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Point-like sources and the scale of quantum gravity

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ABSTRACT

We review the General Relativistic model of a (quasi) point-like particle represented by a massive shell of neutral matter which has vanishing total energy in the small-volume limit. We then show that, by assuming a Generalised Uncertainty Principle, which implies the existence of a minimum length of the order of the Planck scale, the total energy instead remains finite and equal to the shell's proper mass both for very heavy and very light particles. This suggests that the quantum structure of space–time might be related to the classical Equivalence Principle and possible implications for the late stage of evaporating black holes are briefly mentioned.

1. Introduction

In classical physics, nothing prevents us from describing elementary particles as being point-like, except that the stress–energy tensor of the electromagnetic and Newtonian gravitational fields diverge. In fact, this problem can actually be avoided in General Relativity, as was first shown in Ref. [1], where localised sources were described as shells of matter whose total energy remains finite in the "point-like" limit. In particular, if the shell is electrically neutral, its total energy vanishes in the small-volume limit (whose precise definition will be given in due course). This result was interpreted as the fact that General Relativity does not allow finite amounts of energy in a vanishingly small volume.

There are however many reasons to believe that, in a quantum mechanical world, the point-like limit is meaningless. To begin with, one has the Heisenberg Uncertainty Principle to prevent complete localisation in the phase space of Minkowskian theories. Moving bottom up to a semiclassical scenario, in which gravity is still described in terms of a background space–time, rigorous results and plausibility arguments suggest the emergence of a Generalised Uncertainty Principle (GUP) [2]. The idea behind all proposed GUPs is that in a scattering experiment with beams of energy E, the minimum accessible length is given by

$$\delta x \geqslant \frac{\ell_p m_p}{2E} + \frac{\alpha^2}{4} R_g(E), \tag{1}$$

where $\alpha \simeq 1$[1] and R_g is the gravitational radius associated with the energy of the scattering process. The latter is given by $R_g(E) = 2\ell_p E/m_p$ in the simplest approximation of the Schwarzschild geometry, with ℓ_p and m_p the Planck length and mass (we use units with $\hbar = c = 1$ and the Newton constant $16\pi G_N = \ell_p/m_p$). On minimising Eq. (1) with respect to E, one obtains

$$E_{min} = \alpha^{-1} m_p, \qquad \delta x_{min} \equiv \lambda = \alpha \ell_p. \tag{2}$$

Similar conclusions are also obtained from top–down approaches starting from more fundamental theories, such as String Theory [4] and Loop Quantum Gravity [5], which hint to space–time noncommutativity [6] at short length scales. From the phenomenological point of view, the existence of a minimum length interestingly leads to universal corrections which might even be within the reach of forthcoming experiments [3].

We shall here review the neutral solution of Ref. [1], also in the more standard approach of Israel [7], and then investigate what consequences follow from the existence of the minimum length λ of Eq. (2).

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[1] Heisenberg's principle is recovered for $\alpha \to 0$. In Ref. [3], the equivalent parameter $\beta_0 = \alpha^2$ was used.

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2. Classical shell model

Following Ref. [1], we consider the space–time generated by a shell of bare mass m_0 and coordinate radius $r = \epsilon$.

2.1. ADM model

For the interior ($0 \leqslant r < \epsilon$), we shall assume flat Minkowski space–time,

$$ds_{\rm i}^2 = -C^2\,dt^2 + A^4\big(dr^2 + r^2\,d\Omega^2\big), \tag{3}$$

and for the exterior ($r > \epsilon$) the isotropic form of the Schwarzschild metric [8],

$$ds_{\rm o}^2 = -\left(\frac{2r-M}{2r+M}\right)^2 dt^2 + \left(1+\frac{M}{2r}\right)^4 \big(dr^2 + r^2\,d\Omega^2\big), \tag{4}$$

where M, C and A are constants determined by the matching conditions at $r = \epsilon$. The metric (4) is the well-known prototype of a wormhole, asymptotically flat both for $r \to \infty$ and $r \to 0$.

