

Thermal unparticles: a new form of energy density in the universe

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Abstract An unparticle \mathcal{U} with scaling dimension $d_{\mathcal{U}}$ has peculiar thermal properties due to its unique phase space structure. We find that the equation of state parameter $\omega_{\mathcal{U}}$, the ratio of pressure to energy density, is given by $1/(2d_{\mathcal{U}} + 1)$, providing a new form of energy in our universe. In an expanding universe, the unparticle energy density $\rho_{\mathcal{U}}(T)$ evolves dramatically differently from that for photons. For $d_{\mathcal{U}} > 1$, even if $\rho_{\mathcal{U}}(T_{\text{D}})$ at a high decoupling temperature T_{D} is very small, it is possible to have a large relic density $\rho_{\mathcal{U}}(T_{\gamma}^0)$ at present photon temperature T_{γ}^0 , large enough to play the role of dark matter. We calculate T_{D} and $\rho_{\mathcal{U}}(T_{\gamma}^0)$ using photon–unparticle interactions for illustration.

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In cosmology the equation of state (EoS) parameter ω , the ratio of pressure (p) to energy density (ρ), for a species of energy carrier, plays a crucial role in determining the properties of the expanding universe [1]. It determines the energy density at a given temperature since ρ evolves with the Friedmann–Robertson–Walker (FRW) metric scale factor R as $R^{-3(1+\omega)}$. It also fixes the rate of deceleration since the deceleration parameter is proportional to $(1 + 3\omega)\rho$. For example, while cold dark matter (CDM) with $\omega_{\text{M}} = 0$ provides a stronger gravitational attraction than a photon whose $\omega_{\gamma} = 1/3$, quintessence with $\omega_{\text{Q}} < -1/3$ and the cosmological constant Λ with $\omega_{\Lambda} = -1$ accelerate the expansion of our universe. It is an important task for modern cosmology to determine various relic energy densities and their EoS parameters [2]. And this has become even more urgent due to the recent discovery in precision cosmological observations that the majority of the energy budget in our universe is carried by dark matter and dark energy instead of ordinary matter.¹ What is the nature of this ‘dark side’ of the universe?

¹For a brief review, see [3].

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And is there any alternative to dark matter besides the often invoked weakly interacting massive particles in particular?

In this work we demonstrate a novel kind of new energy from unparticles whose EoS parameter $\omega_{\mathcal{U}}$ lies between CDM and the photon’s one with $\omega_{\mathcal{U}}$ equal to $1/(2d_{\mathcal{U}} + 1)$. It might be dubbed *unmatter*, to be distinguished from CDM and ordinary matter. Thus we investigate some of its impacts on cosmology and astrophysics. In an expanding universe, the behavior of the unparticle energy density $\rho_{\mathcal{U}}(T)$ is dramatically different from that for photons. For $d_{\mathcal{U}} > 1$, even if the density $\rho_{\mathcal{U}}(T_{\text{D}})$ at a high decoupling temperature T_{D} is very small, it is possible to have a large relic density $\rho_{\mathcal{U}}(T_{\gamma}^0)$ at present photon temperature T_{γ}^0 , large enough to play the role of dark matter.

The concept of the unparticle [4] stems from the observation that certain high-energy theories with a nontrivial infrared fixed point at some scale $\Lambda_{\mathcal{U}}$ may develop a scale-invariant degree of freedom below the scale, named an unparticle. The notion of mass does not apply to such an entity; instead, its kinematics is mainly determined by its scaling dimension $d_{\mathcal{U}}$ under scale transformations. The unparticle must interact with particles, however feebly, to be physically relevant; and the interaction can be well described in effective field theory (EFT). There has been a burst of activities since the seminal work of Georgi [4], on various aspects of unparticle physics from precision tests and collider physics effects [5, 6] to theoretical issues [7, 8] and cosmological and astrophysical implications [9–16], to mention a few.²

In a glut of unparticle phenomenological studies, either unparticles are treated at zero temperature as occurring in ordinary particle physics processes, or the naive arguments of conformal invariance are invoked for the unparticle EoS with the massless photon as an analogue in mind. In this work we work out the thermodynamics of unparticles di-

²For a complete list of work, which we are unable to quote in this brief article, see the citations to [4] at spires.

rectly from their basic properties, which turns out to be generally different from that of photons.

