & \text{if } m \rightarrow 0 \text{ is taken first.} \end{cases} \quad (4.11)$$

From these results, we can then calculate the cross section for a massless scalar at low energy

$$\sigma_{\text{abs}}^0 \equiv \lim_{\omega \rightarrow 0} \frac{\mathcal{T}}{\omega} = \mathcal{A}_H, \quad (4.12)$$

if η is chosen properly

$$\eta = \begin{cases} 1, & \text{if } \omega \rightarrow 0 \text{ is taken first;} \\ \frac{1 \pm \sqrt{1 - \frac{8\pi^2(\Omega_+^{-2} - \Omega_-^{-2})}{3(\nu^2 - 1)}}}{2}, & \text{if } m \rightarrow 0 \text{ is taken first.} \end{cases} \quad (4.13)$$

We comment that this subtle difference arises from the distinct behaviour of the wavefunction in the spatial infinity thanks to the competition between ω^2 and $U^*(\infty)$. If the low energy limit is taken first, the wavefunction is vanishing in the spatial infinity. If it is the other way around then the wavefunction would be nonvanishing there.

5 Greybody factor

In the above computation of the absorption cross section, if one retains the ω dependence instead of setting it to be identically zero, then one can calculate the greybody factor. This has been done for the BTZ black hole [8], but it remains a nontrivial check for the warped AdS₃ black hole. Following procedures similar to those taken to obtain the result in eq. (4.12), we find the relevant part of the absorption cross section for s-wave:²

$$\sigma_{\text{abs}} \propto \frac{1}{\sqrt{1+4\alpha}} \left| \frac{\Gamma\left[\frac{1}{2}(1+\sqrt{1+4\alpha}) - i\frac{\omega}{4\pi\tilde{T}_R}\right] \Gamma\left[\frac{1}{2}(1+\sqrt{1+4\alpha}) - i\frac{\omega}{4\pi\tilde{T}_L}\right]}{\Gamma\left[1 - i\frac{\omega}{2\pi T_H}\right] \Gamma[\sqrt{1+4\alpha}]}\right|^2, \quad (5.1)$$

where α is given in (2.3) and satisfies the reality condition $1+4\alpha \geq 0$. The *effective* temperatures are known to be related to the ones in (1.3) by a proper coordinate transformation [13]:

$$\begin{aligned} \tilde{T}_R &= \frac{T_R}{(\sqrt{r_+} - \sqrt{r_-})^2} \Big|_{\nu=1} = \frac{1}{2\pi} \frac{\sqrt{r_+} + \sqrt{r_-}}{\sqrt{r_+} - \sqrt{r_-}}, \\ \tilde{T}_L &= \frac{T_L}{(\sqrt{r_+} - \sqrt{r_-})^2} \Big|_{\nu=1} = \frac{1}{2\pi}. \end{aligned} \quad (5.2)$$

First we would like to check that one does recover the results in ref. [8] in the BTZ limit, i.e. $\nu \rightarrow 1$. For simplicity, we will restrict our discussion to the massless case. Since $\alpha \rightarrow 0$ in this limit, we have

$$\begin{aligned} a &\rightarrow -i\frac{\omega}{4\pi\tilde{T}_R}, & b &\rightarrow -i\frac{\omega}{4\pi\tilde{T}_L}, \\ \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} &\rightarrow \frac{\Gamma(1+a+b)\Gamma(1)}{\Gamma(1+b)\Gamma(1+a)}. \end{aligned} \quad (5.3)$$

²It is straightforward to generalize to the case $\mu \neq 0$ by shifting terms in eq. 5.1 such that $\frac{\omega}{4\pi T_R} \rightarrow \frac{\omega + \tilde{\mu}}{4\pi T_R}$ and $\frac{\omega}{2\pi T_H} \rightarrow \frac{1}{4\pi} \left(\frac{\omega + \tilde{\mu}}{T_R} + \frac{\omega}{T_L} \right)$, where $\tilde{\mu} \equiv \frac{\mu}{(2\nu(r_+ + r_-) - \sqrt{r_+ r_-}(\nu^2 + 3))}$.