The total energy of this spherically symmetric space–time with asymptotically flat metric (for $r \to \infty$) is given by the surface integral [1,9,10]

$$E = -\lim_{R\to\infty}\left[\int \frac{d\theta\,d\phi}{8\pi}\sqrt{g^{(2)}}(K - K_0)_{r=R}\right], \tag{5}$$

where $g^{(2)}$ and K are, respectively, the determinant of the two-metric and the trace of the extrinsic curvature of a two-sphere ${}^2S$ of radius R; K_0 is the trace of the extrinsic curvature corresponding to embedding the two-dimensional boundary ${}^2S$ in three-dimensional Euclidean space and yields the Minkowski "reference" energy. The curvature tensor K_{ij} can be evaluated by introducing a Gaussian normal coordinate y such that $r(y = 0) = R$ and the spatial part of the metric (4) reads

$$ds_{(3)}^2 = dy^2 + \left(1+\frac{M}{2r(y)}\right)^4 r^2(y)\,d\Omega^2. \tag{6}$$

One then finds

$$K = 2g^{\theta\theta}K_{\theta\theta} = -\frac{dr}{dy}\frac{\partial g_{\theta\theta}}{\partial r}, \tag{7}$$

and

$$\begin{aligned} E &= -\lim_{R\to\infty}\left\{\frac{1}{2}\left(1+\frac{M}{2R}\right)^{-2}\frac{\partial}{\partial r}\left[r^2\left(1+\frac{M}{2r}\right)^4\right]_{r=R} + R\right\} \\ &= M, \end{aligned} \tag{8}$$

which shows that M is the Arnowitt–Deser–Misner (ADM) mass of the system, as expected [1].

The shell matter at $r = \epsilon$ is represented by a $\delta^{(3)}$-function energy density,

$$8\pi G_{\rm N}\sqrt{g^{(3)}}T_t^t = -\frac{M_0}{2}\sqrt{\eta^{(3)}}\delta^{(3)}(r), \tag{9}$$

where

$$M_0 = \frac{\ell_{\rm p} m_0}{16\pi m_{\rm p}}, \tag{10}$$

$g^{(3)}$ is the determinant of the spatial metric, $\eta^{(3)} = r^4\sin^2\theta$, and

$$4\pi\int_0^\infty \delta^{(3)}(r)r^2\,dr = 1, \tag{11}$$

with

$$\delta^{(3)}(r) = 0 \quad \text{for } |r-\epsilon| > \rho, \tag{12}$$

where $0 < \rho \ll \epsilon$ (and the limit $\rho \to 0$ at the end of the computations is understood).

The relevant equation is given by the tt-component of the Einstein equations,

$$\sqrt{\frac{g^{(3)}}{\eta^{(3)}}}\left(R^t{}_t - \frac{1}{2}R\right) = \frac{\chi\nabla^2\chi}{4\pi} = -\frac{M_0}{2}\delta^{(3)}(r), \tag{13}$$

where

$$\chi = \begin{cases} A & \text{for } 0 \leqslant r < \epsilon, \\ (1+\frac{M}{2r}) & \text{for } r > \epsilon, \end{cases} \tag{14}$$

and

$$\nabla^2\chi = r^{-2}\partial_r\big(r^2\partial_r\chi\big) \tag{15}$$

is the flat space Laplacian in spherical coordinates. Continuity of the metric across the shell then implies

$$A = 1 + \frac{M}{2\epsilon}, \tag{16}$$

and, upon integrating both sides of Eq. (13) in a spherical volume around $r = \epsilon$, one obtains

$$\begin{aligned} \frac{M_0}{2} &= -\lim_{\rho\to 0}\int_{\epsilon-\rho}^{\epsilon+\rho}\chi\frac{\partial}{\partial r}\left(r^2\frac{\partial\chi}{\partial r}\right)dr \\ &= \lim_{\rho\to 0}\left\{\int_{\epsilon-\rho}^{\epsilon+\rho}\left(\frac{\partial\chi}{\partial r}\right)^2 r^2\,dr - \left[r^2\chi\frac{\partial\chi}{\partial r}\right]_{r=\epsilon-\rho}^{r=\epsilon+\rho}\right\} \\ &= -\left[\frac{r^2}{2}\frac{\partial}{\partial r}\left(1+\frac{M}{2r}\right)^2\right]_{r=\epsilon}. \end{aligned} \tag{17}$$

One can finally write Eq. (17) as [1]

$$M = -\epsilon + \sqrt{\epsilon^2 + 2M_0\epsilon} \tag{18}$$

which implies that $2M \sim \sqrt{M_0\epsilon}$ for $\epsilon \ll M_0$.