The thermodynamics of a gas of bosonic particles with mass μ is determined by the partition function:

$$\ln Z(\mu^2) = -g_s V \int \frac{d^4 p}{(2\pi)^4} 2\pi^2 p^0 \theta(p^0) \times \delta(p^2 - \mu^2) \ln(1 - e^{-p^0 \beta}), \tag{1}$$

where V , $\beta = T^{-1}$ are the volume and inverse temperature in natural units respectively, and g_s accounts for degrees of freedom like spin. The density of states in four-momentum space is proportional to the δ function due to the dispersion relation for particles. There is no such a constraint in the case of unparticles, whose density of states is dictated by the scaling dimension d_U of the corresponding field to be proportional to [4]

$$\frac{d^4 p}{(2\pi)^4} \theta(p^0) \theta(p^2) (p^2)^{d_U - 2}. \tag{2}$$

Nevertheless, we can interpret it in terms of a continuous collection of particles with the help of a spectral function $\varrho(\mu^2) \propto \theta(\mu^2) (\mu^2)^{d_U - 2}$ [5]:

$$2\pi \theta(p^0) \delta(p^2 - \mu^2) \frac{d^4 p}{(2\pi)^4} \varrho(\mu^2) d\mu^2. \tag{3}$$

In this construction, compared to the case of particles of a definite mass, μ^2 serves as a new quantum number to be summed over with the weight $\varrho(\mu^2)$.

To write down the partition function for unparticles, we have to normalize ϱ correctly. Since unparticles exist only below the scale Λ_U , the spectrum must terminate there.³ Beyond the scale, unparticles can be resolved and are no more the suitable degrees of freedom to cope with. This also implies that we should require $\beta \Lambda_U > 1$ for self-consistency. We thus find the normalized spectrum,

$$\varrho(\mu^2) = (d_U - 1) \Lambda_U^{2(1-d_U)} \theta(\mu^2) (\mu^2)^{d_U - 2}, \tag{4}$$

which has the correct limit $\delta(\mu^2)$ as $d_U \rightarrow 1^+$. Note that integrability at the lower end of μ^2 requires $d_U \geq 1$. The partition function for unparticles is

$$\begin{aligned} \ln Z &= \int_0^{\Lambda_U^2} d\mu^2 \varrho(\mu^2) \ln Z(\mu^2) \\ &= -\frac{g_s V (d_U - 1)}{4\pi^2 \beta^3 (\beta \Lambda_U)^{2(d_U - 1)}} \int_0^{(\beta \Lambda_U)^2} dy y^{d_U - 2} \\ &\quad \times \int_y^\infty dx \sqrt{x - y} \ln(1 - e^{-\sqrt{x}}). \end{aligned} \tag{5}$$

³We assume scale invariance all the way up to Λ_U throughout the paper, though it could potentially be broken by couplings to standard model fields at lower energies.

For $\beta \Lambda_U > 1$, the above integrals factorize to good precision due to the exponential:

$$\begin{aligned} \ln Z &= \frac{g_s V (d_U - 1) 2B(3/2, d_U - 1)}{4\pi^2 \beta^3 (\beta \Lambda_U)^{2(d_U - 1)} (2d_U + 1)} \\ &\quad \times \Gamma(2d_U + 2) \zeta(2d_U + 2), \end{aligned} \tag{6}$$

where Γ , B , ζ are standard functions and integration by parts has been used for $2d_U + 1 > 0$. Using the definition of the B function, the apparent singularity at $d_U = 1$ can be removed explicitly:

$$\ln Z = \frac{g_s V}{\beta^3 (\beta \Lambda_U)^{2(d_U - 1)}} \frac{\mathcal{C}(d_U)}{4\pi^2}, \tag{7}$$

with $\mathcal{C}(d_U) = B(3/2, d_U) \Gamma(2d_U + 2) \zeta(2d_U + 2)$. It is now straightforward to work out the quantities:

$$p_U = g_s T^4 \left(\frac{T}{\Lambda_U} \right)^{2(d_U - 1)} \frac{\mathcal{C}(d_U)}{4\pi^2}, \tag{8}$$

$$\rho_U = (2d_U + 1) g_s T^4 \left(\frac{T}{\Lambda_U} \right)^{2(d_U - 1)} \frac{\mathcal{C}(d_U)}{4\pi^2}.$$

Again the case of massless particles is recovered correctly by setting $d_U = 1$ and $\mathcal{C}(1) = 2\pi^4/45$. The above results imply the following EoS parameter for unparticles:

$$\omega_U = (2d_U + 1)^{-1}. \tag{9}$$

The results for fermionic unparticles can be obtained by replacing $\mathcal{C}(d_U)$ by $(1 - 2^{-(2d_U + 1)}) \mathcal{C}(d_U)$.