Using the identity

$$|\Gamma(1 - ia)|^2 = \frac{\pi a}{\sinh \pi a}, \quad (5.4)$$

we then recover the greybody factor

$$\sigma_{\text{abs}} \propto \omega \frac{e^{\omega/T_H} - 1}{\left(e^{\omega/2\tilde{T}_L} - 1\right) \left(e^{\omega/2\tilde{T}_R} - 1\right)}. \quad (5.5)$$

It can be seen that the above derivation still holds for $\nu > 1$, as long as α remains approaching zero. Replacing the effective temperature by

$$\begin{aligned} \tilde{T}_L &= \frac{T_L}{\delta} = \frac{\nu^2 + 3}{8\pi\nu}, \\ \tilde{T}_R &= \frac{T_R}{\delta} = \frac{(\nu^2 + 3)(r_+ - r_-)}{8\pi \left[\nu(r_+ + r_-) - \sqrt{r_+ r_- (\nu^2 + 3)} \right]}, \\ \text{with } \delta &\equiv \nu(r_+ + r_-) - \sqrt{r_+ r_- (\nu^2 + 3)}, \end{aligned} \quad (5.6)$$

we are able to obtain the same greybody factor as before. We would like to point out that this observation is consistent with the results obtained by computing the quasinormal modes in ref. [13].³ We remark that although we mainly focus on the massless limit in computation of absorption cross section at low energy limit, the greybody factor (5.1) can also survive for arbitrary large m in the near extremal limit such that $r_+ - r_- \rightarrow 0$ and $\omega \rightarrow 0$, but keeping $\frac{\omega}{\tilde{T}_R}$ finite.⁴

6 Effective CFT description

In this section, we would like to comment on the greybody factor (5.1) from a viewpoint of an effective CFT₂ on the (τ, u) -plane dual to the warped AdS₃ space. Having learnt from the usual AdS₃/CFT₂ dictionary, we propose a thermal correlator in the following form [15]:⁵

$$\begin{aligned} \left\langle \mathcal{O}(x_+, x_-) \mathcal{O}(0, 0) \right\rangle_T &\sim (2h_+ - 1) \left[\frac{\pi \tilde{T}_R}{\sinh(\pi \tilde{T}_R x_+)} \right]^{2h_+} \left[\frac{\pi \tilde{T}_L}{\sinh(\pi \tilde{T}_L x_-)} \right]^{2h_+}, \\ x_{\pm} &\equiv u \pm \tau, \\ h_+ &\equiv \frac{1}{2} \left[1 + \sqrt{1 - \frac{3(\nu^2 - 1)}{\nu^2} k^2 + \frac{4m^2 \ell^2}{\nu^2 + 3}} \right], \end{aligned} \quad (6.1)$$

³A alternative viewpoint is that $T_{L,R}$ remains intact but instead ω is rescaled to $\tilde{\omega} = \delta \omega$. This is just a matter of convention that depends on which coordinate system is chosen. In our convention, the effective temperature is the same as that of BTZ at $\nu = 1$ limit, nevertheless the convention in [10] gives rise to correct entropy with conjectured central charges. We thank the referee for bringing up this point.

⁴This limit should be taken before $\nu \rightarrow 1$ to be taken, if desired, in order to avoid ambiguity since two limits do not commute.

⁵We have normalized the coefficient of correlator to agree with the computation in black hole. In generic, there could be an ambiguity for the coupling between scalar field and boundary operator, say $\int \phi_0 \mathcal{O}$.

where h_+ has been derived from various aspects in the appendix. Now we would like to claim that a particular operator $\mathcal{O}_*(x_+, x_-)$ carrying the conformal dimension $h_+^* \equiv h_+|_{k=k_*}$ with⁶

$$k_* = \frac{2\nu}{\nu^2 + 3}\omega, \tag{6.2}$$

might be responsible for the greybody factor of the warped AdS₃ black hole. To see this, we recall that the absorption cross section is given by

$$\begin{aligned} & \int dx_+ dx_- e^{-i\omega(x_+ + x_-)} \left\langle \mathcal{O}_*(x_+, x_-) \mathcal{O}_*(0, 0) \right\rangle_T \\ & \sim \frac{(2h_+^* - 1) \sinh\left(\frac{\omega}{2\tilde{T}_H}\right)}{\omega \Gamma^2(2h_+^*)} \left| \Gamma\left(h_+^* + i\frac{\omega}{4\pi\tilde{T}_R}\right) \Gamma\left(h_+^* + i\frac{\omega}{4\pi\tilde{T}_L}\right) \right|^2. \end{aligned} \tag{6.3}$$

