Widening the Axion Field Range from Mixings

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Based on the work with
Gary Shiu and Wieland Staessens
Introduction-Axions

- Axions: **CP-odd real scalars with continuous shift symmetry**; first introduced to solve strong CP problem

\[ a(x) \rightarrow a(x) + \delta(x) \]

- Rich applications in particle physics and cosmology
Introduction-Axions

- Continuous shift symmetry → Derivative couplings
- Possible light dark matter (DM) and inflaton?

- Continuous shift symmetry is exact at most perturbatively.

- Break continuous shift symmetry down to discrete shift symmetry by non-perturbative instantons

- Axion mass given by NP effects

\[ m_a^2 = \frac{\partial^2 V_{\text{eff}}(a)}{\partial a^2} = \frac{\Lambda^4}{f_a^2}, \quad \Lambda = \text{NP scale} \]

Axion decay constant: periodicity, field range
Introduction-Scenario 1: Axions as DM

$10^9 \text{GeV} \leq f_a^{DM} \leq 10^{12} \text{GeV}$

(From PDG 2013)
Introduction-Scenario 2: Axions as Inflaton

- **Natural inflation**: periodic inflaton potential
  
  \[
  V(a) = \Lambda^4 [1 - \cos \frac{a}{f_a}]
  \]

- Slow roll:
  
  \[
  \epsilon \equiv \frac{M_P^2}{2} \left( \frac{V'}{V} \right)^2 \ll 1, \quad |\eta| \equiv M_P^2 \left| \frac{V''}{V} \right| \ll 1 \Rightarrow f_a > M_P
  \]

Large field inflation
Introduction-Stringy Axions

• Arise from dimensional reduction of higher forms (bulk fields)

• Shift symmetry as remnant of gauge symmetry of the forms in extra dimensions

• Constraint from string compactifications (no axion mixing):
  \[ f_a \lesssim \frac{g^2}{8\pi^2} M_P \]

  Sub-Planckian!

  typically a little lower than string scale

  Not in ADM or slow-roll inflation range
Introduction-Scenario 2: Axions as Inflaton

To have large field inflation:

- N-flation (Dimopoulos-Kachru-McGreevy-Wacker (2005))
- Axion monodromy
- ?

Silverstein-Westphal (2008), McAllister-Silverstein-Westphal (2008),
Kaloper-(Lawrence)-(Sorbo) (2008/11/14), Marchesano-Shiu-Uranga (2014),
Hebecker-Kraus-Witkowski (2014), Blumenhagen-Herschmann-Plauschinn (2014),
Introduction-Motivation

- Widen the axion window without violating the string axion bound
- Apply both to field and string theories
Axion Mixings

• General lagrangian

\[ S^{\text{eff}} = - \int \left[ \sum_{\alpha, \beta=1}^{M} f_{\alpha \beta} F^{\alpha} \wedge \star_{4} F^{\beta} + \sum_{A=1}^{P} \frac{1}{g_{A}^{2}} \text{Tr} \left( G^{A} \wedge \star_{4} G^{A} \right) - \frac{1}{2} \sum_{i,j=1}^{N} G_{ij} \left( da^{i} - \sum_{\alpha=1}^{M} k_{\alpha}^{i} A^{\alpha} \right) \wedge \star_{4} \left( da^{j} - \sum_{\beta=1}^{M} k_{\beta}^{j} A^{\beta} \right) \right] . \]

- \frac{1}{8\pi^{2}} \sum_{A=1}^{P} \left( \sum_{i=1}^{N} r_{iA} a^{i} \right) \text{Tr} \left( G^{A} \wedge G^{A} \right) - \frac{1}{8\pi^{2}} \sum_{\alpha, \beta=1}^{M} \left( \sum_{i=1}^{N} s_{i\alpha \beta} a^{i} \right) F^{\alpha} \wedge F^{\beta} + \ldots \]

Abelian

Non-Abelian

Winding number, anomalous coefficient etc

“Axion charge”

• Basis and normalization

Axion charge

Fermion and/or GCS terms to ensure the gauge invariance

Compactness of U(1)’s

\[ U(1)^{\alpha} : A^{\alpha} \rightarrow A^{\alpha} + d\eta^{\alpha}, \quad a^{i} \rightarrow a^{i} + k_{\alpha}^{i} \eta^{\alpha} \]
Axion Mixings

\[ S_{\text{eff}} = -\int \left[ \sum_{\alpha, \beta=1}^{M} f_{\alpha\beta} F^{\alpha} \wedge \ast_4 F^{\beta} + \sum_{A=1}^{P} \frac{1}{g_A^2} \text{Tr}(G^A \wedge \ast_4 G^A) - \frac{1}{2} \sum_{i,j=1}^{N} G_{ij}(da^i - \sum_{\alpha=1}^{M} k^i_\alpha A^\alpha) \wedge \ast_4 (da^j - \sum_{\beta=1}^{M} k^j_\beta A^\beta) \right] \]

- 3 types of axion mixings:

- Mixing due to **non-diagonal metric** (referred as kinetic mixing or metric mixing)

- Mixing due to **Stueckelberg couplings** (referred as Stueckelberg mixing)

- Mixing due to mismatch between kinetic eigenstates and mass eigenstates
Axion-Mixings (minimal)

\[ S^{N=2}_{\text{axion}} = \int \left[ -\frac{1}{2} \sum_{i,j=1}^{2} g_{ij} (d a^i - k^i A) \wedge \star_4 (d a^j - k^j A) + \frac{1}{8\pi^2} (r_1 a^1 + r_2 a^2) \text{Tr} G \wedge G + ... \right] \]