It is clear that ω_U is very different from that for photons or CDM, and generically lies in between for $d_U > 1$. This is in contrast to the naive expectation based on conformal theory arguments and the massless photon analogue. This arises essentially from the fact that unparticles exist only below a finite energy scale Λ_U as reflected in the spectral function $\varrho(\mu^2)$, while a conventional conformal theory is not characterized by such a scale. If the limit $\Lambda_U \rightarrow \infty$ were naively taken, which means there would be no unparticles in the infrared, ρ_U would vanish trivially. This is indeed not the case of interest here. The factor $\Lambda_U^{2(1-d_U)}$ in p_U and ρ_U acts as an effective parameter in the low-temperature theory, and the presence of Λ_U reflects its connection to the underlying theory that produces the unparticle. This connection between low- and high-energy theories is completely expected, like, for instance, in the thermodynamics of solids, where one can take the view of atomic physics.

The ensemble of unparticles thus provides a new form of energy density in our universe, which will have important repercussions for cosmology. We now study their implications in our expanding universe by concentrating on their contribution to the energy density in the universe. The unparticle energy density at present is determined by its initial value at the decoupling temperature T_D , where unparticles drop out of the thermal equilibrium with standard model

(SM) particles, and its evolution thereafter, which is closely related to the EoS parameter.

In an FRW expanding universe, the energy density $\rho(T)$ (or $\rho(R)$) of a species after decoupling from equilibrium is given by

$$\rho(R) = \rho(R_D) \left(\frac{R_D}{R}\right)^{3(1+\omega)}, \tag{10}$$

where R_D is the scale factor of the expanding universe at decoupling. From now on, we will interchange the notation $\rho(T)$ and $\rho(R)$ freely. Since photon expansion follows $R_D/R = T_\gamma/T_D$, we have

$$\begin{aligned} \rho_{\mathcal{U}}(T_\gamma) &= \rho_{\mathcal{U}}(T_D) \left(\frac{T_\gamma}{T_D}\right)^{3(1+\omega_{\mathcal{U}})}, \\ \rho_\gamma(T_\gamma) &= \rho_\gamma(T_D) \left(\frac{T_\gamma}{T_D}\right)^4, \end{aligned} \tag{11}$$

where ρ_γ, T_γ are the quantities for photons. For $d_{\mathcal{U}} > 1$, the unparticle energy density decreases more slowly than the photon’s as the universe cools down.

If the unparticle is always in thermal equilibrium with the photons, its energy density drops faster than the energy density of the photon when temperature goes down. However, after the unparticle freezes out of equilibrium, the situation is different. The ratios of the energy densities, $r_\gamma(T) = \rho_{\mathcal{U}}(T)/\rho_\gamma(T)$, at two temperatures T_1 and T_2 are related by

$$r_\gamma(T_2) = r_\gamma(T_1) \left(\frac{T_1}{T_2}\right)^{\frac{2d_{\mathcal{U}}-2}{2d_{\mathcal{U}}+1}}. \tag{12}$$

A dramatic consequence of this is that even if the unparticle density is small compared with the photon’s at a high temperature T_D , it may become larger or even comparable to the critical density at a lower temperature. For illustration, we show in Fig. 1 how the ratio $r_\gamma(T_\gamma^0)$ at the present photon temperature $T_\gamma^0 = 2.725 \pm 0.002$ K [17] changes with $d_{\mathcal{U}}$