Making use of the identity $x\Gamma(x) = \Gamma(x + 1)$ and eq. (5.4), we are able to reproduce the result in eq. (5.1). To understand the relation (6.2) better, we first recall the coordinates adopted in the Poincaré frame (A.17), and then we have to introduce the energy $\omega = 2\omega'$ for $t = \tau/2$. The relation (6.2) then simply reduces to

$$k = \pm\omega' \tag{6.4}$$

for $\nu = 1$. This implies that \mathcal{O}_* can be seen as a conventional CFT operator but in a rotating frame (t, u) with speed

$$c \equiv \frac{\partial u}{\partial t} = \frac{\omega'}{k} = \pm \frac{\nu^2 + 3}{4\nu}, \tag{6.5}$$

which reduces to unit speed for undeformed AdS₃. This agrees with a known fact that for $\nu = 1$ the metric (1.1) reduces to the BTZ metric in a rotating frame.

The key formula (6.3) can also be understood in terms of the correlator (1.9) obtained in the appendix. We may first rescale⁷ and rotate the (t, u) -plane into (u_+, u_-) -plane either clockwise (right sector) or counterclockwise (left sector):

$$\begin{aligned} \left\langle \mathcal{O}_*(u_+, u_-) \mathcal{O}_*(0, 0) \right\rangle^L & \sim \frac{e^{i\omega u_+}}{|u_-|^{2h_+^*}}, & \text{counterclockwise: } & (t, u) \rightarrow (u_-, u_+); \\ \left\langle \mathcal{O}_*(u_+, u_-) \mathcal{O}_*(0, 0) \right\rangle^R & \sim \frac{e^{i\omega u_-}}{|u_+|^{2h_+^*}}, & \text{clockwise: } & (t, u) \rightarrow (u_+, u_-), \end{aligned} \tag{6.6}$$

$$u_{\pm} \equiv u \pm t/2c.$$

Assigning temperatures \tilde{T}_L and \tilde{T}_R to the left and right sectors respectively, and we then define the composite thermal correlator as the product of the thermal correlators in the

⁶This relation is hinted by the coordinate transformation between black hole and warped AdS in the asymptotic infinity [10]. We thank the referee for pointing this out.

⁷Before the rotation, we have to rescale $u \rightarrow \frac{\nu^2 + 3}{2\nu}u$ in order to normalize g_{uu} to unity in the Poincaré metric A.17. This is to assure that after coordinate transformation, we have $k_*(u \pm t) \rightarrow \omega u_{\pm}$. The rescaling of t is relatively not important here.

two sectors, i.e.

$$\left\langle\left\langle\mathcal{O}_*(u_+, u_-)\mathcal{O}_*(0, 0)\right\rangle\right\rangle_T \sim (2h_+ - 1)e^{i\omega(u_+ + u_-)} \left[\frac{\pi\tilde{T}_R}{\sinh(\pi\tilde{T}_R u_+)}\right]^{2h_+^*} \left[\frac{\pi\tilde{T}_L}{\sinh(\pi\tilde{T}_L u_-)}\right]^{2h_+^*}. \tag{6.7}$$

This is exactly the integrand in (6.3), and therefore the same result is expected. This construction can be seen as an effective CFT₁ coupled with a free U(1) field. This is similar to an earlier description of dimensional reduction from AdS₃ to AdS₂ for the BTZ black hole with the Chern-Simons and other higher derivative terms [16], which has recently been extended to the warped AdS₃ black hole [17].

7 Superradiant modes

Another unusual feature for the warped AdS₃ black hole is the appearance of the superradiant modes when

$$\omega^2 > \frac{(\nu^2 + 3)^2}{12(\nu^2 - 1)} + \frac{\nu^2 + 3}{3(\nu^2 - 1)}m^2\ell^2, \tag{7.1}$$

and $\nu \neq 1$. Note that this phenomenon is particular to the warped AdS₃ black hole and would not occur in the BTZ case where $\nu = 1$. In eqs. (4.3) and (4.7), the exponents of r for the wavefunction in spatial infinity becomes complex. Therefore, both terms of A_{in} in eq. (4.8) will contribute to the absorption cross section given by eq. (4.10). From the CFT side, the existence of the superradiant modes indicates that conformal dimension h_{\pm} become complex conjugates and the *effective* mass of scalar is below the Breitenlohner-Freedman bound

$$m_{\text{eff}}^2 \equiv \frac{4m^2}{\nu^2 + 3} - \frac{12(\nu^2 - 1)\omega^2}{(\nu^2 + 3)^2\ell^2} < -\frac{1}{\ell^2}. \tag{7.2}$$