- Minimal setup: 2 axions, 1 Stueckelberg U(1) and 1 non-Abelian gauge group
Axion Mixings (minimal)

W/O Stueckelberg coupling

\[ S_{\text{kin}}^{\text{axion}} = - \int \frac{1}{2} \sum_{i,j=1}^{2} g_{ij}(\sigma) \, da^i \wedge *_4 da^j. \]

flat direction whose continuous shift symmetry is not broken has no potential
axionic couplings scale inversely with the axion decay constant.

can’t have super-Planckian excursion

have to invoke additional physical effects

1. Monodromy effects. \( V(\xi) \sim \xi^p \)

Through torsional monodromy effects \( (p=2) \), or flux induced monodromies \( (p>2) \)

2. Alignment effects.

Add a second non-Abelian gauge group anomalously coupling to both axions

3. Abelian \( U(1) \) gauge symmetry. Will study this in detail
Axion Mixings (minimal)

In the presence of Stueckelberg coupling

Kinetic eigenstates:

- **Axion eaten by U(1):** part of the massive U(1) in appropriate gauge

- **Uneaten axion:** obtain effective potential for inflation by integrating out massive U(1), chiral fermions and non-Abelian gauge field

\[ V_{\text{eff}}(\xi) = \Lambda^4 \left[ 1 - \cos \left( \frac{\xi}{f_\xi} \right) \right] \]

\[ f_\xi = \frac{\cos \frac{\theta}{2} (\lambda_+ k + r_2 + \lambda_- k - r_1) + \sin \frac{\theta}{2} (\lambda_- k - r_2 - \lambda_+ k + r_1)}{\sqrt{\lambda_+ \lambda_- M_{\text{st}}}} \]

**mixing angle:** amount of metric mixing

Single field potential for the remaining axion
Axion Mixings (minimal)

• Explore axion field range - intermediate kinetic mixing

\[ f_\xi = \frac{\sqrt{\lambda^+\lambda^- M_{st}}}{\cos \frac{\theta}{2} (\lambda^+_k + r_2 + \lambda^-_k - r_1) + \sin \frac{\theta}{2} (\lambda^-_k - r_2 - \lambda^+_k + r_1)} \]

White region \( f_\xi > 10^2 \sqrt{g_{11}} \)

FIG. 1. Contour plots of decay constant \( f_\xi(\theta, \varepsilon) \) for \( 2r_1 = 2r_2 = 2k^1 = k^2 \) (left) and \( r_1 = 2r_2 = k^1 = 2k^2 \) (right). The \( f_\xi \)-values range from small (purple) to large (red) following the rainbow contour colors. Unphysical regions with complex \( f_\xi \) are located in the black band.
Axion Mixings (minimal)

Other axion large field inflation scenarios:


  \[ f_{\text{eff}} \propto \sqrt{N}, \quad N = \text{number of axions} = \text{number of non-Abelian gauge instantons} \]

  \[ f_{\text{eff}} \propto \sqrt{N!}n^N, \quad \text{N>2 version} \]

  \( N = \text{number of axions} = \text{number of non-Abelian gauge instantons}, \)

  \( n \in \mathbb{Z} \) anomalous coefficients

• **Aligned natural inflation**: J. E. Kim, H. P. Nilles and M. Peloso, JCAP 0501, 005 (2005)

  \( K. \text{ Choi, H. Kim and S. Yun, Phys. Rev. D} \ 90, \text{ no. 2, 023545 (2014)} \)

  \( \delta M_{Pl}^2 \sim N \)

  Both tied to number of DOF

• Planck mass renormalization

Spoil the slow roll condition?
Axion Mixings (minimal)

Our approach to get a super-Planckian inflation:

• Axion field range enhancement *not tied to the number of DOF*

• Relying on tuning *continuous parameters* in moduli space, not much on discrete parameters

• Minimal (fewer DOF) setup works, which has less severe Planck mass renormalization issue.
Axion Mixings (ADM)

**Minimal setup**

\[ f_\xi = \frac{\sqrt{\lambda_+ \lambda_-} M_{st}}{\cos \frac{\theta}{2} (\lambda_+ k r_2 + \lambda_- k r_1) + \sin \frac{\theta}{2} (\lambda_- k r_2 - \lambda_+ k r_1)} \]

- **To lower axion decay constants**: by tuning continuous parameters and choosing appropriate discrete parameters
  
  e.g. assuming \( \varepsilon = 1, \theta = \pi, r_1 = -r_2, k^1 = k^2 \)

  \[ f_\xi = \frac{\sqrt{g_{11}^2 - g_{12}^2}}{\sqrt{2r_2} \sqrt{g_{11} + g_{12}}} = \frac{\sqrt{g_{11}} \sqrt{1 - \varepsilon^2}}{\sqrt{r_2}} \]  
  \[ r_2 \sim \mathcal{O}(1 - 10), \sqrt{g_{11}} \sim \mathcal{O}(10^{15} - 10^{17}) \text{GeV}, \text{ large mixing } 1 - \rho^2 \sim \mathcal{O}(10^{-4} - 10^{-8}) \]

**Non-minimal setup**

For large N, eigenvalue repulsion

- Not require the intrinsic axion field range to be too small
String Theory Embedding

- Approaching Planck scale: **physics is sensitive to Planck scale**
- Need a full theory description → String Theory
String Theory Embedding

