Black holes and Thunderbolt singularities with Lifshitz Scaling terms

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Introduction

- Horava-Lifshitz gravity (P. Horava, 2009)

\[ \mathcal{L}_{HL} = \mathcal{L}_{IR} \left[ \partial_0^2, \partial_x^2 \right] + \mathcal{L}_{UV} \left[ \partial_x^4, \partial_x^6 \right] \]

Sixth order spatial derivatives make theory power-counting renormalizable.

(\textit{projectable}) HL gravity seems to be truly renormalizable.

- Sixth order spatial derivatives lead unusual dispersion relation:

\[ \omega^2 \sim k^6 \quad \text{for large } k \]

\[ c \sim k^2 \rightarrow \infty \quad \text{for large } k \]

\textbf{Question}: Can we construct BH without Lorentz invariance?
Instantaneous mode should be taken into account.
Previous works: theory

- IR limit of HL gravity (without UV correction)

**The action**

Gravitational dynamical fields: $g_{\mu\nu}$, aether $u^\mu$ (timelike, fixed-norm, twistless)

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left[ R - c_{13} (\nabla_\alpha u_\beta)(\nabla^\beta u^\alpha) - c_2 (\nabla_\alpha u^\alpha)^2 + c_{14} u^\alpha u^\beta (\nabla_\alpha u_\gamma)(\nabla_\beta u^\gamma) \right]$$

$$u^\mu := \nabla^\mu \varphi / \sqrt{- (\nabla_\alpha \varphi)(\nabla^\alpha \varphi)} : \text{aether, } \varphi : \text{khronon}$$

- Spacetime with preferred foliation via khronon $\varphi$

$(\mathcal{M}, g, \varphi)$

$u^\mu \propto \nabla_\mu \varphi$

$\Sigma_\varphi : \varphi = \text{const.}$

"time" direction

"spacial" direction
Previous works: universal horizon

- If there were instantaneous mode, can we find event horizon?

**Local structure**

Light cone angle:
- $= 90^\circ$ (null)
- $> 90^\circ$ (superluminal)
- $= 180^\circ$ (instantaneous)

**Global structure**

Universal horizon: static limit for the particle with $c=\infty$

Steady structure:

- Local structure
- Global structure

Universal horizon: any particle cannot escape from UH even if $c=\infty$

Static and spherical symmetric BH with universal horizon.

(Barausse et al. 2011; Blas and Sibiryakov 2011; Berglund et al. 2012)
The theory we consider

**motivation**: We want to consider backreaction to BH by UV correction.

\[ \mathcal{L}_{\text{HL}} = \mathcal{L}_{\text{IR}} \left[ \partial_t^2, \partial_{x^i}^2 \right] + \mathcal{L}_{\text{UV}} \left[ \partial_{x^i}^4, \partial_{x^i}^6 \right] \]

Solving equation in static, spherically symmetric and asymptotic flat spacetime

**HL gravity with khronon**

\[
S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} (\mathcal{L}_{\text{IR}} + \mathcal{L}_{\text{UV}})
\]

\[
\mathcal{L}_{\text{IR}} := R - c_{13} (\nabla_\alpha u_\beta) (\nabla_\beta u^\alpha) - c_2 (\nabla \cdot u)^2 + c_{14} (\dot{u}^\alpha \dot{u}_\alpha)
\]

\[
\mathcal{L}_{\text{UV}} := -m_{\text{pl}}^{-2} \left[ \beta_1 (\dot{u}_\alpha \dot{u}^\alpha)^2 + \beta_2 \mathcal{R}^2 \right]
\]

where, \( \mathcal{R}_{\mu\nu}[g, u] \) is a 3-Ricci tensor and \( \dot{u}^\mu := u^\alpha \nabla_\alpha u^\mu \).

\[
\omega_G^2 = \frac{1}{1 - c_{13}} k^2 \quad \omega_S^2 = \frac{(c_{13} + c_2)(2 - c_{14})}{c_{14}(1 - c_{13})(2 + c_{13} + 3c_2)} k^2 + \frac{8(c_{13} + c_2)\beta_2}{2 + c_{13} + 3c_2} \left( \frac{k^2}{m_{\text{pl}}} \right)^2
\]

scalar graviton becomes instantaneous if \( \mathcal{R}^2 \) term joins

\( \mathcal{R}^2 \) term effect
Substituting the following Eddington-Finkelstein type ansatz into e.o.m.

\[ ds^2 = -T(r)dv^2 + 2B(r)dvdr + r^2d\Omega^2, \quad u^\mu = \left( a(r), \frac{a(r)^2T(r) - 1}{2a(r)B(r)}, 0, 0 \right) \]

where,

\[
T(r) = 1 + \frac{T_1}{r} + \frac{T_2}{r^2} + \frac{T_3}{r^3} + \cdots, \quad B(r) = 1 + \frac{B_1}{r} + \frac{B_2}{r^2} + \frac{B_3}{r^3} + \cdots, \quad a(r) = 1 + \frac{a_1}{r} + \frac{a_2}{r^2} + \frac{a_3}{r^3} + \cdots
\]

then, we can solve e.o.m. order by order:

- **mass**
  \[ T_1 = \text{arbitrary}, \quad T_2 = 0, \quad T_3 = \frac{c_{14}T_1^3}{48}, \]