As a result, the scattering process becomes unstable. Because of the connection in (6.2), the condition for ω can be translated into a condition on k_* and condition (7.1) leads to a bound on angular velocity just as in the case of Kerr black holes.⁸

Acknowledgments

The authors would like to thank Pei-Ming Ho, Shunsuke Teraguchi and Feng-Li Lin for helpful discussion. This project was partially supported by the Taiwan’s National Science Council and National Center for Theoretical Sciences under Grant No. NSC95-2112-M-003-013-MY3 (HCK), NSC97-2112-M-002-015-MY3 (WYW) and NSC97-2119-M-002-001.

A Warped AdS/CFT correspondence

A.1 Warped AdS calculation

In this section, we demonstrate the dictionary between the spatial warped AdS₃ and the corresponding CFT₂. In particular, we would like to derive the conformal weight h_{\pm} from

⁸We remark another interesting direction towards CFT calculation of superradiance in Kerr black holes by a recent paper [19]. The near horizon limit of extremal Kerr black holes (NHEK) also possesses the same $SL(2, R) \times U(1)$ symmetry as in the warped AdS black holes.

the viewpoints of spacetime isometry and boundary conformal group as well as the two-point correlator in the Poincaré patch.⁹ We first consider the gravity side with the following warped geometry in the global coordinate:

$$ds^2 = \frac{\ell^2}{\nu^2 + 3} \left[-\cosh^2 \sigma d\tau^2 + d\sigma^2 + \frac{4\nu^2}{\nu^2 + 3} (du + \sinh \sigma d\tau)^2 \right]. \quad (\text{A.1})$$

Let us now consider the Klein-Gordon equation of a massive scalar field $\Phi(\tau, \sigma, u)$ in this background. Assuming a stationary form $\Phi = e^{-i\omega_g \tau + iku} \phi_g(\sigma)$, the equation becomes

$$\frac{d^2 \phi_g}{d\sigma^2} + \tanh \sigma \frac{d\phi_g}{d\sigma} + \left[\omega_g^2 \operatorname{sech}^2 \sigma + 2\omega_g k \operatorname{sech} \sigma \tanh \sigma - \left(\frac{\nu^2 + 3}{4\nu^2} - \tanh^2 \sigma \right) k^2 - \frac{m^2 \ell^2}{\nu^2 + 3} \right] \phi_g = 0 \quad (\text{A.2})$$

after separation of variables. It can be brought into a standard form of the hypergeometric equation

$$z_g(1 - z_g)f'' + [c_g - (1 + a_g + b_g)z_g]f' - a_g b_g f = 0 \quad (\text{A.3})$$

via the following change of variable

$$\begin{aligned} \phi_g &= z_g^{(\omega_g + ik)/2} (1 - z_g)^{(\omega_g - ik)/2} f(z_g), \\ z_g &\equiv \frac{1 + i \sinh \sigma}{2}. \end{aligned} \quad (\text{A.4})$$

Here, prime denotes differentiation with respect to z_g . a_g, b_g are determined by

$$a_g b_g = \omega_g(\omega_g + 1) + \frac{3k^2(\nu^2 - 1)}{4\nu^2} - \frac{m^2 \ell^2}{\nu^2 + 3}, \quad a_g + b_g = 1 + 2\omega_g,$$

and $c_g = 1 + \omega_g + ik$.