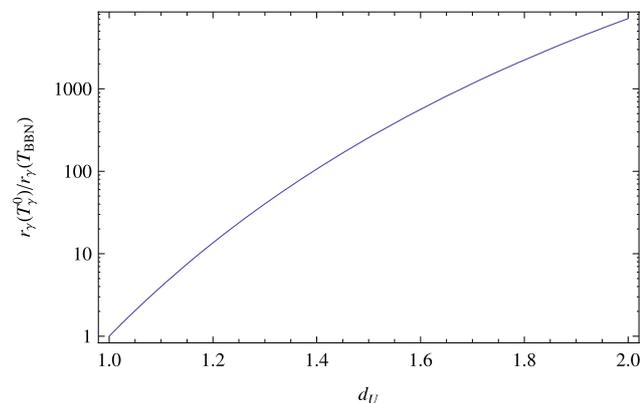


Fig. 1 The double ratio $r_\gamma(T_\gamma^0)/r_\gamma(T_{\text{BBN}})$ as a function of $d_{\mathcal{U}}$

for a given $r_\gamma(T_{\text{BBN}})$ at the big-bang-nucleosynthesis (BBN) temperature, $T_{\text{BBN}} = 1$ MeV, where unparticle and photon are assumed to have decoupled. We see that the double ratio $r_\gamma(T_\gamma^0)/r_\gamma(T_{\text{BBN}})$ is always larger than one for $d_{\mathcal{U}} > 1$.

The above property opens the possibility for the unparticle to play an important role as dark ‘matter’. Such dark matter, or better named, unmatter, is different from the usual one. It provides gravitational attraction, but with EoS deviating from 0. It would be interesting to see whether such a picture fits in a global analysis of various cosmological data. This is, however, beyond the scope of this work.

The temperature T_D depends on unparticle–particle interactions. In EFT below $\Lambda_{\mathcal{U}}$, there could be many possible interactions between unparticles and SM particles even if unparticles are singlets under the SM gauge group [18]. A practical study with a global fitting should make a survey of all such interactions and those induced by thermal effects. For the purpose of illustration here, we consider below the unparticle–photon interactions:

$$\mathcal{L} = \lambda \Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}} F^{\mu\nu} F_{\mu\nu} \mathcal{U} + \tilde{\lambda} \Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}} \tilde{F}^{\mu\nu} F_{\mu\nu} \mathcal{U}, \tag{13}$$

where F, \tilde{F} are respectively the electromagnetic field tensor and its dual, and the coefficients $\lambda, \tilde{\lambda}$ can be expressed in terms of the standard ones in [18]. We will treat the two interactions one by one.

The above interactions can bring photons and unparticles into equilibrium. Taking the λ term as an example, the cross section for $\gamma\gamma \rightarrow \mathcal{U}$ is

$$\sigma(s) = \frac{1}{4} \lambda^2 \left(\frac{s}{\Lambda_{\mathcal{U}}^2}\right)^{d_{\mathcal{U}}} \frac{1}{s} A_{d_{\mathcal{U}}}, \tag{14}$$

where

$$A_{d_{\mathcal{U}}} = \frac{16\pi^{5/2} \Gamma(d_{\mathcal{U}} + 1/2)}{(2\pi)^{2d_{\mathcal{U}}} \Gamma(d_{\mathcal{U}} - 1) \Gamma(2d_{\mathcal{U}})} \tag{15}$$

is a normalization factor for the unparticle density of states, as suggested in [4], and the interaction rate is

$$\Gamma \simeq n_\gamma \sigma(s) c = \frac{\zeta(3) A_{d_{\mathcal{U}}} \lambda^2 T}{8\pi^2} \left(\frac{2T}{\Lambda_{\mathcal{U}}}\right)^{2d_{\mathcal{U}}}, \tag{16}$$

where n_γ is the photon number density, and we have used $s = (2T)^2$.

This rate is compared with the Hubble parameter $H = 1.66 g_*^{1/2} T^2/m_{\text{Pl}}$ in the radiation-dominated era to determine at what temperature unparticles decouple from photons [1]. Here g_* is the total number of degrees of freedom at the decoupling temperature and $m_{\text{Pl}} = 1.22 \times 10^{19}$ GeV is the Planck mass. When $\Gamma < H$, the unparticles will decouple from photons. Taking the equal sign, one obtains the

decoupling temperature,

$$T_D = \frac{1}{2} \left(\frac{1.66 g_*^{1/2} \Lambda_U^{2d_U} 4\pi^2}{m_{pl} \lambda^2 A_{d_U} \zeta(3)} \right)^{1/(2d_U-1)} \tag{17}$$

Replacing λ by $\tilde{\lambda}$, one obtains the decoupling temperature due to the $\tilde{\lambda}$ term. In the following numerical discussions, we will take λ to be non-zero for illustration. The results will be the same for taking $\tilde{\lambda}$ non-zero.

