Parity-violating nuclear force as derived from QCD sum rules

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Parity-violating nuclear force, as may be accessed from parity-violation studies in nuclear systems, represents an area of nonleptonic weak interactions that has been the subject of experimental investigations for several decades. In the simple meson-exchange picture, a parity-violating nuclear force may be parametrized as arising from the exchange of π, ρ, ω, or other mesons with strong meson-nucleon coupling at one vertex and weak parity-violating meson-nucleon coupling at the other vertex. The QCD sum rule method allows for a fairly complicated, but nevertheless straightforward, leading-order loop-contribution determination of the various parity-violating MNN couplings starting from QCD (with the nontrivial vacuum) and Glashow-Salam-Weinberg electroweak theory. We continue our earlier investigation of the parity-violating πNN coupling (by Henley, Hwang, and Kisslinger) to other parity-violating couplings. Our predictions are in reasonable overall agreement with the results estimated on phenomenological grounds, such as in the now classic paper of Desplanques, Donoghue, and Holstein, in the global experimental fit of Adelberger and Haxton, or the effective field theory thinking of Ramsey-Musolf and Page.

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I. INTRODUCTION

Parity-violation (PV) studies in nuclear systems, such as the asymmetry [1,2] in $\vec{p} + p \rightarrow p + p$ and the photon circular polarization measurement in $n + p \rightarrow d + \gamma$, and PV studies in ^{18}F and ^{21}Ne, offer a means of determining the parity-violating nuclear force, which represents a special category of nonleptonic weak interactions accessible experimentally. In the simple meson-exchange picture (as a way to organize the nuclear forces), parity-violating nuclear force arises from the exchange of π, ρ, ω, or other mesons with strong MNN coupling at one vertex and weak parity-violating MNN coupling at the other vertex. In the 1980s, parity-violating meson-nucleon couplings were estimated primarily on phenomenological grounds, such as in the classic paper of Desplanques, Donoghue, and Holstein (DDH) [3]. A global fit to obtain these PV meson-nucleon couplings, making use of the various experimental data available at the time, was performed also (in 1985) by Adelberger and Haxton (AH) [4]. Nevertheless, progress [5], both experimental and theoretical, has been slow since then, although the interest in the problem has grown to some extent in relation to the adopted effective field theory for the few nucleon systems.

Meanwhile, in the 1980s, the standard model of particle physics, which consists of the Glashow-Salam-Weinberg (GSW) electroweak theory and quantum chromodynamics (QCD), had become well established. Although the nonperturbative feature of QCD manifests itself in the formation of a hadron structure and makes it very difficult to quantitatively predict hadron properties including both the strong parity-conserving and weak parity-violating meson-nucleon couplings, the method of QCD sum rules [6] offers a systematic way of taking into account effects caused by the nontrivial nature of the QCD ground state or the QCD vacuum. Since the 1990s, the QCD sum rule method has become quite popular. In view of the laborious efforts in lattice gauge theory, the QCD sum rule method does offer an alternative avenue to obtaining the various predictions, to some extent in the spirit of QCD.

In fact, one should view the QCD sum rule methods as supplementary to the lattice approaches to QCD, as all the condensate parameters fitted in the sum rule methods would eventually be calculated in the lattice QCD. Therefore, it follows that certain QCD models are valid, such as the flux-tube model in a certain form, see, e.g., Ref. [7]. In other words, we view these approaches as trying to solve a very difficult problem (i.e., QCD) via different means—all having made progress through these years.

To "complete" the study of nuclear parity violations, we need to investigate both the parity-conserving and parity-violating meson-nucleon couplings, using the conventional scheme of nuclear forces. As already shown in another paper [8] and recently quoted by Erkol *et al.* [9], it is possible to use the method of QCD sum rules in external fields [10,11] to determine the strong πNN, ρNN, and ωNN couplings. The purpose of this paper is to present a QCD sum rule determination of the parity-violating ρNN and ωNN couplings, which turns out to be fairly complicated but nevertheless straightforward. Our present results, together with a previous study of the parity-violating πNN coupling [12], allow a direct connection to be made between the parity-violating nuclear force and the standard model, despite the complication of QCD.

The external-field QCD sum rule method [10,11] has been used to treat the strong πNN coupling [13], the weak parity-violating πNN coupling [12], and the strong ρNN and ωNN couplings [8]. In all cases considered, quantitative successes have been achieved mainly because the nonperturbative effects of QCD, as expressed in terms of induced condensates, have been taken into account and are found to be of critical importance. Our efforts to treat hyperon weak decays remain in progress [14], with fairly encouraging results (which are beyond the scope of the present article). Therefore, it is natural to follow the same method to treat the weak parity-violating ρNN and ωNN couplings. Nevertheless, it remains difficult to assess properly whether our present calculation may also succeed in a quantitative manner, since after all the nonleptonic amplitudes in question are "notoriously difficult to calculate" (quoting the phrase from the referee on the early version of one of our early articles). However, we have good reason, based on successful experiences with the method of QCD sum rules in general, to believe that the complicated task carried out in this research could be the first important step in establishing a benchmark in the (future) quantitative treatments of this difficult problem. Of course, there are some higher order diagrams, such as the Penguin diagrams (which are under our investigation), that might be of importance; but we have to start from somewhere. Just as for many other nonleptonic weak decays where relevant data exist, we already have experimental information [4,5] which we can use to test our theoretical predictions.

For the sake of completeness, we begin by outlining a few ingredients regarding the external-field QCD sum rule method, without detailed qualifying statements. The external vector field is expressed as

$$Z_\mu = -\tfrac{1}{2} Z_{\mu\nu} x^\nu. \tag{1}$$

We attempt to determine the following polarization function for a nucleon in a small (classical) external vector-meson field Z_μ (where Z represents either a ρ or ω meson):

$$i\int d^4x e^{iq\cdot x}\,\langle 0|T\eta_N(x)\bar{\eta}_N(0)|0\rangle_Z = \Pi(q) + hZ_{\mu\nu}\Pi^{\mu\nu}(q), \tag{2}$$

where the numerical constant h is the coupling between the external field and the up (u) quark field; more explicitly, the coupling between the external field and the down (d) quark is $-h$ for ρ meson and $+h$ for ω meson. The standard form for the composite operator $\eta_N(x)$ is adopted [11] as

$$\eta_p(x) = \epsilon^{abc}[u^{aT}(x)C\gamma_\mu u^b(x)]\gamma_5\gamma^\mu d^c(x), \tag{3}$$
$$\eta_n(x) = \epsilon^{abc}[d^{aT}(x)C\gamma_\mu d^b(x)]\gamma_5\gamma^\mu u^c(x), \tag{4}$$

which transform as the proton and neutron fields, respectively. Here $u^a(x)$ and $d^a(x)$ are the up and down quark fields with the superscript a the color index, and C is the charge conjugation operator.