On the other hand, the asymptotic behaviour of the two independent solutions to this equation is given by

$$\begin{aligned} \phi_g(z_g) &\rightarrow C_+ z_g^{h_+} + C_- z_g^{h_-}, \quad (\text{A.5}) \\ \text{with } h_{\pm} &= \frac{1}{2}(1 \pm \Delta), \quad \Delta \equiv \sqrt{1 - \frac{3(\nu^2 - 1)}{\nu^2} k^2 + \frac{4m^2 \ell^2}{\nu^2 + 3}}, \\ C_+ &= (-1)^{a_g} \frac{\Gamma(c_g)\Gamma(b_g - a_g)}{\Gamma(b_g)\Gamma(c_g - a_g)}, \quad C_- = (-1)^{b_g} \frac{\Gamma(a_g - b_g)}{\Gamma(a_g)\Gamma(c_g - b_g)}. \end{aligned}$$

For non-integer $\Delta < 1$, both solutions are renormalizable and either term in eq. (A.5) can act as a source or fluctuation. For $\Delta > 1$, only the first term is renormalizable. By setting $C_- = 0$, we obtained the following quantization condition:

$$\omega = -(n + h_+), \quad n = 0, 1, 2, \dots \quad (\text{A.6})$$

In the case of $\Delta < 1$, a similar condition may be achieved

$$\omega = -(n + h_-), \quad n = 0, 1, 2, \dots, \quad (\text{A.7})$$

if we choose the second term to be the renormalizable one.

⁹Part of discussion in this appendix has overlap with that in [20].

A.2 Group theory calculation

Now let us turn to the group theory side. Here, the boundary conformal group has been deformed into $SL(2, R)_R \times U(1)_L$. Following the notation in [10], the subgroup $SL(2, R)_R$ can be realized by

$$\begin{aligned}\tilde{J}_0 &= 2\partial_\tau, \\ \tilde{J}_1 &= 2\sin\tau \tanh\sigma\partial_\tau - 2\cos\tau\partial_\sigma + \frac{2\sin\tau}{\cosh\sigma}\partial_u, \\ \tilde{J}_2 &= -2\cos\tau \tanh\sigma\partial_\tau - 2\sin\tau\partial_\sigma - \frac{2\cos\tau}{\cosh\sigma}\partial_u,\end{aligned}\tag{A.8}$$

and the additional $U(1)_L$ by

$$J_2 = 2\partial_u.\tag{A.9}$$

It is convenient to use the following combination:

$$\begin{aligned}\tilde{L}_0 &= i\tilde{J}_0, & \tilde{L}_+ &= -(\tilde{J}_1 - i\tilde{J}_2), & \tilde{L}_- &= \tilde{J}_1 + i\tilde{J}_2, \\ L_0 &= iJ_2, & L_+ &= L_- = 0.\end{aligned}\tag{A.10}$$

The Casimirs operators are given by

$$L^2 = \frac{1}{2}(L_+L_- + L_-L_+) - L_0^2, \quad \tilde{L}^2 = \frac{1}{2}(\tilde{L}_+\tilde{L}_- + \tilde{L}_-\tilde{L}_+) - \tilde{L}_0^2.\tag{A.11}$$

We may rewrite eq. (A.5) in terms of the d’Lambertian, which can in turn be expressed in terms of the left and right Casimirs

$$-\frac{1}{4}(L^2 + \tilde{L}^2)\Phi = \frac{m^2\ell^2}{\nu^2 + 3}\Phi.\tag{A.12}$$

Solutions satisfying $L_+\Phi_+ = 0$ and $\tilde{L}_-\Phi_- = 0$ can be found explicitly and

$$\Phi_\pm = e^{-ih_\pm^0\tau}(\operatorname{sech}\sigma)^{h_\pm^0}, \quad h_\pm^0 = \frac{1}{2}\left[1 \pm \sqrt{1 + \frac{4m^2\ell^2}{\nu^2 + 3}}\right].\tag{A.13}$$

This agrees with the earlier result of h_\pm for $k = 0$. To render the desired k -dependence, we may introduce the *untwist* generators for the right-moving mode in which the ∂_u term is removed:

$$\tilde{J}_1 \rightarrow \tilde{J}_1 - \frac{\sin\tau}{\cosh\sigma}J_2, \quad \tilde{J}_2 \rightarrow \tilde{J}_2 + \frac{\cos\tau}{\cosh\sigma}J_2.\tag{A.14}$$

This will lead to a natural separation between generators in $SL(2, R)$ and $U(1)_L$, which can be seen later in the computation of the correlator.

