New Constraints on Cosmic Polarization Rotation (CPR) including SPTpol B-Mode polarization observations

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Outline

- Introduction
- Equivalence Principles, Electromagnetism, & the Cosmic Connection – Axion, Dilaton and Skewon
- Photon sector
  - Nonbirefingence
  - axion, dilaton, skewon
- Observational constraints on CPR (Cosmic Polarization rotation)
- Summary
Let there be light!

Light abundant at 100 ps (~100 Gev): gamma rays

CMB produced
Light, WEP I, WEP II & EEP

- Light is abundant since 100ps (Electroweak phase trans.) or earlier after Big Bang
- Galileo EP (WEP I) for photon: the light trajectory is dependent only on the initial direction – no splitting & no retardation/no advancement, independent of polarization and frequency
- WEP II, no polarization rotation
- EEP, no amplification/no attenuation, no spectral distortion
The ISSUE
(Why Minkowski Metric? from gravity point of view)

- How to derive spacetime structure/the lightcone from classical, local and linear electrodynamics
- (i) the closure condition
- (ii) The Galileo weak equivalence principle
- (iii) The non-birefringence (vanishing double refraction) and “no amplification/dissipation” condition of astrophysical/cosmological electromagnetic wave propagation from observations
In the historical development, special relativity arose from the invariance of Maxwell equations under Lorentz transformation. In 1908, Minkowski [1] further put it into 4-dimensional geometric form with a metric invariant under Lorentz transformation. The use of metric as dynamical gravitational potential [2] and the employment of Einstein Equivalence Principle for coupling gravity to matter [3] are two important cornerstones to build general relativity. In putting Maxwell equations into a form compatible with general relativity, Einstein noticed that the equations can be formulated in a form independent of the metric gravitational potential in 1916 [5,6]. Weyl [7], Murnaghan [8], Kottler [9] and Cartan [10] & Schrödinger further developed and clarified this resourceful approach.
Metric-Free and Connection-Free

- Maxwell equations for macroscopic/spacetime electrodynamics in terms of independently measurable field strength $F_{kl} (E, B)$ and excitation (density with weight +1) $H^{ij} (D, H)$ do not need metric as primitive concept (See, e. g., Hehl and Obukhov [11]):

$$H^{ij}_{,j} = -4\pi J^i, \quad e^{ijkl} F_{jk,l} = 0, \quad (1)$$

- with $J^k$ the charge 4-current density and $e^{ijkl}$ the completely anti-symmetric tensor density of weight +1 with $e^{0123} = 1$. We use units with the light velocity $c$ equal to 1. To complete this set of equations, a constitutive relation is needed between the excitation and the field:

$$H^{ij} = \chi^{ijkl} F_{kl}. \quad (2)$$
Constitutive relation: \( H^{ij} = \chi^{ijkl} F_{kl} \).

Since both \( H^{ij} \) and \( F_{kl} \) are antisymmetric, \( \chi^{ijkl} \) must be antisymmetric in \( i \) and \( j \), and \( k \) and \( l \). Hence \( \chi^{ijkl} \) has 36 independent components.

- Principal part: 20 degrees of freedom
- Axion part: 1 degree of freedom
- Skewon part: 15 degrees of freedom
  (Hehl-Ohbukhov-Rubilar skewon 2002)

\[
\chi^{ijkl} = (P)\chi^{ijkl} + (Sk)\chi^{ijkl} + (A)\chi^{ijkl}, \quad (\chi^{ijkl} = -\chi^{ikjl} = -\chi^{ijlk})
\]

\[
(P)\chi^{ijkl} = (1/6)[2(\chi^{ijkl} + \chi^{klij}) - (\chi^{iklj} + \chi^{ljik}) - (\chi^{iljk} + \chi^{jkil})],
\]

\[
(A)\chi^{ijkl} = \chi^{ijkl} = \varphi e^{ijkl},
\]

\[
(Sk)\chi^{ijkl} = (1/2)(\chi^{ijkl} - \chi^{klij}),
\]
Related formulation in the photon sector: SME & SMS