At the hadronic level, we define the parity-violating meson-nucleon couplings related to ρ and ω mesons in the standard manner [3]:

$$\begin{aligned} L_{\rm int}^{\rm p.v.} = -\bar{N}\Bigg[& h_\rho^0 \vec{\tau}\cdot\vec{\phi}_\mu^\rho + h_\rho^1 \phi_\mu^{\rho 3} \\ &+ \frac{h_\rho^2}{2\sqrt{6}}\big(3\tau^3\phi_\mu^{\rho 3} - \vec{\tau}\cdot\vec{\phi}_\mu^\rho\big)\Bigg]\gamma^\mu\gamma_5 N \\ &+ h_\rho^{'1}\bar{N}\big(\vec{\tau}\times\vec{\phi}_\mu^\rho\big)^3 \frac{\sigma^{\mu\nu}k_\nu}{2m}\gamma_5 N \\ &- \bar{N}\big[h_\omega^0\phi_\mu^\omega + h_\omega^1\tau^3\phi_\mu^\omega\big]\gamma^\mu\gamma_5 N. \end{aligned} \tag{5}$$

Here the superscripts in the couplings h_ρ^0, h_ρ^1, h_ρ^2, $h_\rho^{\prime 1}$, h_ω^0, and h_ω^1 refer to the isospin character, as the weak interactions do not observe isospin symmetry. We may use the above expression to write the polarization function $\Pi(q)$, with the particular piece that is proportional to h, at the hadronic level. It is clear that we need to combine different channels (i.e., $p \to p$, $n \to n$, $p \to n$, and $n \to p$) in order to extract these PV coupling constants.

At the quark level, we use weak interactions as described by the GSW electroweak theory to determine the polarization function, obtaining in general three-loop diagrams which require regularizations. Here we adopt dimensional regularization in the minimum-subtraction (MS) scheme and introduce suitable counterterms in defining the renormalized operators $[\eta_p]_R$ and $[\eta_n]_R$. As mentioned above, we may parametrize the same polarization function phenomenologically, making use of the parity-violating ρNN and ωNN interaction Lagrangians. Comparing the results obtained through the two ways of evaluating the polarization functions (i.e., at the quark level using GSW theory and QCD and at the hadronic level involving ρNN and ωNN couplings), we have a definitive way to extract the weak PV ρNN and ωNN couplings—the primary objective of this paper.

II. PARITY-VIOLATING ρNN AND ωNN COUPLINGS

As a special feature in relation to the method of QCD sum rules, the kinematic variable q_μ^2 of the polarization function $\Pi(q)$ is translated into the choice of the Borel mass squared M^2, which is in the vicinity of slightly above 1 GeV^2. Such choice of the Borel mass ensures the approximate validity of the operator-product expansion (OPE) augmented with power corrections (as due to the various condensates). In other words, perturbative QCD corrections to the coefficients in such OPE are in principle there but are presumably suppressed by choice of the Borel mass M. Unlike what has been involved in most phenomenological approaches to the problem where some effective weak Hamiltonian at the energy scale relevant to the hadron must be directly invoked, we have in the QCD sum rule method the nice feature that the GSW electroweak theory is called for at the scale set by the Borel mass squared M^2 where effects to order $O(G_F\alpha_S)$ are suppressed (due to the running of the strong coupling α_S), and it is the intrinsic smooth extrapolation of the results to lower q^2 that helps explain the successes of the predictions.

Accordingly, it should be possible to use in the QCD sum rule method the effective GSW electroweak Lagrangian at tree level while leaving terms in $O(G_F\alpha_S)$ as corrections. That is, we may use, as a good starting point,

$$\begin{aligned} L_{\rm weak} = -\frac{G_F}{\sqrt{2}}\Bigg[&\bar{u}\gamma_\mu(1-\gamma_5)d\bar{d}\gamma^\mu(1-\gamma_5)u + \sum_{q_1,q_2=u,d} \\ &\times \bar{q}_1\gamma_\mu(A_{q_1} - B_{q_1}\gamma_5)q_1\bar{q}_2\gamma^\mu(A_{q_2} - B_{q_2}\gamma_5)q_2\Bigg], \end{aligned} \tag{6}$$

where G_F is the Fermi coupling constant ($G_F = 1.166 \times 10^{-5}$ GeV^{-2}), and the other constants are defined through $A^u = \frac{1}{2} - \frac{4}{3}\sin^2\theta_W$, $B^u = \frac{1}{2}$, $A^d = -\frac{1}{2} + \frac{2}{3}\sin^2\theta_W$, and $B^d = -\frac{1}{2}$ with $\sin^2\theta_W$ the electroweak mixing parameter ($\sin^2\theta_W = 0.2315$). Note that the first term in the above equation comes from exchange of the $W^{\pm}$ boson, while the second term is due to Z^0 exchange. In view of our primary goal, which is to identify the sizable role played by the various condensates, it is clear that the issue regarding the renormalization of the tree Lagrangian (as would be suppressed by the choice of the Borel mass) is of secondary importance and could be addressed when higher precision (in theoretical prediction) is called for. As will be explained later, it is in fact sufficient to even use the current-current form, since the difference from what we may obtain by employing the renormalizable gauge throughout, such as the R_ξ gauge, does not matter as far as our final QCD sum rules are concerned.

A. Renormalization of composite operators

Inclusion of the weak interaction in the polarization function leads to sub-divergences even in the lowest order $O(G_F)$. Such divergences may be removed (or regularized) by using suitably defined renormalized composite operators $[\eta]_R$ and $[\bar{\eta}]_R$. Such renormalized operators may be obtained by considering the four-fermion interaction:

$$\begin{aligned}
&\int d^d x d^d y e^{i(p\cdot x+q\cdot y)}\langle u_{i'}^{a'}(0)d_{j'}^{b'}(0)\bar{u}_i^a(x)\bar{d}_j^b(y)\rangle \\
&= \frac{\delta^{a'a}\delta^{b'b}}{p^2q^2}\hat{p}_{i'i}\hat{q}_{j'j} + \frac{G_F}{\sqrt{2}}\frac{1}{48\pi^2(d-4)}\frac{1}{p^2q^2} \\
&\quad\times\left[(p+q)_\rho(p+q)_\sigma + \frac{1}{2}(p+q)^2 g_{\rho\sigma}\right] \\
&\quad\times\{[\gamma^\rho\gamma_\mu(A^u - B^u\gamma_5)\hat{p}]_{i'i}[\gamma^\sigma\gamma^\mu(A^d - B^d\gamma_5)\hat{q}]_{j'j} \\
&\quad+[\gamma^\rho\gamma_\mu(1-\gamma_5)\hat{p}]_{i'i}[\gamma^\sigma\gamma^\mu(1-\gamma_5)\hat{q}]_{j'j}\} \\
&\quad+\text{finite parts},
\end{aligned} \tag{7}$$