Together with a new solution

$$\Phi_\pm = e^{-ih_\pm\tau + i\epsilon ku}(\operatorname{sech}\sigma)^{h_\pm},\tag{A.15}$$

it can be shown to reproduce h_\pm for

$$\epsilon^2 = \frac{3(\nu^2 - 1)}{4\nu^2}.\tag{A.16}$$

A.3 Correlator in Poincarè coordinate

To obtain the correlator of the corresponding operators, it is also worthwhile to work out the same wavefunction in the Poincarè coordinate. Although a general Poincarè coordinate valid for all range of σ is not available as far as we know, it is sufficient to know its behaviour in the limit $\sigma \rightarrow \infty$ where a boundary CFT is conjectured to exist. The local patch can be achieved by the coordinate transformation $x = e^{-\sigma}$ and $t = \tau/2$:

$$ds^2 \rightarrow \frac{\ell^2}{(\nu^2 + 3)x^2} \left[\frac{3(\nu^2 - 1)}{\nu^2 + 3} dt^2 + dx^2 + \frac{4\nu^2}{\nu^2 + 3} x^2 du^2 + \frac{8\nu^2}{\nu^2 + 3} x dt du \right]. \quad (\text{A.17})$$

As in ref. [18], we will solve the following equation:

$$\left(\frac{1}{\sqrt{-g}} \partial_x \sqrt{-g} g^{xx} \partial_x + \partial_u g^{uu} \partial_u - m^2 \right) K(x, u) = 0. \quad (\text{A.18})$$

Using the ansatz $K(x, u) = e^{iku} \tilde{K}(x)$, we obtain

$$\tilde{K}(x) \sim x^{h_+}, \quad (\text{A.19})$$

where we have chosen the operator with conformal weight h_+ as the renormalizable mode. The group transformation of $SL(2, R)$ acting on the (t, x) -plane will map

$$K(x, u) \rightarrow K(x, u, t) = e^{iku} \left(\frac{x}{|x^2 - t^2|} \right)^{h_+}. \quad (\text{A.20})$$

After performing a translation on $u \in U(1)$ and t , we define the bulk-to-boundary Green function as

$$K_b(x, u, t, u', t') \equiv e^{ik(u-u')} \left[\frac{x}{|x^2 - (t-t')^2|} \right]^{h_+}. \quad (\text{A.21})$$

Therefore we obtain a bulk field which is determined in the following way

$$\phi(x, t, u) \sim \int dt' du' K_b(x, t, u, t', u') \phi_0(t', u'). \quad (\text{A.22})$$

Its derivative in the limit $x \rightarrow 0$ is given by

$$\frac{\partial}{\partial x} \phi(x, t, u) \sim x^{h_+-1} \int dt' du' \frac{e^{ik(u-u')} \phi_0(t', u')}{|t-t'|^{2h_+}}. \quad (\text{A.23})$$

The on-shell action only gets contribution from the boundary term,

$$S_{\text{eff}} = \lim_{x \rightarrow 0} -\frac{1}{2} \int dt du \sqrt{-g} g^{xx} \phi \partial_x \phi \sim \frac{1}{2} \int dt du dt' du' \frac{e^{ik(u-u')}}{|t-t'|^{2h_+}} \phi_0(t, u) \phi_0(t', u'). \quad (\text{A.24})$$

From the AdS/CFT dictionary between the effective action in the bulk and the generating function on the boundary, we have

$$e^{-S_{\text{eff}}(\phi)} = \left\langle e^{\int \phi_0 \mathcal{O}} \right\rangle. \quad (\text{A.25})$$

As a result, the correlator of the operators is given by

$$\left\langle \mathcal{O}(t, u) \mathcal{O}(t', u') \right\rangle \sim \frac{e^{ik(u-u')}}{|t-t'|^{2h_+}}. \quad (\text{A.26})$$

We remark that this correlator does not take the usual form expected from the CFT_2 . Instead, it is a product of the correlator in CFT_1 and the free propagator along the u -direction, reflecting the breaking of symmetry from $\text{SL}(2, R) \times \text{SL}(2, R)$ to $\text{SL}(2, R) \times \text{U}(1)$ due to the deformation.

In summary, we have seen that in addition to mass m , spin k is also involved in the definition of conformal weight h_{\pm} , thanks to the deformation $\nu \neq 1$. This may be traced back to the nontrivial fibrated coordinate adopted in (A.1).

