Effective Field Theory of Anisotropic Inflation

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mainly based on a paper in preparation with J.Gong, G.Shiu, J.Soda, M.Yamaguchi
see also arXiv:1412.5601 (published in PRD) with Y.Hidaka and G.Shiu

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introduction
single field slow-roll inflation is perfect!

search for small deviations from the standard single field inflation?

1. inflation as a probe of new particles & new physics
   cf. cosmological collider physics
   [Chen-Wang ’09, Baumann-Green ’11, TN-Yamaguchi-Yokoyama ’12, ArkaniHamed-Maldacena ’15, …]

2. precision test of our homogeneity & isotropy assumptions
test of isotropy

anisotropic universe (Bianchi type 1):

\[ ds^s = -dt^2 + a^2 e^{2\sigma} (dx^2 + dy^2) + a^2 e^{-4\sigma} dz^2 \]

- \( \dot{\sigma}(t) \) characterizes spacetime anisotropy (FRW for \( \sigma = \text{constant} \))
- isometry: spatial translations & x-y rotation
test of isotropy

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no rotational symmetries in x-z, y-z planes!
\( \rightarrow \) deviations from predictions in standard inflation
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→ deviations from predictions in standard inflation

- statistical anisotropy

\[ \langle \zeta_k \zeta_{k'} \rangle = (2\pi)^3 \delta^{(3)}(k + k') \frac{(2\pi)^3}{k^3} P_\zeta(k)(1 + g_\ast \sin^2 \theta) \]

※ current constraint by Planck: \( |g_\ast| < 0.03 \) (95% CL)
test of isotropy

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- primordial gravitational waves

mixing between scalar & tensor \( \langle \zeta \gamma_{ij} \rangle \neq 0 \)

→ affect the tensor sector in more general
Nambu-Goldstone modes

spontaneous breaking of rotational symmetries

→ there appear corresponding NG modes
Nambu-Goldstone modes

spontaneous breaking of rotational symmetries

→ there appear corresponding NG modes

NG mode $\pi$ (inflaton)
for time translations

anisotropic mixing

NG modes for “broken rotation”

primordial tensor mode
Nambu-Goldstone modes

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can we have generic relations between spacetime anisotropy and cosmological observables from the spacetime symmetry viewpoint?
→ develop EFT for such fluctuations around anisotropic universe
Nambu-Goldstone modes

spontaneous breaking of rotational symmetries
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anisotropic mixing

NG mode spectrum is not unique for spacetime symmetry breaking!

can we have generic relations between spacetime anisotropy and cosmological observables from the spacetime symmetry viewpoint?
→ develop EFT for such fluctuations around anisotropic universe
global vs local viewpoints of spacetime symmetry

[cf. Hidaka-TN-Shiu ’14]
general arguments

in contrast to internal symmetry breaking,

# of NG modes ≠ # of broken global symmetries

for spacetime symmetry breaking

ex. conformal symmetry breaking → 1 NG mode called dilaton
general arguments

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ex. conformal symmetry breaking \(\rightarrow\) 1 NG mode called dilaton

NG modes = local transformations of broken symmetry

\(\rightarrow\) \# of NG modes = \# of broken local symmetries

(diffs, local Lorentz, ...)

※ several NG mode contents can appear

for a given global symmetry breaking pattern
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for the correct identification of NG modes and EFT construction,

it is convenient to gauge the (broken) spacetime symmetries

by introducing curved coordinates, vierbein, etc.
isotropic inflation and symmetry breaking

- global symmetry breaking pattern for isotropic inflation:
  
  \[ \text{dS/Minkowski isometry} \rightarrow \text{FRW isometry} \]
  
  ※ \(10 - 6 = 4\) broken global symmetries
isotropic inflation and symmetry breaking

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  \[
  \text{dS/Minkowski isometry} \rightarrow \text{FRW isometry}
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  \[
  \Rightarrow 10 - 6 = 4 \text{ broken global symmetries}
  \]

- various local symmetry breaking patterns

1. single field inflation: \( \langle \phi \rangle = \bar{\phi}(t) \)
   broken time diffs \( \Leftrightarrow \) 1 NG mode for time translation