where the first term is the free piece, while the second term, which is divergent at $d = 4$, comes from the insertion of the weak interaction. This equation enables us to define the renormalized operator $[u(x)d(x)]_R$. Using the minimum-subtraction (MS) scheme, we obtain the renormalized operator $[u(x)d(x)]_R$ as

$$\begin{aligned}
&\left[u_i^a(x)d_j^b(x)\right]_R \\
&= u_i^a(x)d_j^b(x) + \frac{G_F}{\sqrt{2}}\frac{\mu^{d-4}}{48\pi^2(d-4)}\left(\partial_\rho\partial_\sigma + \frac{1}{2}g_{\rho\sigma}\Box\right) \\
&\quad\times\{[\gamma^\rho\gamma_\mu(A^u - B^u\gamma_5)u^a(x)]_i[\gamma^\sigma\gamma^\mu(A^d - B^d\gamma_5)d^b(x)]_j \\
&\quad+[\gamma^\rho\gamma_\mu(1-\gamma_5)u^a(x)]_i[\gamma^\sigma\gamma^\mu(1-\gamma_5)d^b(x)]_j\}.
\end{aligned} \tag{8}$$

Here and in what as follows, the notations are defined in accord with the previous equation such as $(\partial_\rho\partial_\sigma + \frac{1}{2}g_{\rho\sigma}\Box)$ as coming from $[(p+q)_\rho(p+q)_\sigma + \frac{1}{2}(p+q)^2 g_{\rho\sigma}]$. Similarly, the renormalized operators $[u(x)u(x)]_R$ and $[d(x)d(x)]_R$ are given by

$$\begin{aligned}
\left[u_i^a(x)u_j^b(x)\right]_R &= u_i^a(x)u_j^b(x) \\
&\quad+\frac{G_F}{\sqrt{2}}\frac{\mu^{d-4}}{48\pi^2(d-4)}\left(\partial_\rho\partial_\sigma + \frac{1}{2}g_{\rho\sigma}\Box\right) \\
&\quad\times[\gamma^\rho\gamma_\mu(A^u - B^u\gamma_5)u^a(x)]_i \\
&\quad\times[\gamma^\sigma\gamma^\mu(A^u - B^u\gamma_5)u^b(x)]_j.
\end{aligned} \tag{9}$$

$$\begin{aligned}
\left[d_i^a(x)d_j^b(x)\right]_R &= d_i^a(x)d_j^b(x) \\
&\quad+\frac{G_F}{\sqrt{2}}\frac{\mu^{d-4}}{48\pi^2(d-4)}\left(\partial_\rho\partial_\sigma + \frac{1}{2}g_{\rho\sigma}\Box\right) \\
&\quad\times[\gamma^\rho\gamma_\mu(A^d - B^d\gamma_5)d^a(x)]_i \\
&\quad\times[\gamma^\sigma\gamma^\mu(A^d - B^d\gamma_5)d^b(x)]_j.
\end{aligned} \tag{10}$$

The renormalized composite operators $[\eta_n]_R$ and $[\eta_p]_R$ may now be deduced directly from the above three equations. This corresponds to the introduction of counterterms that cancel the subdivergences. We find that $[\eta]_R$ is given by

$$\begin{aligned}
[\eta_p]_R &= \epsilon^{abc}[u^{aT}(x)C\gamma_\mu u^b(x)]\gamma_5\gamma_\mu d^c(x) \\
&\quad+\frac{G_F}{\sqrt{2}}\frac{\mu^{d-4}\epsilon^{abc}}{24\pi^2(d-4)}\left(\partial_\rho^y\partial_\sigma^y + \frac{1}{2}g_{\rho\sigma}(\Box)_y\right) \\
&\quad\times\{-[u^{aT}(y)C\gamma^\sigma\gamma_\mu\gamma^\rho u^b(y)]\gamma_5\gamma_\mu d^c(x) \\
&\quad+[u^{aT}(x)C\gamma_\mu\gamma^\rho\gamma_\nu(A^u - B^u\gamma_5)u^b(y)] \\
&\quad\times\gamma_5\gamma_\mu\gamma^\sigma\gamma^\nu(A^d - B^d\gamma_5)d^c(y) \\
&\quad+[u^{aT}(x)C\gamma_\mu\gamma^\rho\gamma_\nu(1-\gamma_5)u^b(y)] \\
&\quad\times\gamma_5\gamma_\mu\gamma^\sigma\gamma^\nu(1-\gamma_5)d^c(y)\}|_{y=x}.
\end{aligned} \tag{11}$$

An analogous expression for $[\eta_n]_R$ may be obtained by exchanging all the u's and d's. We note that once the subdivergences are removed in this way, the overall divergence (associated with the polarization function) becomes local and thus is removed upon Borel transformation (as employed in the context of QCD sum rules). The difference in our result from what we may obtain by working throughout with the renormalizable R_ξ gauge also disappears upon Borel transformation.

B. QCD sum rules for the parity-violating couplings

The various diagrams that we need to consider are illustrated in Fig. 1, where the propagator with a thick external-line mark is the quark propagator in the presence of an external vector field [8,10,11]. At the quark level, we calculate the polarization function by considering only the leading two terms in the operator product expansion: the first one is associated with the operator $\langle 1\rangle$ and the second with the operator $\langle\bar{q}\sigma_{\mu\nu}q\rangle$. As explained immediately below, the resultant expressions are already extremely complicated algebraically, preventing us from carrying out a better and more complete calculation (unless a considerable amount of time is invested).