The photon sector of the SME Lagrangian is given by $\mathcal{L}_{\text{photon total}}^{\text{total}} = -(1/4) F_{\mu\nu} F^{\mu\nu} - (1/4) (k_F)_{\kappa\lambda\mu\nu} F^{\kappa\lambda} F^{\mu\nu} + (1/2) (k_{AF})^\kappa \varepsilon_{\kappa\lambda\mu\nu} A^\lambda F^{\mu\nu}$ (equation (31) of [7]). The CPT-even part $-(1/4) (k_F)_{\kappa\lambda\mu\nu} F^{\kappa\lambda} F^{\mu\nu}$ has constant components $(k_F)_{\kappa\lambda\mu\nu}$ which correspond one-to-one to our $\chi$’s when specialized to constant values minus the special relativistic $\chi$ with the constant axion piece dropped, i.e. $(k_F)_{\kappa\lambda\mu\nu} = \chi^{\kappa\lambda\mu\nu} - (1/2) (\eta^{\kappa\mu} \eta^{\lambda\nu} - \eta^{\kappa\nu} \eta^{\lambda\mu})$. The CPT-odd part $(k_{AF})^\kappa$ also has constant components which correspond to the derivatives of axion $\phi,^\kappa$ when specialized to constant values.

- SMS in the photon sector due to Bo-Qiang Ma is different from both SME and $\chi^{\kappa\lambda\mu\nu}$-framework.
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Skewonless case: EM wave propagation

Since our galactic Newtonian potential $U$ is of the order of $10^{-6}$, we use weak field approximation in the $\chi$-$g$ framework. The vacuum Maxwell equation, derived from the Lagrangian (9), is

\[(\chi_{ijkl} \frac{\partial A_k}{\partial t}, \ell)_j = 0.\] (19)

Neglecting $\chi_{ijkl}^{p}$ in slowly varying field, (19) becomes

\[\chi_{ijkl} \frac{\partial A_k}{\partial t}, \ell_j = 0.\] (20)

For weak field, we assume

\[\chi_{ijkl} = \chi^{(0)}_{ijkl} + \chi^{(1)}_{ijkl},\] (21)

where

\[\chi^{(0)}_{ijkl} = \frac{1}{2} \eta_{ik} \eta_{jl} - \frac{1}{2} \eta_{il} \eta_{kj}.\] (22)

with $\eta_{ij}$ the Minkowski metric and $|\chi^{(1)}_{ijkl}| << 1$. 

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CPR including SPTpol B-Mode  
Ni
Dispersion relation and Nonbirefringence condition

B. Conditions for gravitational nonbirefringence — Photons propagate along a metric $H_{ik}$

Using eikonal approximation, we look for plane-wave solution propagating in the z-direction. Imposing radiation condition in the zeroth order and solving the dispersion relation for $\omega$, we obtain

$$\omega_{\pm} = k \left\{ 1 + \frac{1}{4} \left[ (K_1 + K_2)^2 + \sqrt{(K_1 - K_2)^2 + 4K^2} \right] \right\}$$  \hspace{1cm} (23)

where

$$K_1 = \chi^{(1)1010} - 2\chi^{(1)1013} + \chi^{(1)1313},$$
$$K_2 = \chi^{(1)2020} - 2\chi^{(1)2023} + \chi^{(1)2323},$$
$$K = \chi^{(1)1020} - \chi^{(1)1023} - \chi^{(1)1320} + \chi^{(1)1323}.$$  \hspace{1cm} (24)

Photons with two different polarizations propagate with different speed $v_{\pm} = \frac{\omega_{\pm}}{k}$ and would split in 4-dimensional spacetime. The conditions for no splitting (no retardation) is $\omega_+ = \omega_-$, i.e.

$$K_1 = K_2, \quad K = 0.$$  \hspace{1cm} (25)