- **Closed string axions (Ramond-Ramond axions):**
  dimensional reduction of p-forms

\[ a^i \equiv \frac{1}{2\pi} \int_{\gamma_i} C_p \]

- Number of RR axions: Hodge numbers
String Theory Embedding

• **Axion metric** $G_{i,j}$: moduli field dependent (stabilization of the saxion, e.g. volume of the internal cycle wrapped by $p$-forms)

• **U(1)**: Abelian factor of **world volume gauge group** $U(N)$ for a stack of $N$ D-branes

• **Gauge Dp-branes**: extending along Minkowski space and wrapping internal $(p-3)$-cycles

• **Wrapping numbers** $→$ Discrete parameters $k^i_\alpha$, $r_{i\alpha\beta}$
String Theory Embedding

• Instantons: **Gauge instantons** or **stringy instantons**

• **Gauge instanton**: on world volume of Dp-brane extending Minkowski space and wrapping internal (p-3)-cycles

  (Graph from 1003.4867)

• **Stringy instanton**: e.g. E(p-1)-instanton (D(p-1)-brane wrapping internal p-cycles, pointlike in 4d).

  \[ e^{-S_{\text{gauge}}} = e^{-|I_n| \left( \frac{8\pi^2}{g_Y^2} + i \theta \right)} \]

• **Instantons contribute only when fermonic zero modes are saturated.**
String Theory Embedding

- Introducing orientifold planes
- Consistency: tadpole cancellation
- Chiral spectrum: bi-fundamentals at intersections
- Green-Schwarz (GS) mechanism to ensure gauge invariance.

\[ U(1) + U(1) = 0 \]
String Theory Embedding

Explicit D6-brane model to realize super-Planckian excursion

2 stacks of D6’s wrapping *non-factorisable* 3-cycles

Internal space \( T^2_{(1)} \times T^2_{(2)} \times T^2_{(3)} \).

Other explicit D-brane models to realize super-Planckian excursion

Factorizable D6-branes in Type IIA on Toroidal Orientifolds

Not all the RR tadpoles and the related gauge anomalies vanish. Need to be remediated.

D7-branes in Type IIB on Swiss-Cheese Calabi-Yau’s

Consistency (tadpole condition) to be checked in the future
String Theory Embedding

Other related questions

- Moduli stabilization  many attempts
  recent  Blumenhagen-Fuchs-Herschmann: 1510.04058 [hep-th]
- Higher instanton corrections: not ruin the enhancement for the axion decay constant; but add modulations on the potential
  Kappa-Nilles-Winkler:1511.05560 [hep-th]
  + all stringy instanton corrections: super-Planckian excursion compatible with WGC (mild)
  Choi-Kim: 1511.07201 [hep-th]
  Not all instantons participate in alignment; some generate small modulations instead, consistent with WGC (mild).
Conclusion

Kinetic and Stueckelberg mixings can enlarge axion field range

• Applies to both field theory and string theory models with limited intrinsic axion field ranges.

• Axion field range enhancement is mainly through **tuning continuous parameters** (with discrete parameters properly chosen).

• **To lower the axion decay constant (for ADM): no large compact cycles** needed (alleviate the requirement for intermediate string scale). **Allow high fundamental string scale -> new possibilities to detect string axions** through astrophysical, cosmological and laboratory ways

• **To increase the axion decay constant (for inflation): not require large DOF** (-> minimal setup) and mainly depends on tuning continuous parameters.

• Model-dependent **higher instanton corrections: deviation from a cosine potential -> measurable effect on the inflationary perturbation spectrum**. Quantifying such deviation needs understanding of UV completion of inflation and moduli stabilization.
Thank you!
Backup
Stueckelberg Mechanism

4d QFT with a massive U(1) and an axion

\[ \mathcal{L}_0 = -\frac{1}{4g^2}(F_{\mu\nu})^2 - \frac{1}{2}(cA_\mu + \partial_\mu a)^2 \]

Add a “vanishing” term

[Image]

E.O.M for \( a \)

\[ \partial_\mu (cA^\mu + \partial_\mu a) + \frac{1}{6}\epsilon^{\mu\nu\rho\sigma} \partial_\mu H_{\nu\rho\sigma} = 0. \]

Up to a total derivative

[Image]

Eliminating the axion field

\[ \mathcal{L}_0 = -\frac{1}{4g^2}(F_{\mu\nu})^2 - \frac{1}{12}(H_{\mu\nu\rho})^2 + \frac{c}{4}\epsilon^{\mu\nu\rho\sigma} B_{\mu\nu} F_{\rho\sigma}. \]

Terms read from string compactifications
Eta Problem

- Eta parameter
  \[ |\eta| \equiv M_{pl}^2 \frac{|V''|}{V} \]

- Quantum corrections tend to drive the scalar mass to the cutoff
  \[ \Delta m^2 \sim \Lambda^2 \]

- Consistency of EFT:
  \[ \Lambda > H \]

- \( \Delta \eta \sim \frac{\Lambda^2}{H^2} \gtrsim 1 \)

- Snow-roll unnatural

- In fact, eta parameter is sensitive to dim-6 operator correction
  \[ O_6 = eV_i(\phi) \frac{\phi^2}{\Lambda^2} \quad \Delta \eta \approx 2c \left( \frac{M_{pl}}{\Lambda} \right)^2 \]
RR Tadpole Cancellation

Emission of a closed string out of a vacuum -> a tadpole

10d Poincare invariance: the source fields can be graviton and dilaton in NS-NS sector, and 10-form in RR sector

RR 10-form is not propagating (its field strength is 11d) in 10d -> can’t appear in physical spectrum of spacetime particles

But it appears in 10d action as a source term

Need cancellation

Bonus: RR tadpole cancellation leads to mixed gauge anomaly cancellation
Chiral Spectrum at D-brane Intersections in Type IIA

• Orientifold planes O6 are introduced to ensure RR tadpole (e.g. emission of a closed string out of vacuum, reflecting uncancelled gravitational/gauge anomalies) cancellation. O6 has -4 units of D6-brane charge.