2.2. Israel's junction equations

The above result for the ADM mass can also be obtained from Israel's junction equations for a static spherically symmetric shell [7] in Schwarzschild coordinates. Let us define the usual areal radius ($\bar r \geqslant 0$) as $\bar r = r$ for $r < \epsilon$ and

$$\bar r = r\left(1+\frac{M}{2r}\right)^2, \quad \text{for } r > \epsilon. \tag{19}$$

For $r > \epsilon$, we then find the two solutions

$$r_\pm = \frac{1}{2}\big(\bar r - M \pm \sqrt{\bar r(\bar r - 2M)}\big), \tag{20}$$

which are both real for $\bar r > 2M \geqslant 0$. Note that $r = r_+(\bar r)$ and $r = r_-(\bar r)$ are respectively increasing and decreasing in $\bar r$ (see Fig. 1). Therefore, $r_+ > M/2$ spans the Schwarzschild manifold outside the horizon, $\mathcal{M}^+ = \{\bar r(r_+) > 2M\}$, whereas the region $0 < r_- < M/2$ is a second copy of the same Schwarzschild manifold, $\mathcal{M}^- = \{\bar r(r_-) > 2M\}$. The complete manifold $\mathcal{M} = \mathcal{M}^- \cup \mathcal{M}^+$ represents a "wormhole" whose "throat" has a minimum areal radius equal to $2M$.

We recall that wormhole metrics are usually given in the form [10]

$$ds_w^2 = e^{-2\phi(\bar r(x))}\,dt^2 + dx^2 + \bar r(x)^2\,d\Omega^2, \tag{21}$$

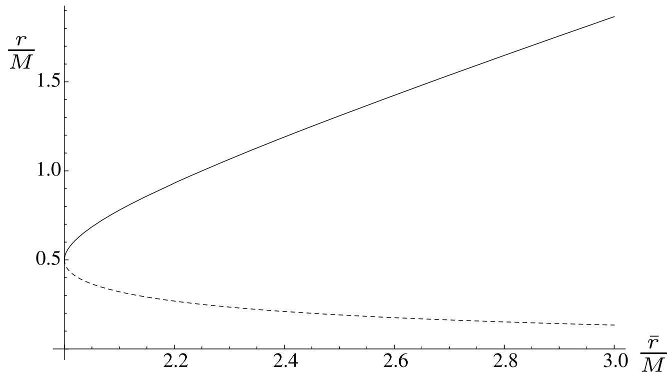

Fig. 1. Radial coordinates r_+ (solid line) and r_- (dashed line) in Eq. (20) as functions of areal radius $\bar{r}$ for fixed M.

where

$$dx = \frac{\sigma_\pm \, d\bar{r}}{\sqrt{1 - b(\bar{r})/\bar{r}}}, \tag{22}$$

with $\sigma_+ = +1$ in $\mathcal{M}^+$ and $\sigma_- = -1$ in $\mathcal{M}^-$; $b(\bar{r})$ is the "shape function" subjected to the condition $b(\bar{r}_t) = \bar{r}_t$, where $\bar{r}_t = 2M$ is the throat corresponding to $x = 0$, and $\phi(x)$ is the "redshift function".

The metrics inside and outside the shell can now be written as

$$ds^2_{i/o} = -f_{i/o} \, dt^2 + f^{-1}_{i/o} \, d\bar{r}^2 + \bar{r}^2 \, d\Omega^2, \tag{23}$$

with $f_i = 1$ and

$$f_o = 1 - \frac{b(\bar{r})}{\bar{r}} = 1 - \frac{2M}{\bar{r}}. \tag{24}$$

One of the junction equations for $\bar{r} > 2M$ then reads [7,11]

$$\begin{aligned} M_0 &= \bar{r}(\epsilon)[\sqrt{f_i} - \sigma_\pm \sqrt{f_o}\,]_{\bar{r}=\bar{r}(\epsilon)} \\ &= \bar{r}(\epsilon) - \sigma_\pm \sqrt{\bar{r}^2(\epsilon) - 2M\bar{r}(\epsilon)}, \end{aligned} \tag{25}$$

with $\sigma_+ = +1$ for $\epsilon > M/2$ and $\sigma_- = -1$ for $0 < \epsilon < M/2$. The above expression exactly yields the relation (18) in isotropic coordinates after using Eq. (19). In the Schwarzschild frame, it is also easier to see that Eq. (25) requires that $\bar{r}(\epsilon)$ remain finite for $\epsilon \to 0$ in order for M_0 to be finite in $\mathcal{M}^-$ (where $\sigma_- = -1$). This, together with the definition (19), implies that M must vanish for $\epsilon \to 0$, again in agreement with Eq. (18).