2. solid inflation [Endlich et al ’12]: \( \langle \phi^I \rangle = x^I \ (I = 1, 2, 3) \)
   3 broken spatial diffs \( \Leftrightarrow \) 3 NG modes for spatial translations

3. inflaton + local boost breaking (cf. Einstein-Aether theory)
   broken time diffs & local boosts \( \Leftrightarrow \) 1 + 3 NG modes
isotropic inflation and symmetry breaking

- global symmetry breaking pattern for isotropic inflation:
  
  \[ \text{dS/Minkowski isometry} \rightarrow \text{FRW isometry} \]
  
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     \[ \times \text{ see Delacretaz-TN-Senatore to appear on arXiv tomorrow!} \]
anisotropic inflation and symmetry breaking

- # of isometric symmetries

dS/Minkowski (4 + 6) → FRW (3 + 3) → Bianchi type 1 (3 + 1)
anisotropic inflation and symmetry breaking

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- local symmetry breaking patterns in anisotropic models

1. inflaton + vector [Watanabe et al ’09, Emami et al ’10, ...]

  \langle \phi \rangle = \bar{\phi}(t), \langle A_3 \rangle = v(t) \quad \rightarrow \text{time diffs, local 0–3, 1–3, 2–3 Lorentz}
anisotropic inflation and symmetry breaking

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2. inflaton + two form field [Ohashi-Soda-Tsujikawa ’13]
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3. solid inflation [Bartolo-Matarrese-Peloso-Ricciardone ’13]
   \[ \langle \phi^1 \rangle = x, \langle \phi^2 \rangle = y, \langle \phi^3 \rangle = \alpha z \rightarrow 3 \text{ spatial diffs} \]
anisotropic inflation and symmetry breaking

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2’. Hodge dual of two form model
   \[\langle \phi \rangle = \bar{\phi}(t), \langle \varphi^2 \rangle = Az \rightarrow \text{time & z diffs} \]
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  2'. Hodge dual of two form model
     \[ \langle \phi \rangle = \bar{\phi}(t), \langle \varphi^2 \rangle = A z \rightarrow \text{time & z diffs} \]

focus in this talk
scalar type source of anisotropy
Hodge dual of two form model

two form model of anisotropic inflation [Ohashi-Soda-Tsujikawa ‘13]

\[ S = \int d^4x \sqrt{-g} \left[ \frac{M_{Pl}^2}{2} R - \frac{1}{2} (\partial_{\mu} \phi)^2 - V(\phi) - \frac{1}{12} f^2(\phi) H^{\mu\nu\rho} H_{\mu\nu\rho} \right] \]

- \( H = dB \) is the field strength
- backgrounds are \( \langle \phi \rangle = \bar{\phi}(t) \), \( \langle H_{012} \rangle = v(t) = Af^{-2}(\bar{\phi})ae^{4\sigma} \)
Hodge dual of two form model

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Hodge dual

\[ f^2(\phi) dB = *d\phi \]
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- backgrounds are \( \langle \phi \rangle = \bar{\phi}(t) \), \( \langle \partial_z \varphi \rangle = A \)

\[
\langle \varphi \rangle = A z + \text{constant}
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\[ \langle \varphi \rangle = Az + \text{constant} \]

※ z-diffs are broken, but spatially homogeneous
EFT for broken time and z diffs

in unitary gauge, 1. NG modes are eaten by the metric
2. time & z diffs are broken by gauge conditions
EFT for broken time and z diffs

in unitary gauge, 1. NG modes are eaten by the metric

2. time & z diffs are broken by gauge conditions

just as original EFT of inflation, the general action is given by

\[ S = S_t + S_z + S_{tz} \]

\[
S_t = \int d^4x \sqrt{-g} \left[ \frac{M_{Pl}^2}{2} R - \Lambda - cg^{00} + \frac{M_2^4}{2} (\delta g^{00})^2 + \ldots \right]
\]

\[
S_z = \int d^4x \sqrt{-g} \left[ -\alpha_1 \delta g^{33} + \alpha_2 (\delta g^{33})^2 + \ldots \right]
\]

\[
S_{tz} = \int d^4x \sqrt{-g} \left[ \beta_1 g^{03} + \beta_2 (g^{03})^2 + \ldots \right]
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\[ S_{tz} = \int d^4x \sqrt{-g} \left[ \beta_1 g^{03} + \beta_2 \left( g^{03} \right)^2 + \ldots \right] \]