• Branes are mapped to image branes (denoted by prime) under orientifold action.

• For intersecting D-brane stacks a and b, with brane numbers $N_a$ and $N_b$, open strings stretching between 2 stacks arise as bi-fundamentals.

Spectrum

- $aa + a'a'$: Contains $U(N_a)$ gauge bosons (plus possible additional adjoint fields).
- $ab + ba + b'a' + a'b'$: Gives $I_{ab}$ chiral fermions in the representation $(N_a, \bar{N}_b)$, plus light (possibly massless) scalars.
- $ab' + b'a + ba' + a'b$: Contains $I_{ab'}$ chiral fermions in the representation $(N_a, N_b)$, plus light (possibly massless) scalars.
- $aa' + a'a'$: Contains $n_{\text{sym},a}$ 4d chiral fermions in the representation $\bigtimes_a$ and $n_{\text{asym},a}$ in the $\bigoplus_a$, with $n_{\text{sym},a} = \frac{1}{2}(I_{aa'} - I_{a,06})$, $n_{\text{asym},a} = \frac{1}{2}(I_{aa'} + I_{a,06})$. 
Factorisable Tori

$T^2 \times T^2 \times T^2$

A stack $A$ of D-branes wraps the $i$-th torus $(T^2)^i$ ($i = 1, 2 \text{ or } 3$), with wrapping numbers $(n^i_A, m^i_A)$. 

$[a_i]$ and $[b_i]$ be the even and odd 1-cycles on $(T^2)^i$ with respect to the antiholomorphic involutions $y^i \to -y^i$.

A basis of even and odd 3-cycles:

- $[\alpha_0] = [a_1][a_2][a_3]$, 
- $[\beta_0] = [b_1][b_2][b_3]$, 
- $[\alpha_1] = [a_1][b_2][b_3]$, 
- $[\beta_1] = [b_1][a_2][a_3]$, 
- $[\alpha_2] = [b_1][a_2][b_3]$, 
- $[\beta_2] = [a_1][b_2][a_3]$, 
- $[\alpha_3] = [b_1][b_2][a_3]$, 
- $[\beta_3] = [a_1][a_2][b_3]$,

$[\alpha_i] \cdot [\beta_j] = \delta_{ij} = -[\beta_j] \cdot [\alpha_i]$, 

$[\alpha_i] \cdot [\alpha_j] = [\beta_i] \cdot [\beta_j] = 0$.

3-cycles $[\pi_A]$, can be expanded in terms of the basis $[\pi_A] = S_A^i [\alpha_i] + R_A^i [\beta_i]$.

$S_A^0 = n_A^1 n_A^2 n_A^3$, 
$S_A^1 = n_A^1 m_A^2 m_A^3$, 
$S_A^2 = m_A^1 n_A^2 m_A^3$, 
$S_A^3 = m_A^1 m_A^2 n_A^3$. 

$R_A^0 = m_A^1 m_A^2 m_A^3$, 
$R_A^1 = m_A^1 n_A^2 n_A^3$, 
$R_A^2 = n_A^1 m_A^2 n_A^3$, 
$R_A^3 = n_A^1 n_A^2 m_A^3$. 

Rectangular and Tilted Tori

Notice that $[a_3]$ does not represent a closed one-cycle on $T^2_3$, only linear combinations such as $2[a_3]$ or $[a_3] + \frac{1}{2}[b_3]$ do.

$y^i \rightarrow -y^i$

$(n^i_A, m^i_A) \rightarrow (n^i_A, -m^i_A)$

$m^i_A = \tilde{m}^i_A + \frac{n^i_A}{2}$ (tilted)

$\tilde{m}^i_A$ and $n^i_A$ being integers.

Half integer wrapping number
Rectangular and Tilted Tori

gcd \( (n, m) = 1 \)  
\[ T^2 \]

\((n, m)\) represents closed line (oriented) on \( T^2 \).

E.g. \((n, m) = (2, 3)\) w/ \( T^2 \) defined by \( z \sim z + Rx \) \( z \sim z + i Ry \).

Orientifold action: \( y \rightarrow -y \).