(25) gives two constraints on $\chi^{(1)}$'s.
The conditions for no splitting (no retardation) of electromagnetic waves propagating in every direction give the following ten constraints on $\chi^{(1)}$'s:

\[
\begin{align*}
\chi^{(1)}_{1220} &= \chi^{(1)}_{1330}, \\
\chi^{(1)}_{2330} &= \chi^{(1)}_{2110}, \\
\chi^{(1)}_{3110} &= \chi^{(1)}_{3220}, \\
\chi^{(1)}_{1020} &= -\chi^{(1)}_{1323}, \\
\chi^{(1)}_{2030} &= -\chi^{(1)}_{2131}, \\
\chi^{(1)}_{3010} &= -\chi^{(1)}_{3212}, \\
\chi^{(1)}_{1320} &= -\chi^{(1)}_{1230}, \\
\chi^{(1)}_{1320} &= -\chi^{(1)}_{2310}.
\end{align*}
\]

\[
\begin{align*}
\chi^{(1)}_{1010} + \chi^{(1)}_{1313} &= \chi^{(1)}_{2020} + \chi^{(1)}_{2323}, \\
\chi^{(1)}_{1010} + \chi^{(1)}_{1212} &= \chi^{(1)}_{3030} + \chi^{(1)}_{3232}.
\end{align*}
\]

\[\text{where } H^{(1)ij}, \psi, \text{ and } \phi \text{ as}
\]

\[
\begin{align*}
H^{(1)10} &= H^{(1)01} = -2\chi^{(1)}_{1220}, \\
H^{(1)20} &= H^{(1)02} = -2\chi^{(1)}_{2330}, \\
H^{(1)30} &= H^{(1)03} = -2\chi^{(1)}_{3110}, \\
H^{(1)12} &= H^{(1)21} = -2\chi^{(1)}_{1020}, \\
H^{(1)23} &= H^{(1)32} = -2\chi^{(1)}_{2030}, \\
H^{(1)31} &= H^{(1)13} = -2\chi^{(1)}_{3010}, \\
H^{(1)11} &= 2\chi^{(1)}_{1020} + 2\chi^{(1)}_{2131} - H^{(1)00}, \\
H^{(1)22} &= 2\chi^{(1)}_{3030} + 2\chi^{(1)}_{3232} - H^{(1)00}, \\
H^{(1)33} &= 2\chi^{(1)}_{1010} + 2\chi^{(1)}_{1313} - H^{(1)00},
\end{align*}
\]

\[
\begin{align*}
\psi &= 1 + 2\chi^{(1)}_{1212} + \frac{1}{2\mu_0}(H^{(1)00} - H^{(1)11} - H^{(1)22}), \\
\phi &= \chi^{(1)0123}.
\end{align*}
\]
Note that in these definitions $H^{(1)00}$ is not defined and free. It is straightforward to show that if the ten constraints (26) are satisfied then $\chi$ can be written to first-order in $\chi^{(1)}$'s in the form

$$
\chi^{ijkl} = (-H)^{\frac{1}{2}} \left( \frac{1}{2} H^{ik} H^{jl} - \frac{1}{2} H^{il} H^{kj} \right) \psi + \phi e^{ijkl},
$$

(28)

where

$$
H^{ij} = \eta^{ij} + H^{(1)ij},
$$

$$
H = \text{det}(H^{ij}),
$$

$$
H^{ij} H^{jk} = \delta^k_i,
$$

and

$$
e^{ijkl} = \begin{cases} 
1, & \text{if } (ijkl) \text{ is an even permutation of } (0123), \\
-1, & \text{if } (ijkl) \text{ is an odd permutation of } (0123), \\
0, & \text{otherwise.}
\end{cases}
$$

(30)
Table I. Constraints on the spacetime constitutive tensor $\chi^{ijkl}$ and construction of the spacetime structure (metric + axion field $\phi$ + dilaton field $\psi$) from experiments/observations in the skewonless case