To get a better feeling toward the extensiveness of the problem, we may look at the following expression, which is

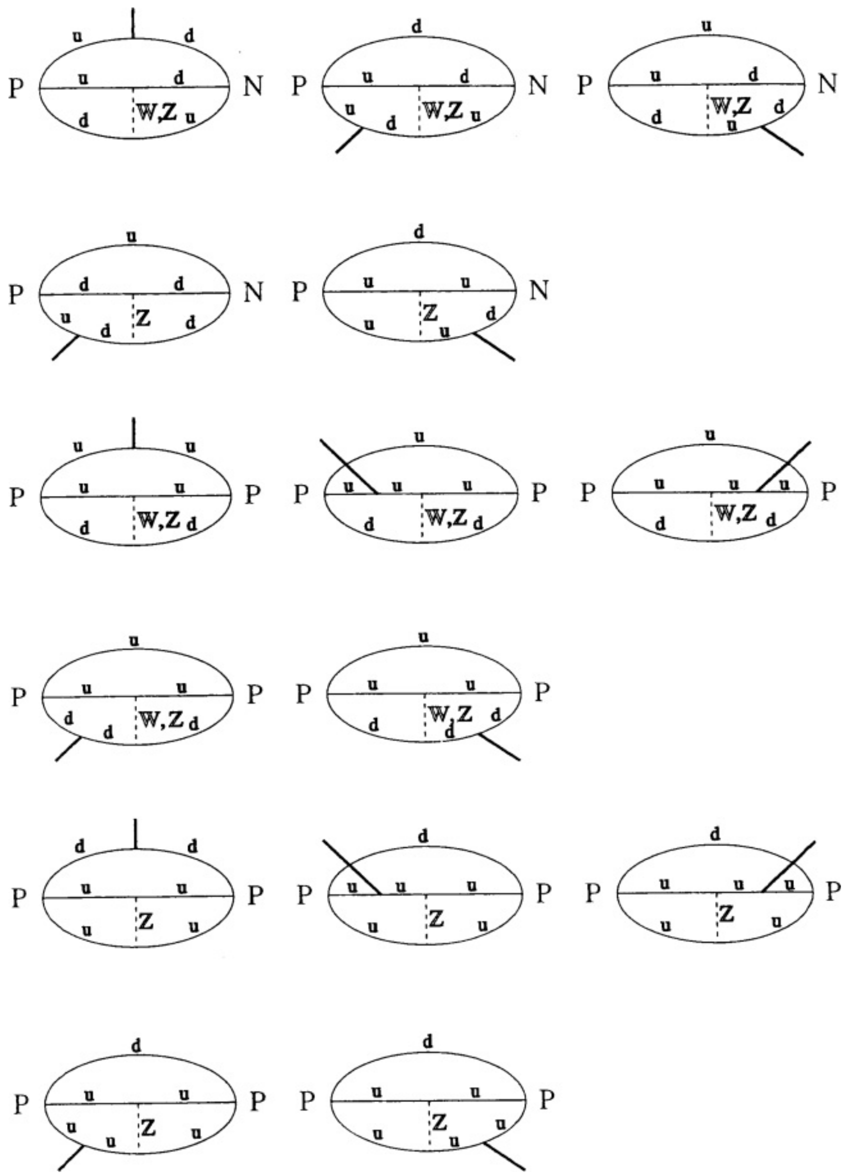

FIG. 1. Loop diagrams for PV-coupling sum-rule calculations.

associated with the coefficient of the operator $\langle 1 \rangle$:

$$\int \frac{d^d p_1}{(2\pi)^d} \frac{d^d p_2}{(2\pi)^d} \frac{d^d p_3}{(2\pi)^d} \frac{p_1^a p_2^b (q - p_3)^c (p_3 - p_1)^d (p_3 - p_2)^e}{p_1^2 p_2^2 (q - p_3)^4 (p_1 - p_3)^2 (p_2 - p_3)^2} \times \gamma^\mu \gamma_a \gamma_\alpha (A^d - B^d \gamma_5) \gamma_b \gamma_\nu (\gamma_c \sigma_{\rho\sigma} + \sigma_{\rho\sigma} \gamma_c) \times \gamma_\mu \gamma_d \gamma_\alpha (A^u - B^u \gamma_5) \gamma_e \gamma^\nu, \tag{12}$$

which involves taking the trace of the product of 13 γ matrices, a task that needs a good algebraic software package (such as Mathematica). It also involves three integration variables p_1, p_2, and p_3, and dimensional regularization is required during the integration. To work out the problem, we chose to simplify these three-loop expressions by devising algebraic programs making use of Mathematica. It still takes up considerable amount of computer time to solve the problem. In practice, we perform the calculation in two steps. First, we do the integrations in the order of p_1 and p_2, then p_3. This needs a small program written in Mathematica to handle it (using the various formulas suitable in d dimensions). The result contains about a hundred terms with different tensor structures. Second, we contract the result with the γ matrices and simplify it. For this step, we use the Mathematica package FeynCalc 1.0 (written by Rolf Mertig). The final result is

$$-\frac{q^4}{\pi^6}(-\hat{q} g_{\rho\sigma} + \hat{q} \gamma_\rho \gamma_\sigma - \gamma_\sigma q_\rho + \gamma_\rho q_\sigma) \times [(A^d A^u + B^d B^u) - (A^u B^d + A^d B^u)\gamma_5] \times \left\{ \frac{1}{384(d-4)^2} + \frac{1}{27648(d-4)}[-275 + 108X] + \frac{1}{165888}[4345 - 27\pi^2 - 2475X + 486X^2] \right\}, \tag{13}$$

where $X = \gamma - \log(4\pi) + \log(-q^2)$. The presence of the pole term $X/(d-4)$ indicates that this diagram contains subdivergences. The subdivergences come from the p_1 and p_2 integrals. The removal of these subdivergences is done by using the renormalized operators $[\eta_p]_R$ and $[\eta_n]_R$.

We have examined the question of how to define γ_5 in d dimensions, since the definition of γ_5 is a tricky issue in the d dimension ('t Hooft & Veltman in 1972). In the present case, however, what we may do is to ignore this fact and anticommute it with all the γ_μ's to reach the utmost right position. This is what we have done in the above expresion, i.e., by moving all the γ_5 matrices to the right of all γ matrices. After removing all the subdivergences, the difference between using the proper procedure and using the naive method turns out to be only a polynomial of q^2. The difference does not contain nonlocal terms such as $\ln(-q^2)$ or $1/q^2$, which we must handle with care.

At the hadron level, there are four coupling constants for the ρ meson and two for the ω meson, so the calculation should be carried out for the various channels including $\rho^+ np$, $\rho^- pn$, $\rho^0 pp$, $\rho^0 nn$, ωpp, and ωnn, and suitable linear combinations allow the determination of the various PV couplings.