\( \overrightarrow{OA} = \overrightarrow{OB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{D'E} \)
Rectangular and Tilted Tori

eq 2: \( T^2 \) defined by

\[
\begin{align*}
    z &\sim z + iR_y \\
    z &\sim z + R_x + i\frac{R_y}{2}
\end{align*}
\]

Wrap 2 units along \( \overrightarrow{\mathbf{z}} \), 3 units along \( y \)-axis

Coordinates for \( A \) is

\[
    nR_x + i(\bar{n}R_y + nR_x \tan \theta)
\]

\[
= nR_x + i(\tilde{m}R_y + \frac{1}{2}R_y)
\]

\( (n=2, \tilde{m}=3 \text{ here}) \)

So \((n, \tilde{m} + \frac{n}{2})\) represents \( A \),

\[ \sqrt{n, \tilde{m}} \in \mathbb{Z} \]

Under \( y \to -y \), \((n, \tilde{m} + \frac{n}{2}) \to (n, \tilde{m} - \frac{n}{2}) \)
Axion Mixings (minimal)

W/O Stueckelberg coupling

\[ S_{\text{axion}}^{\text{kin}} = -\int \frac{1}{2} \sum_{i,j=1}^{2} G_{ij}(\sigma) \, da^i \wedge \ast_4 da^j, \]

Metric

\[ g = \begin{pmatrix} g_{11} & g_{12} \\ g_{12} & g_{22} \end{pmatrix}, \quad \text{with } g_{11}, g_{12}, g_{22} \in \mathbb{R}\backslash\{0\}. \]

SO(2) rotation diagonalizing the metric:

\[ \begin{pmatrix} a^- \\ a^+ \end{pmatrix} = \begin{pmatrix} \sin \frac{\theta}{2} & -\cos \frac{\theta}{2} \\ \cos \frac{\theta}{2} & \sin \frac{\theta}{2} \end{pmatrix} \begin{pmatrix} a^1 \\ a^2 \end{pmatrix} \]

\[ \cos \theta = \frac{g_{11} - g_{22}}{\sqrt{4g_{12}^2 + (g_{11} - g_{22})^2}}, \quad \sin \theta = \frac{2g_{12}}{\sqrt{4g_{12}^2 + (g_{11} - g_{22})^2}}, \quad \text{with } 0 \leq \theta < 2\pi. \]

Rescale the axions s.t. the anomalous coupling is in a purely topological term:

\[ \tilde{a}^- \equiv \left( r_1 \sin \frac{\theta}{2} - r_2 \cos \frac{\theta}{2} \right) a^-, \quad \tilde{a}^+ \equiv \left( r_1 \cos \frac{\theta}{2} + r_2 \sin \frac{\theta}{2} \right) a^+ \]

\[ \hat{a}^+ \equiv f_{\tilde{a}^-} \tilde{a}^+, \quad \hat{a}^- \equiv f_{\tilde{a}^+} \tilde{a}^- \]

\[ V_{\text{axion}}^{\text{eff}}(\hat{a}^-, \hat{a}^+) = \Lambda^4 \left[ 1 - \cos \left( \frac{\hat{a}^-}{f_{\tilde{a}^-}} + \frac{\hat{a}^+}{f_{\tilde{a}^+}} \right) \right] \]

\[ f_{\tilde{a}^-} = \frac{\sqrt{\lambda^-}}{|r_1 \sin \frac{\theta}{2} - r_2 \cos \frac{\theta}{2}|}, \quad f_{\tilde{a}^+} = \frac{\sqrt{\lambda^+}}{|r_1 \cos \frac{\theta}{2} + r_2 \sin \frac{\theta}{2}|}. \]

Eigenvalues of the metric
Axion Mixings (minimal)

W/O Stueckelberg coupling

\[ S_{\text{axion}} = - \int \left[ \frac{1}{2} d\hat{\alpha}^- \wedge \ast_4 d\hat{\alpha}^- + \frac{1}{2} d\hat{\alpha}^+ \wedge \ast_4 d\hat{\alpha}^+ + V_{\text{axion}}^{\text{eff}}(\hat{\alpha}^-, \hat{\alpha}^+) \ast_4 1 \right] \cdot \]

\[ V_{\text{axion}}^{\text{eff}}(\hat{\alpha}^-, \hat{\alpha}^+) = \Lambda^4 \left[ 1 - \cos \left( \frac{\hat{\alpha}^-}{f_{\hat{\alpha}^-}} + \frac{\hat{\alpha}^+}{f_{\hat{\alpha}^+}} \right) \right] \quad f_{\hat{\alpha}^-} = \frac{\sqrt{\lambda_-}}{|r_1 \sin \frac{\theta}{2} - r_2 \cos \frac{\theta}{2}|}, \quad f_{\hat{\alpha}^+} = \frac{\sqrt{\lambda_+}}{|r_1 \cos \frac{\theta}{2} + r_2 \sin \frac{\theta}{2}|} \cdot \]

Trans-Planckian excursion possible?

Need to find eigenbasis of the mass square matrix

\[ \left( \begin{array}{c} \xi \\ \zeta \end{array} \right) = \frac{1}{\sqrt{f_{\hat{\alpha}^+}^2 + f_{\hat{\alpha}^-}^2}} \left( \begin{array}{cc} f_{\hat{\alpha}^+} & -f_{\hat{\alpha}^-} \\ f_{\hat{\alpha}^-} & f_{\hat{\alpha}^+} \end{array} \right) \left( \begin{array}{c} \hat{\alpha}^+ \\ \hat{\alpha}^- \end{array} \right) \quad S_{\text{axion}} = - \int \left[ \frac{1}{2} d\xi \wedge \ast_4 d\xi + \frac{1}{2} d\zeta \wedge \ast_4 d\zeta + V_{\text{axion}}^{\text{eff}}(\zeta) \ast_4 1 \right] \cdot \]

\[ V_{\text{axion}}^{\text{eff}}(\zeta) = \Lambda^4 \left[ 1 - \cos \left( \frac{\sqrt{f_{\hat{\alpha}^+}^2 + f_{\hat{\alpha}^-}^2}}{f_{\hat{\alpha}^+} f_{\hat{\alpha}^-}} \zeta \right) \right] \quad f_{\text{eff}} = \frac{f_{\hat{\alpha}^+} f_{\hat{\alpha}^-}}{\sqrt{f_{\hat{\alpha}^+}^2 + f_{\hat{\alpha}^-}^2}} \cdot \]

can’t be super-Planckian!