There are experimental constraints on the coupling $\lambda/\Lambda_U^{d_U}$ from astrophysics [9–16], radiative positronium decay $o\text{-P} \rightarrow \gamma U$ [18] and CERN LEP $e^-e^+ \rightarrow \gamma U$ [18]. Among them, the astrophysical one by energy loss arguments in stars is most stringent. Using the numbers obtained in [16] we can calculate the allowed maximal coupling $(\lambda/\Lambda_U^{d_U})_{\max}$ and the corresponding minimal decoupling temperature T_D^{\min} . The actual decoupling temperature can of course be higher than this minimal value. The results are listed in Table 1. It is seen that T_D^{\min} can vary in a big range from as large as 10^7 GeV to as low as a few 10 GeV, depending on the value of d_U .

In order that the relic density of unparticles is not too large, say as large as the critical energy density (ρ_{cr}) which would over close the universe, for a given T_D one has to choose a big enough Λ_U besides the requirement $\Lambda_U > T_D$. This provides a way to constrain the scale Λ_U directly. We illustrate our results in Table 2 for several representative values of the ratio of energy densities, $\Omega_U(T_\gamma^0) = \rho_U(T_\gamma^0)/\rho_{cr}(T_\gamma^0)$, at T_γ^0 . In our analysis, we assume $T_D = T_D^{\min}$, as shown in Table 1. By equating the $\rho_U(T_D)$ that is obtained via (8) on the one hand with the one from backward evolution via (11) on the other, we can determine Λ_U for each given d_U . Also shown are the values of the ratio $r_\gamma = \rho_U/\rho_\gamma$ at T_D^{\min} and T_{BBN} . Note that we do not assume a value for the dimensionless coupling λ ; instead, it is fixed by Λ_U and $T_D = T_D^{\min}$ via (17).

For $d_U = 4/3$, we find that it is not possible to saturate the critical density, nor the dark matter density $\Omega_{\text{DM}} = 0.2$ [17]. With the constraint that $T_D < \tilde{\Lambda}_U$, the largest $\Omega_U(T_\gamma^0)$ is 0.16, which occurs at $T_D = \tilde{\Lambda}_U$. This, of course, still leaves enough room for the unparticle to play a significant role as dark matter. For $d_U = 5/3$ and 2, we see that the present unparticle relic density can easily saturate the critical density and dark matter density. In all cases, $\rho_U(T_D)$ is smaller

Table 1 Upper bound $(\lambda/\Lambda_U^{d_U})_{\max}$ (in units of GeV^{-d_U}) and the corresponding T_D^{\min} (in units of GeV) for various values for d_U . The appropriate g_* has been used for the given energy with SM particles and a scalar unparticle

d_U	4/3	5/3	2
$(\lambda/\Lambda_U^{d_U})_{\max}$	1.04×10^{-14}	7.17×10^{-13}	5.11×10^{-11}
T_D^{\min}	7.37×10^6	2.70×10^3	3.68×10

(in most cases much smaller) than $\rho_\gamma(T_D)$. Requiring the present relic of unmatter to be less than these densities one obtains conservative lower bounds on $\tilde{\Lambda}_U$ for given $T_D = T_D^{\min}$. For small d_U , the scale $\tilde{\Lambda}_U$ is constrained to be very large, making the low-energy search for unparticle effects difficult. But for large d_U (close to 2), the scale can still be as low as a few hundred GeV, which may be reached at LHC and ILC colliders.

The standard BBN theory explains the data well. It is therefore important to make sure that at T_{BBN} unparticles do not cause problems. A simple criterion is to require that at this temperature the unparticle energy density be less than the photon’s. With this restriction, it is interesting to see whether one can still have large relic unmatter at present. We find this is indeed possible. Although there are many cases shown in Table 2 where $r_\gamma(T_{\text{BBN}})$ is larger than 1, circumstances with sizable $\Omega_U(T_\gamma^0)$ but small $r_\gamma(T_{\text{BBN}})$ also appear at large d_U . This can easily be understood from (12) and from Fig. 1. For $d_U > 1$, a small $r_\gamma(T_{\text{BBN}})$ can result in a sizable $\Omega_U(T_\gamma^0)$. A universe dominated by unparticles between the BBN era and the matter- or dark energy-dominated universe is possible.