We choose to focus on the antisymmetric part of the sum rules (proportional to $Z^A_{\mu\nu}$ with $Z^A_{\nu\mu} = -Z^A_{\mu\nu}$), which already contains enough information to determine the meson-nucleon couplings. Different tensor structures lead to different sum rules. After suitably adding and subtracting between these ρNN (ωNN) sum rules, we obtain the following results (in the MS scheme).

$$\frac{\lambda_N^2 h_\rho^0}{(q^2-m^2)^2} = -\frac{G_F h}{\sqrt{2}}\frac{q^4}{256\pi^6}\left[31\ln\left(-\frac{q^2}{\bar{\mu}^2}\right) - 6\ln^2\left(-\frac{q^2}{\bar{\mu}^2}\right)\right]\left(\frac{1}{2}+\sin^2\theta_W\right), \quad (14)$$

$$\frac{\lambda_N^2 h_\rho^1}{(q^2-m^2)^2} = \frac{G_F h}{\sqrt{2}}\frac{q^4}{256\pi^6}\frac{1}{36}\left[59\ln\left(-\frac{q^2}{\bar{\mu}^2}\right) - 6\ln^2\left(-\frac{q^2}{\bar{\mu}^2}\right)\right]\left(\frac{1}{3}\sin^2\theta_W\right), \quad (15)$$

$$h_\rho^2 = 0, \quad (16)$$

$$\frac{\lambda_N^2 h_\rho^{1'}}{2m(q^2-m^2)^2} = -\frac{G_F h}{\sqrt{2}}\chi\langle\bar{q}q\rangle\frac{q^2}{432\pi^4}\left[13\ln\left(-\frac{q^2}{\bar{\mu}^2}\right) - 3\ln^2\left(-\frac{q^2}{\bar{\mu}^2}\right)\right]\left(\frac{1}{3}\sin^2\theta_W\right), \quad (17)$$

$$\frac{\lambda_N^2 h_\omega^0}{(q^2-m^2)^2} = -\frac{G_F h}{\sqrt{2}}\frac{q^4}{256\pi^6}\left[31\ln\left(-\frac{q^2}{\bar{\mu}^2}\right) - 6\ln^2\left(-\frac{q^2}{\bar{\mu}^2}\right)\right]\left(\frac{1}{2}+\sin^2\theta_W\right), \quad (18)$$

$$\frac{\lambda_N^2 h_\omega^1}{(q^2-m^2)^2} = -\frac{G_F h}{\sqrt{2}}\frac{q^4}{256\pi^6}\frac{5}{36}\left[35\ln\left(-\frac{q^2}{\bar{\mu}^2}\right) - 6\ln^2\left(-\frac{q^2}{\bar{\mu}^2}\right)\right]\left(\frac{1}{3}\sin^2\theta_W\right), \quad (19)$$

where $\ln(1/\bar{\mu}^2) = \gamma + \ln(1/4\pi\mu^2)$. The entity μ is the renormalization scale used in dimensional regularization; $\sin^2\theta_W$ is the electroweak mixing parameter in the GSW electroweak theory.

Performing Borel transformation on both sides, taking into account anomalous dimensions for the various terms in the OPE, and making use of the continuum approximation for contributions from higher exited states, we find

$$\frac{\lambda_N^2 h_\rho^0}{M^2}e^{-\frac{m^2}{M^2}} = -\frac{G_F h}{\sqrt{2}}\frac{M^6}{256\pi^6}L^{-\frac{4}{9}}\left[-62E_2\left(\frac{W}{M}\right) + 24F_2\left(\frac{W}{M},\frac{M}{\bar{\mu}}\right)\right]\left(\frac{1}{2}+\sin^2\theta_W\right), \quad (20)$$

$$\frac{\lambda_N^2 h_\rho^1}{M^2}e^{-\frac{m^2}{M^2}} = \frac{G_F h}{\sqrt{2}}\frac{M^6}{256\pi^6}L^{-\frac{4}{9}}\frac{1}{36}\left[-118E_2\left(\frac{W}{M}\right) + 24F_2\left(\frac{W}{M},\frac{M}{\bar{\mu}}\right)\right]\left(\frac{1}{3}\sin^2\theta_W\right), \quad (21)$$

$$h_\rho^2 = 0, \quad (22)$$

$$\frac{h_\rho^{1'}}{2m}\frac{\lambda_N^2}{M^2}e^{-\frac{m^2}{M^2}} = -\frac{G_F h}{\sqrt{2}}\chi\langle\bar{q}q\rangle\frac{M^4}{432\pi^4}L^{-\frac{16}{27}}\left[-13E_1\left(\frac{W}{M}\right) + 6F_1\left(\frac{W}{M},\frac{M}{\bar{\mu}}\right)\right]\left(\frac{1}{3}\sin^2\theta_W\right), \quad (23)$$

$$\frac{\lambda_N^2 h_\omega^0}{M^2}e^{-\frac{m^2}{M^2}} = -\frac{G_F h}{\sqrt{2}}\frac{M^6}{256\pi^6}L^{-\frac{4}{9}}\left[-62E_2\left(\frac{W}{M}\right) + 24F_2\left(\frac{W}{M},\frac{M}{\bar{\mu}}\right)\right]\left(\frac{1}{2}+\sin^2\theta_W\right), \quad (24)$$

$$\frac{\lambda_N^2 h_\omega^1}{M^2}e^{-\frac{m^2}{M^2}} = -\frac{G_F h}{\sqrt{2}}\frac{M^6}{256\pi^6}L^{-\frac{4}{9}}\frac{5}{36}\left[-70E_2\left(\frac{W}{M}\right) + 24F_2\left(\frac{W}{M},\frac{M}{\bar{\mu}}\right)\right]\left(\frac{1}{3}\sin^2\theta_W\right). \quad (25)$$

The function L [$\equiv \ln(M/\Lambda_{\rm QCD})/\ln(\bar{\mu}/\Lambda_{\rm QCD})$ with $\Lambda_{\rm QCD} = 100$ MeV and $\bar{\mu} = 0.5$ GeV] is introduced to take care of the anomalous dimensions. Unlike our analysis of the QCD sum rules for the strong couplings g_ρ and g_ω [8], loop integrations in the weak parity-violating case yield subdivergences in the form of $p^{2n}[\ln(-p^2/\bar{\mu}^2)]^2$ with n some integer which, upon Borel transformation, yields terms proportional to $M^{2n-2}\ln(M^2/\bar{\mu}^2)$ in the QCD sum rules. Thus, the value for the dimensional parameter $\bar{\mu}$ is of some numerical importance. The standard choice $\bar{\mu} = 0.5$ GeV is used in this paper.

Furthermore, the quantity W (with the standard choice of $W = 1.45$ GeV) is the threshold used in the continuum approximation, in which the contributions due to the excited states and the continuum are approximated by what may be obtained at the quark level (by use of QCD). The continuum approximation introduces into the sum rules (20)–(25), the functions E_i and F_i, which are defined in the Appendix.

We should also mention that the QCD sum rules which we obtained for PV couplings are basically those of leading order in QCD (with the nontrivial vacuum structure). It is a tremendous task to try to include a sufficient number of terms that involve the condensates of higher dimension in nature. Nevertheless, it is important to emphasize that the QCD sum rule approach is a definitive (deductive) procedure for evaluating the various diagrams based on QCD, contrary to the qualitative or semiquantitative nature of the earlier DDH approach [3]. In other words, the present QCD sum approach can be improved upon, albeit a fairly complicated task, in order in QCD and with increasing dimensions.