flat direction whose continuous shift symmetry is not broken

axionic couplings scale inversely with the axion decay constant.
Axion Mixings (minimal)

W/O Stueckelberg coupling

\[ V_{\text{axion}}(\zeta) = \Lambda^4 \left[ 1 - \cos \left( \frac{\sqrt{f_{a+}^2 + f_{a-}^2}}{f_{a+} + f_{a-}} \zeta \right) \right] \]

\[
\left( \frac{\xi}{\zeta} \right) = \frac{1}{\sqrt{f_{a+}^2 + f_{a-}^2}} \left( \begin{array}{c}
 f_{a+} - f_{a-} \\
 f_{a-}^2 - f_{a+}^2
\end{array} \right) \left( \begin{array}{c}
 \dot{a}^+ \\
 \dot{a}^-
\end{array} \right)
\]

can't have super-Planckian excursion

if we interpret the axion \( \xi \) as the inflaton candidate
have to invoke additional physical effects

(1) Monodromy effects. \( V(\xi) \sim \xi^p \)

Through torsional monodromy effects (p=2), or flux induced monodromies (p>2)

(2) Alignment effects.

Add a second non-Abelian gauge group anomalously coupling to both axions

(3) Abelian \( U(1) \) gauge symmetry. Will study this in detail
Axion Mixings (minimal)

How to ensure gauge invariance?

\[ S_{sub} = \int \left[ -\frac{f_{\alpha}^2}{2} (\bar{d}^2 - \bar{k}^2 A) \wedge \star_4 (\bar{d}^2 - \bar{k}^2 A) - \frac{1}{g_1^2} F \wedge \star_4 F + \frac{1}{8\pi^2} \tilde{a}^2 \text{Tr}(G \wedge G) \right] \]

\[ U(1) : A \rightarrow A + d\eta', a^i \rightarrow a^i + k^i \eta' \]

non-Abelian: \( B \rightarrow B + D\eta \)

- **Solution 1:** Triangle anomalies + Variance in instanton coupling = 0 (GS mech.)

- **Solution 2:** Triangle anomalies + Variance in instanton coupling + Variance in GCS terms = 0

\[ S_{sub}^{GCS} = - \int \frac{1}{8\pi^2} A^{GCS} A \wedge \Omega \]

\[ d\Omega = \text{Tr}(G \wedge G) \]
Axion Mixings (minimal)

Choose gauge to eat an axion

\[ S_{\text{axion}}^{\text{full}} = \int \left[ -\frac{f_{\tilde{a}}^2}{2} d\tilde{a}_1 \wedge \ast_4 d\tilde{a}_1 - \frac{f_{\tilde{a}}^2}{2} (d\tilde{a}_2 - \tilde{k}^2 A) \wedge \ast_4 (d\tilde{a}_2 - \tilde{k}^2 A) - \frac{1}{g_1^2} F \wedge \ast_4 F \\
- \frac{1}{g_2^2} \text{Tr}(G \wedge \ast_4 G) + \frac{1}{8\pi^2} [\tilde{a}^1 + \tilde{a}^2] \text{Tr}(G \wedge G) - \frac{1}{8\pi^2} A^{\text{GCS}} A \wedge \Omega + \ldots \right] \]

\[ A \to A + \frac{1}{\tilde{k}^2} d\tilde{a}^2 \]

\[ \tilde{k}^2 = M_A \left( \frac{r_1}{\sqrt{G_{11}}} \sin \varphi + \frac{r_2}{\sqrt{G_{22}}} \cos \varphi \right) = r_1 k^1 + r_2 k^2 \]

\[ S_{\text{axion}}^{\text{full,unitary}} = \int \left[ -\frac{f_{\tilde{a}}^2}{2} d\tilde{a}_1 \wedge \ast_4 d\tilde{a}_1 - \frac{(f_{\tilde{a}}^2 \tilde{k}^2)^2}{2} A \wedge \ast_4 A - \frac{1}{g_1^2} F \wedge \ast_4 F - \frac{1}{g_2^2} \text{Tr}(G \wedge \ast_4 G) \\
+ \frac{1}{8\pi^2} \tilde{a}^1 \text{Tr}(G \wedge G) - \frac{1}{8\pi^2} A^{\text{GCS}} A \wedge \Omega + A \wedge \ast_4 J_\psi \right. \\
+ \frac{1}{8\pi^2} \left( \tilde{k}^2 + A^{\text{GCS}} \right) \tilde{a}^2 \text{Tr}(G \wedge G) + \ldots \left. \right] \]

\[ M_A^2 = G_{11} (k^1)^2 + G_{22} (k^2)^2 \]

\[ J_\psi^\mu = \sum_i \left[ (q_i^L)\bar{\psi}_L \gamma^\mu \psi^i_L + (q_i^R)\bar{\psi}_R \gamma^\mu \psi^i_R \right] \]

from chiral rotation of chiral fermions
Axion Mixings (minimal)

**Integrate out the massive U(1)**

**EOM for massive U(1):**

\[ \frac{1}{g_1^2} d(\star_4 dA) - (f_{a_2} \tilde{k}^2)^2 \star_4 A = \frac{A_{GCS}}{8\pi^2} \Omega - \star_4 J_\psi \]