background equations of motion:

\[ \Lambda = M_{Pl}^2 \left( 3H^2 + \dot{H} - \frac{3}{2} H \dot{\sigma} - \frac{1}{2} \ddot{\sigma} \right) \quad c = M_{Pl}^2 \left( -\dot{H} + \frac{3}{2} H \dot{\sigma} + \frac{1}{2} \ddot{\sigma} - 3\ddot{\sigma} \right) \]

\[ \alpha_1 = -\frac{1}{2} M_{Pl}^2 a^2 e^{-4\sigma} \left( 9H \dot{\sigma} + 3\ddot{\sigma} \right) \quad \beta_1 = 0 \]
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revisiting Hodge dual of two form model

the Hodge dual of two form model is captured by

$$S = \int d^4 x \sqrt{-g} \left[ \frac{M_{Pl}^2}{2} R - \Lambda - c g^{00} - \alpha_1 \delta g^{33} \right]$$

※ \((\partial_\mu \phi)^2 = \dot{\phi}^2 g^{00}, (\partial_\mu \varphi)^2 = A^2 g^{33}\)

※ all the EFT coefficients are expressed in terms of \(H\) and \(\dot{\varphi}\)
revisiting Hodge dual of two form model

the Hodge dual of two form model

\[
S^{(2)} = \int d^4 x \ a^3 \ [c \ (\pi^2 - (\partial_i \pi)^2) + \alpha_1 \ (\chi^2 - (\partial_i \chi)^2) - 2 \dot{\alpha}_1 a^{-2} e^{4\sigma} \pi \partial_3 \chi]
\]

※ (\partial_\mu \phi)^2 = \dot{\phi}^2 g^{00}, (\partial_\mu \varphi)^2 = A^2 g^{33}

※ all the EFT coefficients are expressed in terms of \( H \) and \( \dot{\sigma} \)

introduce NG modes \( \pi \) & \( \chi \) for broken t & z diffs
revisiting Hodge dual of two form model

the Hodge dual of two form model

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S^{(2)} = \int d^4 x \, a^3 \left[ c (\dot{\pi}^2 - (\partial_i \pi)^2) + \alpha_1 (\dot{\chi}^2 - (\partial_i \chi)^2) - 2\dot{\alpha}_1 a^{-2} e^{4\sigma} \pi \partial_3 \chi \right]
\]

\[\star (\partial_\mu \phi)^2 = \dot{\phi}^2 g^{00}, \quad (\partial_\mu \varphi)^2 = A^2 g^{33}\]

\[\star \text{ all the EFT coefficients are expressed in terms of } H \text{ and } \dot{\sigma}\]

introducing canonically normalized fields \( \pi_c = \sqrt{2c} \pi, \chi_c = \sqrt{2\alpha_1} \chi \),

\[
S^{(2)} = \int d^4 x \, a^3 \left[ \frac{1}{2} (\dot{\pi}_c^2 - (\partial_i \pi_c)^2) + \frac{1}{2} (\dot{\chi}_c^2 - (\partial_i \chi_c)^2 - m^2 \chi_c^2) + \beta \pi \frac{\partial_3 \chi}{a e^{-2\sigma}} \right]
\]
revisiting Hodge dual of two form model

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\( \star (\partial_\mu \phi)^2 = \dot{\phi}^2 g^{00}, \quad (\partial_\mu \varphi)^2 = A^2 g^{33} \)

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\[ m_\chi^2 = -2(1 - 2\Sigma)(2 - \Sigma)H^2, \quad \beta = 3\sqrt{2} H \frac{(-\Sigma)^{1/2}(1 - 2\Sigma)}{\sqrt{\epsilon + \frac{3}{2} \Sigma - 3\Sigma^2}} \]

\( \star \Sigma = \frac{\dot{\sigma}}{H} \) characterizes spacetime anisotropy
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\[ S^{(2)} = \int d^4x \, a^3 \left[ c (\dot{\pi}^2 - (\partial_i \pi)^2) + \alpha_1 (\dot{\chi}^2 - (\partial_i \chi)^2) - 2\dot{\alpha}_1 a^{-2} e^{4\sigma} \pi \partial_3 \chi \right] \]

\[ \times (\partial_\mu \phi)^2 = \dot{\phi}^2 g^{00}, (\partial_\mu \varphi)^2 = A^2 g^{33} \]