- **additional parameter**
  \[ B_1 = 0, \quad B_2 = \frac{c_{14}T_1^2}{16}, \quad B_3 = -\frac{c_{14}T_1^3}{12}, \]
  \[ a_1 = -\frac{T_1}{2}, \quad a_2 = \text{arbitrary}, \quad a_3 = -\left( \frac{c_{14} - 6}{96}T_1^3 + T_1a_2 \right), \]

**T1 and a2 gives boundary condition at spatial infinity**
If we consider $\mathcal{L}_{\text{UV}} = -m_{\text{pl}}^{-2} \beta_1 (\dot{u}^\alpha \dot{u}_\alpha)^2$ as UV correction term, BH can be found.

$$ds^2 = -T(r) dv^2 + 2B(r) dv dr + r^2 d\Omega^2,$$

$u^\mu = \left( a(r), \frac{a(r)^2 T(r) - 1}{2a(r) B(r)}, 0, 0 \right)$

$$(c_{13}, c_2, c_{14}) = (0.100, -6.135 \times 10^{-4}, c_{14} = 0.100)$$

$$(a_2, \beta_1/m_{\text{pl}}^2, \beta_2/m_{\text{pl}}^2) = (0.200, 1.000, 0) \text{ normalized by } G_N M := G \left(1 - \frac{c_{14}}{2}\right)^{-1} M = 0.5$$
If we consider $\mathcal{L}_{UV} = -m_{pl}^{-2}\beta_1 (\dot{u}^\alpha \dot{u}_\alpha)^2$ as UV correction term, BH can be found.

At Universal horizon ($r = r_{\text{UH}}$), $\Sigma$ coincides with $r$ constant surface.

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Presumably, $a_2$ is not a Noether charge, but represents aether distribution:

$$ds^2 = -T(r)dv^2 + 2B(r)dvdr + r^2d\Omega^2, \quad u^\mu = \left( a(r), \frac{a(r)^2T(r) - 1}{2a(r)B(r)}, 0, 0 \right)$$

when we choose large value of $a_2$, aether field collapses.

$$(c_{13}, c_2, c_{14}) = (0.100, -6.135 \times 10^{-4}, c_{14} = 0.100)$$

$$(a_2, \beta_1/m_{pl}^2, \beta_2/m_{pl}^2) = (0.500, 1.000, 0) \text{ normalized by } G_NM := G \left(1 - \frac{c_{14}}{2}\right)^{-1}M = 0.5$$
If $R^2$ term is joined, universal horizon turns to be physical singularity:

$$ds^2 = -T(r) dv^2 + 2B(r) dvdr + r^2 d\Omega^2, \quad u^\mu = \left( a(r), \frac{a(r)^2 T(r) - 1}{2a(r)B(r)}, 0, 0 \right)$$

\[
\begin{align*}
(c_{13}, c_2, c_{14}) &= (0.100, -6.135 \times 10^{-4}, c_{14} = 0.100) \\
(a_2, \beta_1/m_{pl}^2, \beta_2/m_{pl}^2) &= (1.112 \times 10^{-3}, 0, -1.000) \quad \text{normalized by } G_NM := G \left(1 - \frac{c_{14}}{2}\right)^{-1} M = 0.5
\end{align*}
\]
If $\mathcal{R}^2$ term is joined, universal horizon turns to be physical singularity:

- $r \neq 0$: physical singularity
- $\Sigma_{\varphi}$: $\varphi = \text{const.}$
- $r = \infty$ and $\mathcal{I}^+$
- Outgoing particle with $c=\infty$

Since signal from singularity cannot escape, Cosmic Censorship is not violated.
results

• Black hole solution with universal horizon which is a event horizon can be constructed including only $(\dot{u}^{\alpha} \dot{u}_{\alpha})^2$ term.

• If $R^2$ term is considered, **BH cannot exist**. Universal horizon turn to be physical singularity. However, it does not mean violation of Cosmic Censorship.

• The solution with singular universal horizon has similar property to thunderbolt as a final state of BH evaporation in 2 dim. quantum gravity.

(Hawking and Stewart,1992; Ishibashi and Hosoya,2002)

future works

• Although we have considered 2 terms as UV correction, the number of possible terms in $\mathcal{L}_{UV}$ are over 30.

• Since, at least, projectable HL gravity seems to be truly renormalizable, we should promote quantum analysis rather than "semi-classical" way.
Appendix
Lifshitz scaling terms as quantum correction

- Lorentz violation in HL gravity is due to $z>1$ Lifshitz scaling.

- Scaling dimension: $[t]=-z$, $[x^i]=-1$.

- Breaking equivalence between space and time.

- If $z>3$, gravity acquires (power-counting) renormalizability.

- $z>1$ terms are interpreted into quantum effect (renormalization counter-term)

$$\mathcal{L}_{\text{HL}} = \mathcal{L}_{\text{IR}}[\partial_t^2, \partial_{x_i}^2] + \mathcal{L}_{\text{LS}}[\partial_{x_i}^4, \cdots, \partial_{x_i}^{2z}]$$
New solution : “Thunderbolt singularity”

What is Thunderbolt singularity?
- One of the final state of BH evaporation in 2 dim. quantum gravity. (Hawking and Stewart,1992; Ishibashi and Hosoya,2002)

Reconsider our solution
- If UV correction becomes dominant, universal horizon turns to be singular.
- the BH singularity may be captured on the universal horizon.
- Modification in gravitational dispersion relation may leads this singularity.

At the end, singularity spreads across spacelike or null surface.

"It hit you and wipes you out", as it were, hitting with a thunderbolt.