III. NUMERICAL ANALYSIS

The input parameters for our numerical analysis are those commonly adopted in standard QCD sum rule analyses [6,8, 10,12].

$$\lambda_N^2 = 1.2 \times 10^{-3}\,\text{GeV}^6, \quad a = 0.546\,\text{GeV}^3, \quad b = 0.47\,\text{GeV}^4,$$
$$\chi = -6\,\text{GeV}^{-2}, \quad m_0^2 = 0.8\,\text{GeV}^2,$$
$$h = 4.65, \quad g_\rho = 2.79, \quad g_\omega = 8.37.$$

Variations in some of these parameter values may result in errors of numerical importance. Neglect of higher dimensional terms brings in uncertainties that are difficult to quantify (unless some such terms can explicitly be taken into account). Nevertheless, we may use our experience from analyzing other QCD sum rules, such as those for the strong ρNN and ωNN couplings [8], to assess these errors.

Before performing the standard numerical analysis, we wish to first use the QCD sum rules (20)–(25), together with some data from the nuclear parity-violation experiments, to provide some estimates of the various parity-violating (PV) couplings. We note that both the couplings h_ρ^0 and h_ω^0 receive contributions from both $W^\pm$ and Z^0 exchanges, while other PV couplings are dictated by Z^0 exchange. As a result, the couplings h_ρ^0 and h_ω^0 are much larger than the other PV coupling constants, which are suppressed by a factor of $(1/3)\sin^2\theta_W = 0.07$. This means that, as an approximation, we may neglect the couplings $h_\rho^1, h_\rho^2, h_\rho^{\prime 1}$, and h_ω^1. Another important approximate relation is $h_\rho^0 = h_\omega^0$.

There are many parity-violation measurements in processes involving complex nuclei, leading to the determination of the quantity X_N^P, which characterizes the strength of the PV interaction. This quantity, with an experimental value of about 3×10^{-6} [15], can be expressed in terms of the PV coupling constants as

$$X_N^P = 5.5 f_\pi - 0.25 g_\rho h_\rho^1 - 0.62 g_\rho h_\rho^0 - 0.05 g_\rho h_\rho^{'1} - 0.17 g_\omega h_\omega^1 - 0.19 g_\omega h_\omega^0. \quad (26)$$

The PV πNN coupling f_π may be obtained via the QCD sum rule method, and it is about $(3.0 \pm 0.5) \times 10^{-7}$ [12,16]. Thus, we have $h_\rho^0 \approx h_\omega^0 \approx -4.1 \times 10^{-7}$, by assuming that the other PV couplings $(h_\rho^1, h_\rho^2, h_\rho^{'1}, h_\omega^1)$ are considerably smaller in comparison.

Analogously, we may also use the expression of the asymmetry A_{pp}, as observed in polarized proton-proton scattering, to determine the PV couplings:

$$A_{pp}(15\,\text{MeV}) = 0.01 g_\omega\left(h_\omega^0 + h_\omega^1\right) + 0.03 g_\rho\left(h_\rho^0 + h_\rho^1 + \frac{h_\rho^2}{\sqrt{6}}\right), \quad (27)$$

with an experimental value of $-(1.7 \pm 0.85) \times 10^{-7}$ [3]. It yields a value of about $-(10 \pm 5) \times 10^{-7}$ for h_ρ^0 and h_ω^0, the magnitude of which is somewhat larger than the previous value but remains consistent in light of large errors.

We proceed to analyze the various QCD sum rules as a function of the Borel mass squared M^2. The choice of the range for the Borel mass is guided by the study of the QCD sum rules for other properties of the nucleon, such as the nucleon mass, magnetic moments, axial couplings, or strong and weak πNN couplings. Specifically, it has been found in general that the nucleon sum rules should give rise to reasonable predictions in the Borel mass range of $0.9 \leqslant M^2 \leqslant 1.1\,\text{GeV}^2$. Such a choice for the Borel mass squared also enables us to estimate the errors of our QCD sum rule predictions by inferring from the analysis of the other QCD sum rules for the nucleon. The error could be as large as about 25% in some special cases.

In Figs. 2–5, the various couplings are shown as a function of the Borel mass squared M^2 (in units of GeV^2). It is of some importance to note that the scales in these figures are in fact different, resulting in errors of different magnitudes. As long as we have faith in these sum rules when a sufficient number of higher dimensional terms are included, it makes sense to make (semiquantitative) predictions with the aid of only a couple of leading terms. The following predictions have been obtained

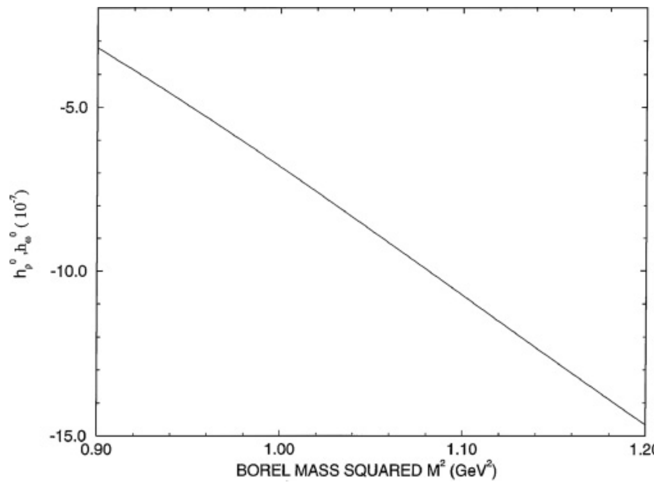

FIG. 2. PV coupling h_ρ^0 (or h_ω^0) as a function of the Borel mass squared M^2 in the range of 0.9–1.2 GeV^2.

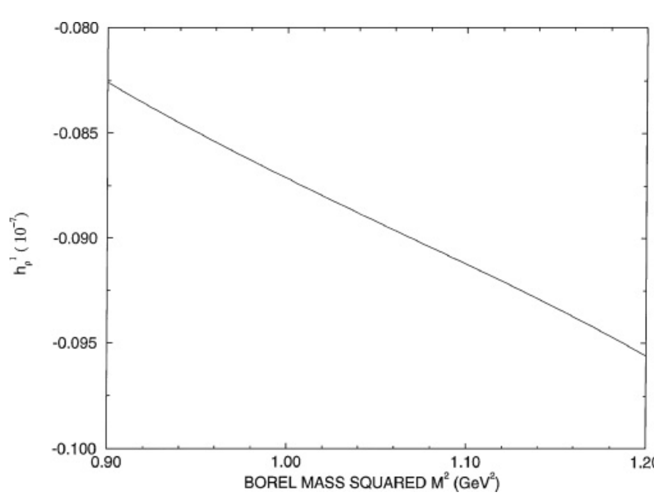

FIG. 3. PV coupling h_ρ^1 as a function of the Borel mass squared M^2 in the range of 0.9–1.2 GeV^2.