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introducing canonically normalized fields \( \pi_c = \sqrt{2c} \pi, \chi_c = \sqrt{2\alpha_1} \chi \),

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\[ \times \Sigma = \frac{\dot{\sigma}}{H} \text{ characterizes spacetime anisotropy} \]

in particular, the mass of \( \chi \) is tachyonic \( m_\chi^2 \simeq -4H^2 \) when \(|\Sigma| \ll 1|\).
statistical anisotropy

time diffs NG mode

$\pi_c$

$\beta \pi \frac{\partial_3 \chi}{a}$

anisotropic mixing

z diffs NG mode

$\chi_c \left( m^2 \simeq -4H^2 \right)$
statistical anisotropy

time diffs NG mode
\[ \pi_c \]

\[ \beta \pi \frac{\partial_3 \chi}{a} \]
anisotropic mixing

z diffs NG mode
\[ \chi_c \left( m^2 \simeq -4H^2 \right) \]

two point function of \( \pi_c \)
\[
\left\langle \pi_c \mathbf{k} \pi_c - \mathbf{k} \right\rangle =
\begin{array}{cccccc}
\pi_c & \pi_c & \pi_c & \pi_c \chi_c & \pi_c \chi_c & \pi_c \\
\end{array}
\]
statistical anisotropy

time diffs NG mode
$\pi_c$

anisotropic mixing

$\beta \pi \frac{\partial_3 \chi}{a}$

z diffs NG mode
$\chi_c \ (m^2 \simeq -4H^2)$

two point function of $\pi_c$

$$\langle \pi_c \mathbf{k} \pi_c \mathbf{-k} \rangle = \begin{array}{cccc}
\pi_c & & \pi_c & & \pi_c & & \pi_c & & \pi_c & & \pi_c & & \pi_c \\
\times & & \times & & + & & \times & & \times & & \times & & \times & & \times & & \times \\
\pi_c & & \pi_c & & \pi_c & & \pi_c \chi_c & & \pi_c \chi_c & & \pi_c \chi_c & & \pi_c & & \pi_c \\
\end{array}$$

$$= (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{k}') \frac{H^2}{2k^3} \left[ 1 + \cos^2 \theta \frac{-18\Sigma}{\epsilon + \frac{2}{3}\Sigma} \right] \left( N_k^2 \right)$$

※ $N_k = \ln(-k\tau)$ is the e-folding number after horizon crossing
statistical anisotropy

two point function of $\pi_c$

$$\langle \pi_c k \pi_c - k \rangle = \begin{array}{ccccccc} x & x & + & x & \cdots & x & \cdots & x \\ \pi_c & \pi_c & \pi_c & \pi_c \chi_c & \pi_c \chi_c & \pi_c \chi_c & \pi_c \end{array}$$

$$= (2\pi)^3 \delta^{(3)}(k + k') \frac{H^2}{2k^3} \left[ 1 + \cos^2 \theta \frac{-18\Sigma}{\epsilon} + \frac{2}{3}\Sigma \frac{N_k^2}{N_k} \right]$$

※ $N_k = \ln(-k\tau)$ is the e-folding number after horizon crossing

constraint on $\Sigma = \frac{\dot{\sigma}}{H}$ is $|\Sigma| < 4.6 \times 10^{-9} \times \frac{\epsilon}{0.01} \times \frac{60^2}{N_k^2}$
summary and prospects

# summary of the talk
- how accurate our assumption of isotropic universe?
  → spacetime symmetry based EFT of anisotropic inflation
- global vs local symmetry viewpoints of space time symmetry breaking
  → various local symmetry breaking patterns for anisotropic universe
    ※ various NG mode contents
- EFT construction for Hodge dual of two form model
  1. characterized by time and z diffs breaking
  2. mixing, mass etc are directly related to the spacetime geometry
  3. direct relation between the spacetime share and statistical anisotropy
summary and prospects

# in our paper in preparation...
- more general setups for scalar type source of anisotropy
- other symmetry breaking including vector type ones

# future directions
- primordial gravitational waves in our approach
- test of homogeneity during inflation
Thank you!