2 possible sources

Take derivative

\[ d(\star_4 J_\psi) = -\frac{1}{8\pi^2} A_{\text{mix}} d\Omega = -\frac{1}{8\pi^2} A_{\text{mix}} \text{Tr}(G \wedge G) \]

Up to a total derivative

**Now**

\[ S_{\text{axion}}^{\text{full, unitary}} = \int \left[ -\frac{f_{a_1}^2}{2} d\tilde{a}^1 \wedge \star_4 d\tilde{a}^1 + \frac{1}{8\pi^2} \tilde{a}^1 \text{Tr}(G \wedge G) \right. \]

\[ + \left. \left( A_{GCS} + A_{\text{mix}} \right)^2 \frac{1}{2(A_{\text{mix}})^2} \star_4 J_\psi \wedge \star_4 J_\psi + \ldots \right] \]

4-point interaction suppressed by the U(1) mass
Axion Mixings (minimal)

Integrate out chiral fermions and non-Abelian gauge group

Similar to QCD

- Chiral fermions condensate into meson-like states.
- 4-point couplings give masses of fermions of $\sim \Xi^3/f_{\alpha^2}$
- Use non-linear sigma model techniques to integrate out heavy mesons.
Chiral Rotation

\[ \mathcal{L}_{\text{fermion}} = \bar{\psi}^i_L i \left( \partial^\mu A^\mu \right) \psi^i_L + \bar{\psi}^i_R i \left( \partial^\mu A^\mu \right) \psi^i_R \]

Under a chiral transformation of the type,

\[ \psi^i_L \rightarrow e^{iq_L \bar{a}^2} \psi^i_L, \quad \psi^i_R \rightarrow e^{iq_R \bar{a}^2} \psi^i_R, \]

\[ \mathcal{L}^{\text{eff}}_{\text{fermion}} = \mathcal{L}_{\text{fermion}} + \bar{a}^2 \left( \partial_\mu \mathcal{J}_\psi^\mu + \frac{1}{32\pi^2} \mathcal{A}_{\text{mix}}^{\epsilon \mu \nu \rho \sigma} \text{Tr}(G_{\mu \nu}G_{\rho \sigma}) \right) \]

Adler-Bell-Jackiw anomaly

Variance from the measure
Another Way to Generate the Sinusoidal Potential

- Way 2. **Gaugino condensates**: break U(1) R symmetry. **Supersymmetry obtains a NP correction**

Moduli dependent, assumed to be stabilized at some higher scale

\[ W = W_{\text{per}} + A e^{-\frac{2\pi}{N} T} \]

Superfield \( T \) as \( t + i a \)

- Rank of the non-Abelian group
- Axion

\[ K(T, \overline{T}) = -3 \ln(T + \overline{T}) \]

F-term potential for N=1 SUGRA

\[
V_{\text{axion}}(a) = \frac{8\pi}{N} \frac{\langle t \rangle}{T^2} |A| |W_{\text{per}}| e^{-2\frac{\pi}{N} t} \cos \left( \frac{2\pi}{N} a + i \gamma \right)
\]

\[ \gamma = \text{Arg} (W_{\text{per}} A^*) \quad T \text{ the dimensionless volume of the internal space} \]
Aligned Natural Inflation

\[ V(\Phi^1, \Phi^2) = \Lambda_1^4 \left[ 1 - \cos \left( \frac{n_1 \Phi^1}{f_1} + \frac{n_2 \Phi^2}{g_1} \right) \right] + \Lambda_2^4 \left[ 1 - \cos \left( \frac{m_1 \Phi^1}{f_2} + \frac{m_2 \Phi^2}{g_2} \right) \right], \quad n_i, m_i \in \mathbb{Z} \]

**Perfect alignment**

\[
\frac{f_1}{n_1} = \frac{g_1}{n_2} = \frac{f_2}{m_1} = \frac{g_2}{m_2}
\]

**Derivation from alignment**

\[
\alpha = g_2/m_2 - \frac{f_2}{f_1/n_1} g_1/n_2
\]

**Along flat direction:**

\[
f_{\text{eff}} \propto \frac{1}{\alpha}
\]

- e.g. \( f_1/n_1 = f_2/m_1 \ll g_1/n_2, g_2/m_2 \)

If want enhancement \( \alpha \sim 10^{-2} \mathcal{O}(f_i, g_i) \), will require \( \frac{n_2}{m_2} = \frac{99}{100} \)

**Too large integers!**
Axion Bounds from String Compactifications

- Take weakly heterotic string for instance.

\[ S_{bos} = M_s^8 \int d^{10} x \sqrt{-g} e^{-2\phi} (R - \frac{1}{2}|H_3|^2) - M_s^8 \int d^{10} x \sqrt{-g} e^{-2\phi} \frac{1}{2}|F_2|^2 \]

- \( H_3 = dB_2 \)
- Moduli from \( B_{\mu\nu} \)

\[ \frac{f_a}{M_p^2} = 1 \]

\( M_s^8 l_s^6 = M_P^2 \)

dimensional reducing to 4d
WCG and Swampland

- WGC (weak form) states that \( \exists \) state with \( \left( \frac{M}{Q} \right) \leq M_{Pl} \)
  \( \sim \) conjectured generalisation for 0-forms with \( S_{\text{inst}} \leq \frac{M_{Pl}}{f} \)