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Constraints</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pulsar Signal Propagation</td>
<td>$\chi^{ijkl} \rightarrow \frac{1}{2} (-h)^{1/2} [h^{ik} h^{jl} - h^{il} h^{kj}] \psi + \phi \epsilon^{ijkl}$</td>
<td>$10^{-16}$</td>
</tr>
<tr>
<td>Radio Galaxy Observation</td>
<td></td>
<td>$10^{-32}$</td>
</tr>
<tr>
<td>Gamma Ray Burst (GRB)</td>
<td></td>
<td>$10^{-38}$</td>
</tr>
<tr>
<td>CMB Spectrum Measurement</td>
<td>$\psi \rightarrow 1$</td>
<td>$8 \times 10^{-4}$</td>
</tr>
<tr>
<td>Cosmic Polarization Rotation Experiment</td>
<td>$\phi - \phi_0 (\equiv \alpha) \rightarrow 0$</td>
<td>$</td>
</tr>
<tr>
<td>Eötvös-Dicke-Braginsky Experiments</td>
<td>$\psi \rightarrow 1$</td>
<td>$10^{-10}$</td>
</tr>
<tr>
<td></td>
<td>$h_{00} \rightarrow g_{00}$</td>
<td>$10^{-6}$</td>
</tr>
<tr>
<td>Vessot-Levine Redshift Experiment</td>
<td>$h_{00} \rightarrow g_{00}$</td>
<td>$1.4 \times 10^{-4}$</td>
</tr>
<tr>
<td>Hughes-Drever-type Experiments</td>
<td>$h_{ij} \rightarrow g_{ij}$</td>
<td>$10^{-18}$</td>
</tr>
<tr>
<td></td>
<td>$h_{0i} \rightarrow g_{0i}$</td>
<td>$10^{-13}$ - $10^{-14}$</td>
</tr>
<tr>
<td></td>
<td>$h_{00} \rightarrow g_{00}$</td>
<td>$10^{-10}$</td>
</tr>
</tbody>
</table>
Three approaches to Axions/Pseudoscalar-photon interactions

- Top down approach – string theory
- Bottom up approach – QCD axion
- Phenomenological approach -- gravitation

<table>
<thead>
<tr>
<th>Term</th>
<th>Dimension</th>
<th>Reference</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^{\alpha\beta\gamma} A_\alpha F_{\beta\gamma}$</td>
<td>3</td>
<td>Chern–Simons (1974)</td>
<td>Intergrand for topological invariant</td>
</tr>
<tr>
<td>$e^{ijkl} \varphi F_{ij} F_{kl}$</td>
<td>4</td>
<td>Ni (1973, 1974, 1977)</td>
<td>Pseudoscalar-photon coupling</td>
</tr>
<tr>
<td>$e^{ijkl} \varphi F_{ij}^{\mathrm{QCD}} F_{kl}^{\mathrm{QCD}}$</td>
<td>4</td>
<td>Peccei–Quinn (1977)</td>
<td>Pseudoscalar-gluon coupling</td>
</tr>
<tr>
<td>$e^{ijkl} V_i A_j F_{kl}$</td>
<td>4</td>
<td>Weinberg (1978) Wilczek (1978)</td>
<td>External constant vector coupling</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Carroll–Field–Jackiw (1990)</td>
<td></td>
</tr>
</tbody>
</table>
$\varphi$: Pseudoscalar field or pseudoscalar function of gravitational or relevant fields

\[ \mathcal{L}_{\text{int}} \sim p_{\mu} A_{\nu} \tilde{F}^{\mu \nu} , \]

\[ \approx \xi \varphi_{,\mu} A_{\nu} F^{\sim \mu \nu} \]

\[ \approx \xi (1/2) \varphi F_{\mu \nu} F^{\sim \mu \nu} \]

(Mod Divergence)
Galileo EP \rightarrow Electromagnetism: Charged particles and photons

Special Relativity

\[ L_I = -\left(\frac{1}{16\pi}\right)\eta^{ijl}\eta^{jil} F_{ij} F_{kl} - A_k j^k (-g)^{1/2} - \sum_i m_i \frac{ds_i}{dt} \delta(x - x_I) \]

\[ \chi - g \] framework

\[ L_I = -\left(\frac{1}{16\pi}\right)\chi^{ijkl} F_{ij} F_{kl} - A_k j^k (-g)^{1/2} - \sum_i m_i \frac{ds_i}{dt} \delta(x - x_I) \]

Galileo EP constrains \[ \chi : \]

\[ \chi^{ijkl} = (-g)^{1/2} \left[ \frac{1}{2} g^{ik} g^{jl} - \frac{1}{2} g^{il} g^{kj} + \eta \phi \epsilon^{ijkl} \right] \]