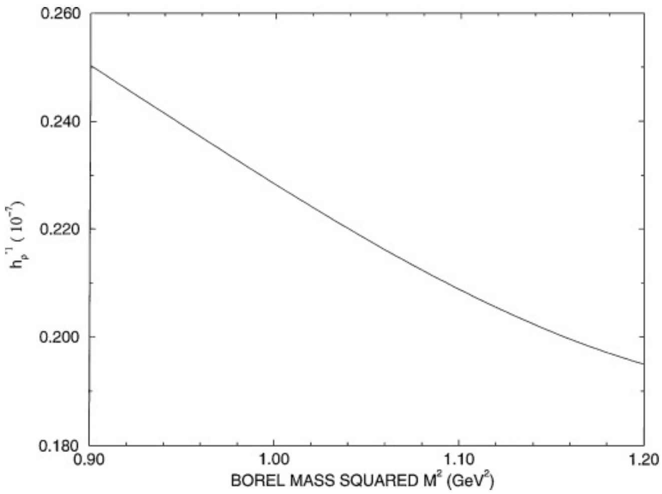

FIG. 4. PV coupling $h_\rho^{\prime 1}$ as a function of the Borel mass squared M^2 in the range of 0.9–1.2 GeV^2.

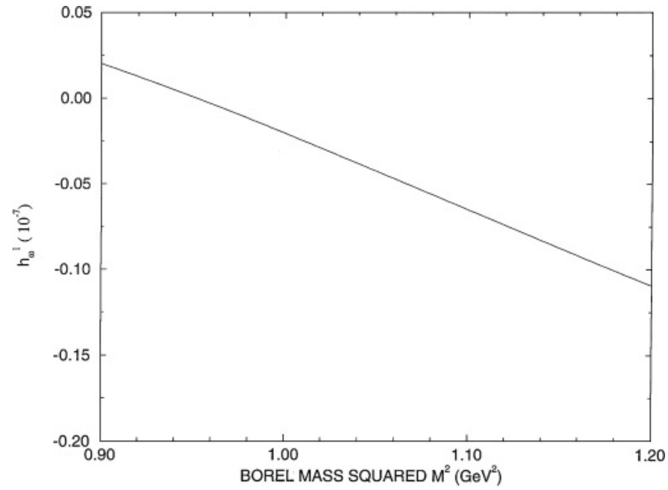

FIG. 5. PV coupling h_ω^1 as a function of the Borel mass squared M^2 in the range of 0.9–1.2 GeV^2.

in this way:

$$\begin{aligned}
h_\rho^0 &: \quad -(6.9 \pm 3.6) \times 10^{-7},\\
h_\rho^1 &: -(0.087 \pm 0.004) \times 10^{-7},\\
h_\rho^2 &: \qquad 0,\\
h_\rho^{1'} &: \quad (0.23 \pm 0.02) \times 10^{-7},\\
h_\omega^0 &: \quad -(6.9 \pm 3.6) \times 10^{-7},\\
h_\omega^1 &: \quad -(0.03 \pm 0.04) \times 10^{-7}.
\end{aligned}$$

The large errors, such as those associated with h_ρ^0 and h_ω^0, are caused primarily by the rapid variation of the prediction in the quoted Borel mass range. Experience with the QCD sum rule method, such as in the weak parity-violating coupling $f_{\pi NN}$ [12], suggests that inclusion of the next couple of terms of higher dimensions could help to smoothen the rapidly varying behavior of the leading term while yielding predictions in the same ballpark.

We note that the predicted values for h_ρ^0 and h_ω^0 are compatible with the above results as extracted from the experimental values of X_N^P and $A_{pp}(15\ \mathrm{MeV})$ (together with the observation from the QCD sum rules that only h_ρ^0 and h_ω^0 are dominant).

In Table I, we compare the various predictions on parity-violating meson-nucleon couplings, including the estimates given by Desplanques, Donoghue, and Holstein (DDH) [3], the values by an overall fit to the existing experiments (AH) [4], and ours making use of the QCD sum rules. Our results, if taken seriously, sharpen the allowed ranges for most of the six couplings. The overall agreement is good, although we should note that from the leading diagrams we included, the isotensor coupling h_ρ^2 vanishes (while it is fairly sizable in Refs. [3,4]) and $h_\rho^{\prime 1}$ is small but is different from zero (as in Refs. [3,4]). [Both h_ρ^2 and $h_\rho^{\prime 1}$ do not contribute in any major way to the existing nuclear PV observables. See, for example, the expressions for X_N^P and $A_{pp}(15\ \mathrm{MeV})$.]

Owing to the tremendous complications caused by including a sufficient number of higher dimensional terms involving the various condensates, we have relied on the experience of analyzing the other QCD sum rules for the nucleon in order to assess the uncertainties. While it is clearly desirable to improve on these derivations (by including more higher order terms, especially for h_ρ^0 and h_ω^0), it is indeed gratifying to note that our overall predictions are fairly consistent with earlier results [3,4].

TABLE I. PV couplings in units of 10^{-7}. The numbers in parentheses are the "best" values.

	DDH (best estimate)	AH (best fit)	QCD sum rules
f_π	$0 \to 11\,(4.6)$	$0 \to 11\,(2.1)$	3.0 ± 0.5[a]
h_ρ^0	$-31 \to 11\,(-11.4)$	$-31 \to 11\,(-5.8)$	$-(6.9 \pm 3.7)$
h_ρ^1	$-0.4 \to 0\,(-0.19)$	$-0.5 \to 0.4\,(-0.22)$	$-(0.087 \pm 0.004)$
h_ρ^2	$-7.6 \to -11\,(-9.5)$	$-6.3 \to -10\,(-7.1)$	0
$h_\rho^{\prime 1}$	0	0	0.23 ± 0.02
h_ω^0	$-10 \to 5.7\,(-1.9)$	$-12 \to 2.6\,(-5.0)$	$-(6.9 \pm 3.7)$
h_ω^1	$-0.8 \to -1.9\,(-1.1)$	$-3.1 \to -1.1\,(-2.4)$	$-(0.03 \pm 0.04)$

[a]Prediction obtained from Ref. [16].