In the above discussions, the interactions of unparticles with photons lead to an interaction rate $\Gamma \sim T^{2d_U+1}$ which brings unparticles and SM particles into equilibrium at a high temperature, and they decouple at a lower temperature if the unparticle dimension d_U is larger than 1. There are many possible ways unparticles can interact with SM particles, but not all interactions will have the same properties as far as thermal equilibrium is concerned. For example, we find that all of the operators involving SM fermions

Table 2 Λ_U and $r_\gamma(T) = \rho_U(T)/\rho_\gamma(T)$ as functions of $\Omega_U(T_\gamma^0)$. We have used $\rho_{cr}(T_\gamma^0) = 8.0992h^2 \times 10^{-47} \text{ GeV}^4$ and taken the central value for $h = 0.73^{+0.04}_{-0.03}$ [17]

$d_U = 4/3$			
$\Omega_U(T_\gamma^0)$	1.0	0.161	0.01
Λ_U (GeV)	–	7.37×10^6	4.78×10^8
$r_\gamma(T_D^{\min})$	–	9.90×10^{-1}	6.13×10^{-2}
$r_\gamma(T_{\text{BBN}})$	–	6.22×10	3.85
$d_U = 5/3$			
$\Omega_U(T_\gamma^0)$	1.0	0.20	0.01
Λ_U (GeV)	1.46×10^4	4.87×10^4	4.61×10^5
$r_\gamma(T_D^{\min})$	2.48×10^{-1}	4.97×10^{-2}	2.48×10^{-3}
$r_\gamma(T_{\text{BBN}})$	2.35×10	4.73	2.36×10^{-1}
$d_U = 2$			
$\Omega_U(T_\gamma^0)$	1.0	0.2	0.01
Λ_U (GeV)	4.31×10^2	9.65×10^2	4.32×10^3
$r_\gamma(T_D^{\min})$	4.57×10^{-2}	9.11×10^{-3}	4.55×10^{-4}
$r_\gamma(T_{\text{BBN}})$	3.06	6.14×10^{-1}	3.05×10^{-2}

listed in [18] will result in an interaction rate proportional to $T^{2d_{\mathcal{U}}-1}$. If unparticles and SM fermions are required to be in thermal equilibrium at a high temperature and then decouple at a lower one, $d_{\mathcal{U}}$ must be larger than $3/2$. On the contrary, if $d_{\mathcal{U}}$ is less than $3/2$, then unparticles and SM fermions will not be in thermal equilibrium at a high temperature in the first place, but will be at a lower temperature till the epoch of a matter-dominated universe. Since when in thermal equilibrium the unparticle density dilutes faster than SM particles, its relic density today will be negligibly small if the equilibrium sets in before or just after BBN with the unparticle relic density not larger than photon density. This is an interesting scenario to study, which may lead to sensitive information about the unparticle's scaling dimension.

There is much to be explored for the roles that thermal unparticles can play in our universe. It is important to analyze the available cosmological and astrophysical data for a global fit with unparticle energy density integrated. We leave this for a detailed future study.

To summarize, we have studied for the first time the thermal properties of unparticles. Due to its peculiar phase space structure we found that the EoS parameter $\omega_{\mathcal{U}}$ is given by $1/(2d_{\mathcal{U}} + 1)$, providing a new form of energy in our universe. In an expanding universe, the behavior of the unparticle's energy density $\rho_{\mathcal{U}}(T)$ is dramatically different from that for photons. For $d_{\mathcal{U}} > 1$, even if its value at a high decoupling temperature T_D is very small, it could evolve into a sizable relic density $\rho_{\mathcal{U}}(T_{\gamma}^0)$ at present, large enough to play the role of dark matter. We have exemplified this with photon–unparticle interactions, and we found that it is in-

deed feasible to obtain a large relic energy density of unparticles with the most stringent constraints saturated.

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