(Pseudo)scalar-Photon Interaction
The birefringence condition in Table I – historical background

netic wave propagation [29–32]. We constructed the relation (8) in the weak-violation/weak-field approximation of the Einstein Equivalence Principle (EEP) and applied to pulsar observations in 1981 [29–31]; Haugan and Kauffmann [32] reconstructed the relation (8) and applied to radio galaxy observations in 1995. After the cornerstone work of Lämmerzahl and Hehl [33], Favaro and Bergamin [34] finally proved the relation (8) without assuming weak-field approximation (see also Dahl [35]). Polarization measurements of
Results -- cosmic propagation of dilaton field and axion field

derived that the amplitude and phase factor of propagation in the cosmic dilaton and cosmic axion field is changed by

\[
\left( \frac{\psi(P_1)}{\psi(P_2)} \right)^{1/2} \exp[i kz - i kt \pm (-i)(\varphi(P_1) - \varphi(P_2))t].
\]

**Constraint from CMB spectrum**

\[ |\Delta \psi|/\psi \leq 4(0.0006/2.7255) \approx 8 \times 10^{-4}. \]  \hspace{1cm} (33)

Direct fitting to the CMB data with the addition of the scale factor \( \psi(P_1)/\psi(P_2) \) would give a more accurate value.
CMB observations

7 orders or more improvement in amplitude, 15 orders improvement in power since 1965

- **1948** Gamow – hot big bang theory; Alpher & Hermann – about 5 K CMB
- **Dicke** -- oscillating (recycling) universe: entropy $\rightarrow$ CMB
- **1965** Penzias-Wilson excess antenna temperature at 4.08 GHz $3.5 \pm 1$ K $2.5 \rightarrow 4.5$ (CMB temperature measurement)
- Precision to $10^{-(3-4)}$ $\rightarrow$ dipolar (earth) velocity measurement to $10^{-(5-6)}$ **1992** COBE anisotropy meas. $\rightarrow$ acoustic osc.
- **2002** Polarization measurement (DASI)
- **2013** Lensing B-mode polarization (SPTpol)
- **2014** POLARBEAR, BICEP2 and PLANCK (lensing & dust B-mode)
Three processes can produce CMB B-mode polarization observed

- (i) gravitational lensing from E-mode polarization (Zaldarriaga & Seljak 1997),
- (ii) local quadrupole anisotropies in the CMB within the last scattering region by large scale GWs (Polnarev 1985)
- (iii) cosmic polarization rotation (CPR) due to pseudoscalar-photon interaction (Ni 1973; for a review, see Ni 2010).
  (The CPR has also been called Cosmological Birefringence)
- (iv) Dust alignment

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Cosmic Polarization Rotation

\[ \alpha = \varphi(P_2) - \varphi(R) = \left[ \varphi(P_2) - \varphi(R) \right]_{\text{mean}} + \delta \varphi(R) = \langle \alpha \rangle + \delta \alpha, \]

\[ \alpha^2 \equiv \langle \alpha^2 \rangle = \left( \left[ \varphi(P_2) - \varphi(R) \right]_{\text{mean}} \right)^2 + \delta \varphi^2(R) = \langle \alpha \rangle^2 + \delta \alpha^2, \]

\[ C_{l,\text{BB,obs}} = C_{l,\text{BB}} \cos^2(2\alpha) + C_{l,\text{EE}} \sin^2(2\alpha), \quad (6a) \]

\[ C_{l,\text{EB,obs}} = (C_{l,\text{EE}} - C_{l,\text{BB}}) \sin(2\alpha) \cos(2\alpha), \quad (6b) \]

\[ C_{l,\text{TB,obs}} = -\sin(2\alpha) C_{l,\text{TE}}, \quad (6c) \]

\[ C_{l,\text{EE,obs}} = C_{l,\text{BB}} \sin^2(2\alpha) + C_{l,\text{EE}} \cos^2(2\alpha), \quad (6d) \]

\[ C_{l,\text{TE,obs}} = \cos(2\alpha) C_{l,\text{TE}}, \quad (6e) \]