Finally, the shell's surface tension is given by the second junction equation [7,11],

$$P = \left.\frac{\partial M_0}{4\pi \, \partial \bar{r}^2}\right|_{\bar{r}(\epsilon)} = \frac{1}{8\pi \bar{r}(\epsilon)}\left[1 - \frac{\sigma_\pm[\bar{r}(\epsilon) - M]}{\sqrt{\bar{r}^2(\epsilon) - 2M\bar{r}(\epsilon)}}\right]. \tag{26}$$

2.3. Small-volume limit

Due to the dependence of M on ϵ, the shell areal radius

$$\bar{r}(\epsilon) = \frac{\epsilon}{4}\left(1 + \sqrt{1 + 2\frac{M_0}{\epsilon}}\right)^2, \tag{27}$$

is now a (single valued) monotonously increasing function of ϵ (see Fig. 2). It also remains finite for $\epsilon \to 0$, thus yielding a minimum shell volume $\propto \bar{r}^3(0)$ and area $\propto \bar{r}^2(0)$ in the classical theory, where

$$\bar{r}(0) = \frac{M_0}{2} \tag{28}$$

is of the order of the Schwarzschild radius of the mass m_0.

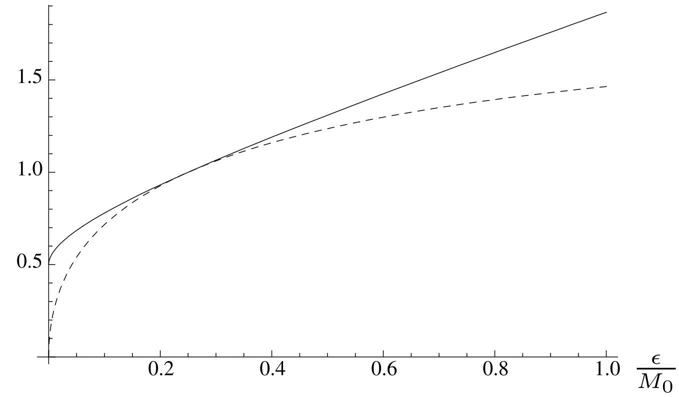

Fig. 2. Shell's areal radius $\bar{r}/M_0$ (solid line) and throat radius $2M/M_0$ (dashed line) *vs* shell's coordinate radius.

This does not mean that the shell never enters the region $\mathcal{M}^-$ parameterized by r_-. In fact, one finds that $\bar{r}(\epsilon) = 2M(\epsilon)$ for

$$\epsilon = r_c \equiv \frac{M_0}{4} \quad \text{and} \quad \bar{r}_c \equiv \bar{r}(r_c) = M_0 = 2\bar{r}(0), \tag{29}$$

which only depend on m_0. Thus, the shell is in $\mathcal{M}^+$ for $\epsilon > r_c$ and in $\mathcal{M}^-$ for $0 \leqslant \epsilon < r_c$. In the limit $\epsilon \to 0$, Eq. (18) yields $M(\epsilon) \to 0$, corresponding to Minkowski space–time for $r > 0$: the throat pinches off and the shell becomes gravitationally inaccessible to observers in $\mathcal{M}^+$. From Eq. (26) with $\sigma_- = -1$, one also sees that the surface tension remains finite in this limit, namely

$$P \simeq \frac{1}{2\pi M_0}. \tag{30}$$

3. Quantum theory

If one views m_0 as the inertial mass and M as the gravitational mass, the result of Ref. [1] for neutral sources clearly conflicts with the expectation that the Equivalence Principle (EP) holds for fundamental particles. Let us then suppose that the shell is described by an effective quantum theory with the minimum length λ of Eq. (2).

Given the existence of λ, the areal radius of a quantum-mechanically corrected shell of bare mass m_0 should be bounded from below, that is

$$\bar{r}(\epsilon) \gtrsim \bar{r}(\epsilon_\lambda) \equiv \bar{r}_\lambda \simeq \lambda, \tag{31}$$

and Eq. (27) provides a minimum value for its coordinate radius,

$$\epsilon \gtrsim \epsilon_\lambda \equiv \lambda\left(1 - \frac{M_0}{2\lambda}\right)^2 \equiv \lambda\left(1 - \frac{x}{2}\right)^2. \tag{32}$$