IV. SUMMARY

The problem of parity-violating nuclear force has been a very difficult one. Progress has been made in the past [3,4] in the simple meson-exchange picture, in which the parity-violating nuclear force arises from exchange of π, ρ, ω, or other mesons with strong MNN coupling at one vertex and weak parity-violating MNN coupling at the other vertex. The QCD sum rule method allows one to determine the various parity-violating MNN couplings starting from QCD (with the nontrivial vacuum) and Glashow-Salam-Weinberg (GSW) electroweak theory. We believe that the QCD sum rule methods, which make explicit connection between the underlying theory (i.e., the QCD and GSW electroweak theory) and the predictions, offer us a systematic method to tackle. We wish to note that our QCD sum rule predictions are fairly consistent with earlier results [3–5].

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APPENDIX: BOREL TRANSFORMATION

For the invariant functions $f(p^2)$ appearing in the polarization function, we may apply the dispersion relation

$$f(s) = \frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im} f(p'^2)}{p'^{\,2}+s}dp'^2, \quad s=-p^2, \tag{A1}$$

with a necessary number of subtractions.

Borel transformation is a general tool employed in the method of QCD sum rules. It is defined by

$$Bf(s) = \lim_{n,s\to\infty,\ s/n=M^2} \frac{s^{n+1}}{n!}\left(-\frac{d}{ds}\right)^n f(s). \tag{A2}$$

Applying Borel transformation to the dispersion relation, we obtain

$$Bf(s) = \frac{1}{\pi}\int_0^\infty e^{-p^2/M^2}\,\mathrm{Im} f(p^2)dp^2. \tag{A3}$$

In general, we may take into account the contribution due to exited states and assume that it is equal to the quark-level contribution of $f(p^2)$ starting from some cutoff value $p^2 = W^2$. That is, we use a modified Borel transformation B_W with the definition:

$$B_W f(s) = \frac{1}{\pi}\int_0^{W^2} e^{-p^2/M^2}\,\mathrm{Im} f(p^2)dp^2, \tag{A4}$$

which involves some cutoff W.

Next, we list all the modified Borel transformation functions needed in the text.

$$\begin{aligned}-M^2E_0\left(\frac{W}{M}\right) &\equiv B_W \ln\frac{s}{\mu^2},\\ &= -M^2(1-e^{-W^2/M^2}),\end{aligned} \tag{A5}$$

$$\begin{aligned}M^4E_1\left(\frac{W}{M}\right) &\equiv B_W s\ln\frac{s}{\mu^2}\\ &= M^4\left[1-e^{-W^2/M^2}\left(1+\frac{W^2}{M^2}\right)\right],\end{aligned} \tag{A6}$$

$$\begin{aligned}-2M^6E_2\left(\frac{W}{M}\right) &\equiv B_W s^2\ln\frac{s}{\mu^2} = -2M^6\left[1-e^{-W^2/M^2}\right.\\ &\quad\left.\times\left(1+\frac{W^2}{M^2}+\frac{W^4}{2M^4}\right)\right],\end{aligned} \tag{A7}$$

$$\begin{aligned}2M^4F_1\left(\frac{W}{M},\frac{M}{\mu}\right) &\equiv B_W s\ln^2\frac{s}{\mu^2}\\ &= 2M^4\left\{1-\gamma-e^{-W^2/M^2}\right.\\ &\quad + Ei\left(-\frac{W^2}{M^2}\right)+\left[1-e^{-W^2/M^2}\right.\\ &\quad\left.\left.\times\left(1+\frac{W^2}{M^2}\right)\right]\ln\frac{M^2}{\mu^2}\right\},\end{aligned} \tag{A8}$$

$$\begin{aligned}-4M^6F_2\left(\frac{W}{M},\frac{M}{\mu}\right) &\equiv B_W s^2\ln^2\frac{s}{\mu^2}\\ &= -4M^6\left\{\frac{3}{2}-\gamma-\frac{3}{2}e^{-W^2/M^2}\right.\\ &\quad\times\left(1+\frac{W^2}{3M^2}\right)+Ei\left(-\frac{W^2}{M^2}\right)\\ &\quad+\left[1-e^{-W^2/M^2}\left(1+\frac{W^2}{M^2}\right.\right.\\ &\quad\left.\left.\left.+\frac{W^4}{2M^4}\right)\right]\ln\frac{M^2}{\mu^2}\right\}.\end{aligned} \tag{A9}$$

Here we have adopted the conventional symbols E_0, E_1, and E_2, and introduced two new symbols F_1 and F_2. The exponential integral function $Ei(x)$ is defined by

$$Ei(x) = -\int_{-x}^\infty \frac{e^{-t}}{t}dt. \tag{A10}$$

[1] W. T. H. van Oers (private communication).
[2] J. Carlson, R. Schiavilla, V. R. Brown, and B. F. Gibson, Phys. Rev. C **65**, 035502 (2002).
[3] B. Desplanques, J. F. Donoghue, and B. R. Holstein, Ann. Phys. (NY) **124**, 449 (1980).
[4] E. G. Adelberger and W. C. Haxton, Annu. Rev. Nucl. Part. Sci. **35**, 501 (1985).
[5] M. J. Ramsey-Musolf and S. A. Page, Annu. Rev. Nucl. Part. Sci. **56**, 1 (2006).
[6] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385, 448 (1979).
[7] N. Isgur and J. Paton, Phys. Rev. D **31**, 2910 (1985); N. Isgur, G. Karl, and J. Soffer, *ibid.* **35**, 1665 (1987); R. Kokoski and N. Isgur, *ibid.* **35**, 907 (1987); J. Merlin and J. Paton, *ibid.*

35, 1668 (1987); A. Le Yaouanc, L. Oliver, O. Pène, and J. C. Raynal, *ibid*. **8**, 2223 (1973).
[8] Chih-Yi Wen and W.-Y. P. Hwang, Phys. Rev. C **56**, 3346 (1997).
[9] G. Erkol, R. G. Timmermans, and Th. A. Rijken, Phys. Rev. C **74**, 045201 (2006).
[10] B. L. Ioffe and A. V. Smilga, Nucl. Phys. **B232**, 109 (1984).
[11] B. L. Ioffe, Z. Phys. C **18**, 67 (1983).
[12] E. M. Henley, W.-Y. P. Hwang, and L. S. Kisslinger, Phys. Lett. **B367**, 21 (1996); [Erratum-*ibid*. **B440**, 449 (1998)].
[13] W.-Y. P. Hwang, Ze-sen Yang, Y. S. Zhong, Z. N. Zhou, and Shi-lin Zhu, Phys. Rev. C **57**, 61 (1998).
[14] E. M. Henley, W.-Y. P. Hwang, and L. S. Kisslinger, manuscript in preparation.
[15] B. Desplanques and J. Missimer, Nucl. Phys. **A300**, 286 (1978).
[16] W.-Y. P. Hwang, Z. Phys. C **75**, 701 (1997).