\[ C_{l,\text{BB,obs}} = C_{l,\text{BB}} \langle \cos^2(2\alpha) \rangle + C_{l,\text{EE}} \langle \sin^2(2\alpha) \rangle \]
\[ \approx C_{l,\text{BB}} \left( 1 - 4\langle \alpha^2 \rangle \right) + 4C_{l,\text{EE}} \langle \alpha^2 \rangle \]
\[ \approx C_{l,\text{BB}} + 4\langle \alpha^2 \rangle C_{l,\text{EE}} \]
\[ = C_{l,\text{BB}} + 4\alpha^2 C_{l,\text{EE}}, \]

\[ C_{l,\text{EB,obs}} \approx (C_{l,\text{EE}} - C_{l,\text{BB}}) \langle \sin(2\alpha) \cos(2\alpha) \rangle \]
\[ \approx 2 \left( \langle \alpha \rangle - \frac{8}{3} \langle \alpha^3 \rangle \right) (C_{l,\text{EE}} - C_{l,\text{BB}}) \]
\[ \approx 2 \langle \alpha \rangle C_{l,\text{EE}}, \]

\[ C_{l,\text{TB,obs}} = -\langle \sin(2\alpha) \rangle C_{l,\text{TE}} \approx -2 \langle \alpha \rangle C_{l,\text{TE}}, \]

\[ C_{l,\text{EE,obs}} \approx C_{l,\text{BB}} \langle \sin^2(2\alpha) \rangle + C_{l,\text{EE}} \langle \cos^2(2\alpha) \rangle \]
\[ \approx C_{l,\text{EE}}, \]

\[ C_{l,\text{TE,obs}} = \langle \cos(2\alpha) \rangle C_{l,\text{TE}} \approx \left( 1 - 2 \langle \alpha^2 \rangle \right) C_{l,\text{TE}} \]
\[ \approx C_{l,\text{TE}}. \]
The current combined evidence so far is consistent with a null CPR and upper limits are of the order of 1 degree with fluctuations and mean is constrained to about 1 degrees.
BB power spectrum from SPTpol, ACTpol, BICEP2/Keck, and POLARBEAR. The solid gray line shows the expected lensed BB spectrum from the Planck+lensing+WP+highL best-fit model. The dotted line shows the nominal 150 GHz BB power spectrum of Galactic dust emission derived from an analysis of polarized dust emission in the BICEP2/Keck field using Planck data. The dash-dotted line shows the sum of the lensed BB power and dust BB power.
Data

Collaborators:
Pan, Baccigalupi, Xia, Xu, Ni and Di Serego Alighieri,

Emode:1502.01589(fig.3)(PLANCK)
Lensing--B:1502.01591v1(fig.4)(PLANCK)
Gravity--Bmode:1403.3985v2(fig.14)(BICEP2)
SPT:1503.02315v1; POLARBEAR:1403.2369v1
ACTPol:1405.5524v2; dust:1409.5738(fig.9)(PLANCK)
\[ r \approx -0.05 \pm 0.1 \]

\[ \langle \delta \alpha^2 \rangle = 100 \pm 500 \text{ mrad}^2 \]

- Fluctuation amplitude bound: 17 mrad (1 degree)
Summary

- Pseudoscalar-photon (axion) interactions arisen from the study of EP, QCD, string theory and pre-metric EM
- CPR is a way to probe pseudoscalar-photon interaction and a possible way to probe inflation dynamics,
- New CPR constraints from B-mode are summarized
- Good calibration is a must for measuring mean CPR
- CPR is a means to test EEP, to look into the parity pattern of CMB or to find new physics
- From the empirical route to construct spacetime structure, axion, dilaton and type II skewon are possibilities which could be explored further
One Hundred Years of General Relativity

From Genesis and Empirical Foundations to Gravitational Waves, Cosmology and Quantum Gravity

The aim of this two-volume title is to give a comprehensive review of one hundred years of development of general relativity and its scientific influences. This unique title provides a broad introduction and review to the fascinating and profound subject of general relativity, its historical development, its important theoretical consequences, gravitational wave detection and applications to astrophysics and cosmology. The series focuses on five aspects of the theory:

- Genesis, Solutions and Energy
- Empirical Foundations
- Gravitational Waves
- Cosmology
- Quantum Gravity

The first three topics are covered in Volume 1 and the remaining two are covered in Volume 2. While this is a two-volume title, it is designed so that each volume can be a standalone reference volume for the related topic.

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Thank you