Depending on the bare mass m_0 (or, equivalently, x), the shell will thus be either in $\mathcal{M}^+$ or $\mathcal{M}^-$. In particular, the shell is in $\mathcal{M}^-$ if the ratio

$$\gamma \equiv \frac{\epsilon_\lambda}{r_c} = \frac{4}{x}\left(1 - \frac{x}{2}\right)^2 < 1, \tag{33}$$

that is $1 < x < 4$, otherwise it is in $\mathcal{M}^+$. In Fig. 3 we display both γ and the ratio

$$\Gamma \equiv \frac{2M_\lambda}{\bar{r}_\lambda} = \begin{cases} 2x(1 - \frac{x}{2}) & \text{if } 0 \leqslant x < 2, \\ \frac{8}{x}(1 - \frac{2}{x}) & \text{if } x > 2, \end{cases} \tag{34}$$

where the vanishing total mass of the classical theory is also replaced by the finite expression obtained for $\epsilon = \epsilon_\lambda$, that is

$$M_\lambda = \begin{cases} M_0(1 - \frac{M_0}{2\lambda}) & \text{if } 0 \leqslant M_0 < 2\lambda, \\ M_0(1 - \frac{2\lambda}{M_0}) & \text{if } M_0 > 2\lambda. \end{cases} \tag{35}$$

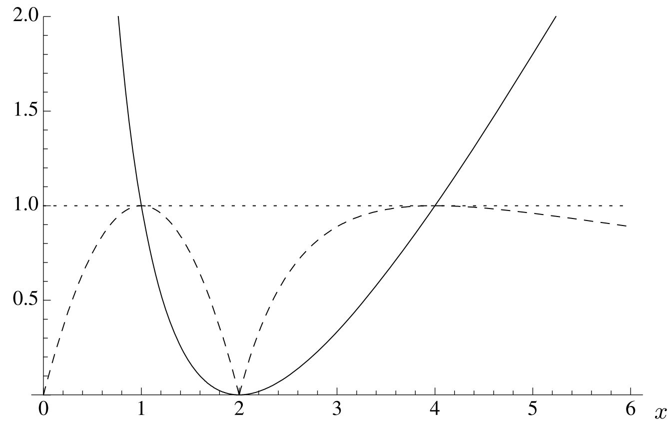

Fig. 3. Ratios γ (solid line) and Γ (dashed line) *vs* rescaled shell's proper mass $m_0 = 16\pi\alpha m_p x$. Intersections of the two curves represent the shell exactly at $r = 2M$.

For an elementary particle and $\lambda = \alpha\ell_p$ (with $\alpha \gtrsim 1$), it is natural to assume $x \sim m_0/m_p \ll 1$. One then finds that the shell is in $\mathcal{M}^+$ and its total energy is

$$m_\lambda \equiv 16\pi m_p \frac{M_\lambda}{\ell_p} \simeq m_0. \tag{36}$$

Unlike in the purely classical theory, the EP therefore holds for this quantum-mechanically corrected model of elementary particles, and does so regardless of the precise value of $\alpha \gtrsim 1$ [3]. We also note that the shell's surface tension (26) in this case (with $\sigma_+ = 1$) is given by

$$P_\lambda \simeq -\frac{\epsilon_\lambda}{\pi M_0^2} \simeq -\frac{\lambda}{\pi M_0^2}. \tag{37}$$

Further, for a mass $m_0 \gg m_p$ ($x \gg 1$), the shell is again in $\mathcal{M}^+$ and its total energy (in units of length) is given by Eq. (36). The EP is thus preserved also for macroscopic objects, as it should.[2]

Significant corrections to the EP would only occur for $M_0 \simeq 2\lambda$, that is for particles with a mass around the Planck scale, which would therefore enter the region $\mathcal{M}^-$. One such option would be micro-black holes, or black holes which have reached the latest stages of their evaporation. Eqs. (35) and (26) then predict vanishing total energy $M \simeq 0$ for a remnant of proper mass $m_0 \simeq 2\alpha m_p$ and surface tension

$$P \simeq \frac{1}{4\pi\alpha\ell_p}. \tag{38}$$

4. Conclusions

We reviewed the classical description of elementary "point-like" neutral particles in General Relativity given long ago in Ref. [1] in terms of shells.[3] A major findings in that paper was that neutral shells of finite "bare" mass m_0 have zero total energy in the small-volume limit and, therefore, they do not interact gravitationally and violate the EP. We applied a GUP type of argument, according to which this limit should be corrected so as to admit a minimum length scale λ of the order of the Planck length. We then found that the energy of this "point-like" source is naturally equal to m_0, regardless of the precise value of λ, and deviations from the EP only occur for objects around the Planck size. Such corrections support the possibility of evaporating black holes leaving remnants of vanishing ADM mass.[4]

It would be interesting to study what further implications our results may have in the context of black hole quantum formation and evaporation [11,15], and the ultra-violet divergences of quantum field theory [16].

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[2] This behaviour of $M = M(M_0)$ for fixed λ somehow resembles the R-duality in String Theory [4].

[3] See also Refs. [14] for more recent analyses.

[4] The remnants should have vanishing specific heat [12], like the non-commutative black holes of Ref. [13].