References

- [1] S.R. Das, G.W. Gibbons and S.D. Mathur, *Universality of low energy absorption cross sections for black holes*, *Phys. Rev. Lett.* **78** (1997) 417 [[hep-th/9609052](#)] [[SPIRES](#)].
- [2] S.R. Das and S.D. Mathur, *Comparing decay rates for black holes and D-branes*, *Nucl. Phys.* **B 478** (1996) 561 [[hep-th/9606185](#)] [[SPIRES](#)].
- [3] J.M. Maldacena and A. Strominger, *Black hole greybody factors and D-brane spectroscopy*, *Phys. Rev.* **D 55** (1997) 861 [[hep-th/9609026](#)] [[SPIRES](#)].
- [4] S.S. Gubser and I.R. Klebanov, *Four-dimensional greybody factors and the effective string*, *Phys. Rev. Lett.* **77** (1996) 4491 [[hep-th/9609076](#)] [[SPIRES](#)].
- [5] J.M. Maldacena and A. Strominger, *Universal low-energy dynamics for rotating black holes*, *Phys. Rev.* **D 56** (1997) 4975 [[hep-th/9702015](#)] [[SPIRES](#)].
- [6] M. Cvetič and F. Larsen, *General rotating black holes in string theory: greybody factors and event horizons*, *Phys. Rev.* **D 56** (1997) 4994 [[hep-th/9705192](#)] [[SPIRES](#)].
- [7] M. Cvetič and F. Larsen, *Greybody factors for rotating black holes in four dimensions*, *Nucl. Phys.* **B 506** (1997) 107 [[hep-th/9706071](#)] [[SPIRES](#)].
- [8] D. Birmingham, I. Sachs and S. Sen, *Three-dimensional black holes and string theory*, *Phys. Lett.* **B 413** (1997) 281 [[hep-th/9707188](#)] [[SPIRES](#)].
- [9] A. Bouchareb and G. Clement, *Black hole mass and angular momentum in topologically massive gravity*, *Class. Quant. Grav.* **24** (2007) 5581 [[arXiv:0706.0263](#)] [[SPIRES](#)].
- [10] D. Anninos, W. Li, M. Padi, W. Song and A. Strominger, *Warped AdS_3 black holes*, *JHEP* **03** (2009) 130 [[arXiv:0807.3040](#)] [[SPIRES](#)].
- [11] J.J. Oh and W. Kim, *Absorption cross section in warped AdS_3 black hole*, *JHEP* **01** (2009) 067 [[arXiv:0811.2632](#)] [[SPIRES](#)].
- [12] J.J. Oh and W. Kim, *Absorption cross section in the topologically massive gravity at the critical point*, [arXiv:0904.0085](#) [[SPIRES](#)].
- [13] B. Chen and Z.-B. Xu, *Quasinormal modes of warped AdS_3 black holes and AdS/CFT correspondence*, [arXiv:0901.3588](#) [[SPIRES](#)].
- [14] S.K. Chakrabarti, P.R. Giri and K.S. Gupta, *Scalar field dynamics in warped AdS_3 black hole background*, [arXiv:0903.1537](#) [[SPIRES](#)].

- [15] H.J.W. Muller-Kirsten, N. Ohta and J.-G. Zhou, *AdS₃/CFT correspondence, Poincaré vacuum state and greybody factors in BTZ black holes*, *Phys. Lett. B* **445** (1999) 287 [[hep-th/9809193](#)] [[SPIRES](#)].
- [16] B. Sahoo and A. Sen, *BTZ black hole with Chern-Simons and higher derivative terms*, *JHEP* **07** (2006) 008 [[hep-th/0601228](#)] [[SPIRES](#)].
- [17] W. Kim and E.J. Son, *Central charges in 2D reduced cosmological massive gravity*, *Phys. Lett. B* **678** (2009) 107 [[arXiv:0904.4538](#)] [[SPIRES](#)].
- [18] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [[hep-th/9802150](#)] [[SPIRES](#)].
- [19] I. Bredberg, T. Hartman, W. Song and A. Strominger, *Black hole superradiance from Kerr/CFT*, [arXiv:0907.3477](#) [[SPIRES](#)].
- [20] D. Anninos, *Sailing from warped AdS₃ to warped dS₃ in topologically massive gravity*, [arXiv:0906.1819](#) [[SPIRES](#)].