- Consistent compactifications for
  \[ M\text{-theory} \xleftrightarrow{S,T} \ M_{1,2} \times S^1_M \times \hat{S}^1 \times X_6 \]
  \[ \text{Type IIB} \ M_{1,2} \times S^1 \times X_6 \]
  \( \sim \) Constraints on effective axion decay constant by applying WGC on 5dim BH in M-theory

- Resolving ambiguity in defining effective axion decay constant
  \( \sim \) possible axionic directions with \( f > M_{Pl} \)

- **BUT** in our simple model: 2 axions + 1 \( U(1) \) + 1 instanton
  - apply WGC at appropriate scale \( \Lambda > M_{gauge} \)
    \[ \Rightarrow \exists \text{ axionic direction with } f_2 = \frac{M_{gauge}}{k^2} < M_{Pl} \ \checkmark \text{WGC} \]
  - Higher harmonics are still troublesome for \( \perp \) axion
    Montero-Uranga-Valenzuela (2015)
Renormalization of the Planck Mass

• Running the Planck mass: 
  \[ M_P \rightarrow M_P(\mu) \] for some scale \( \mu \)
  \[ M_P(0) \sim 10^{19} \text{ GeV} \]

• Graviton propagator:

• Integrating out (propagating) scalars, fermions and gauge bosons

\[ M_P^2(\mu) \sim \frac{1}{G(\mu)} = \frac{1}{G(0)} - \frac{\mu}{12\pi} (n_0 + n_{1/2} - 4n_1) \]

\( n_i \) is the number of particles with spin-\( i \)
Moduli Stabilization

• A modulus field is a scalar field whose potential is vanishing (thus massless).

• The VEVs of those massless fields are moduli (sometimes we don't distinguish modulus and modulus field).

• E.g. in models with 1 extra dimension compactified on a circle, the radius of this curled dimension is a modulus. The value of the radius sets the scale for the extra dimension, and affects the physics spectrum in lower dimension.

• E.g. the metric in the axion moduli space is field (modulus) dependent. For the toroidal case, the metric is diagonal with entries given by gauge coupling constants, determined by the volume of the internal cycles. The volumes are moduli.
Moduli Stabilization

- **Unstabilized moduli can cause serious problems!**

- **Predictions** of the theory **crucially depend on the VEVs of the moduli**. No potential means the VEVs can be arbitrary. Not much to predict.

- **Moduli can be time-dependent**, conflicting with observation.

- Massless scalars **can mediate long range force (~ gravity)**. Conflicting with Newton’s law (**no such long range force exists**).

- **Polonyi problem**: in the early universe, the light scalars can have VEVs ~ Planck scale. Overclosure of the universe, etc.

- Different than a massless goldstone, **moduli exist with or without any symmetry**. Physics depends on the VEVs of moduli. Physically distinct vacua can be connected by varying moduli.
Moduli Stabilization

• The **axion metric is moduli dependent**.

• In the work, **assume the moduli (the volume, the shape of internal cycles, etc) have been stabilized at some higher energy scale**.


• e.g. stabilized on orbifolds, Ruehle & Wieck, 1503.07183

• Rudelius 1409.5793, considered the moduli dependence of the metric, but assumed straight geodesics…
String Theory Embedding

Explicit D6-brane model to realize super-Planckian excursion

Internal geometry \( T^2_{(1)} \times T^2_{(2)} \times T^2_{(3)} \):

\[ x^i \rightarrow x^i + R_1^{(i)}, \ y^i \rightarrow y^i + R_2^{(i)} \]

Symplectic basis of the homology \( (\gamma_i, \delta^i) \)

Orientifold action \((x^i, y^i) \overset{\Omega R}{\rightarrow} (x^i, -y^i)\)

Bulk action -> metric on axion field space

\[ \kappa_{ij} = \text{diag} \left( u_1 u_2 u_3, \frac{u_1}{u_2 u_3}, \frac{u_2}{u_1 u_3}, \frac{u_3}{u_1 u_2} \right) \quad u_i = \frac{R_2^{(i)}}{R_1^{(i)}} \]

three-cycle \( \Pi_x \) wrapped by a D6\(_x\)-brane

\[ \Pi_x = r_x^i \gamma_i + s_x^i \delta^i \]
String Theory Embedding

Explicit D6-brane model to realize super-Planckian excursion

Cohomology basis

\[ \alpha_0 = dx^1 \wedge dx^2 \wedge dx^3, \quad \beta^0 = dy^1 \wedge dy^2 \wedge dy^3, \]
\[ \alpha_1 = dx^1 \wedge dy^2 \wedge dy^3, \quad \beta^1 = dy^1 \wedge dx^2 \wedge dx^3, \]
\[ \alpha_2 = dy^1 \wedge dx^2 \wedge dy^3, \quad \beta^2 = dx^1 \wedge dy^2 \wedge dx^3, \]
\[ \alpha_3 = dy^1 \wedge dy^2 \wedge dx^3, \quad \beta^3 = dx^1 \wedge dx^2 \wedge dy^3, \]

Tadpole

\[ \sum_x N_x (\Pi_x + \Pi'_x) = 4\Pi_{O6}. \]
String Theory Embedding

Explicit D6-brane model to realize super-Planckian excursion

Setup: a stack of D6\_a branes and a single D6\_b brane

\[ U(N_a) : \Pi_a = r_a^2 \gamma_2 + r_a^3 \gamma_3 + s_a^2 \delta^2 + s_a^3 \delta^3, \quad U(1)_b : \Pi_b = r_b^i \gamma_i. \]

\[ S_{\text{axion}} = \int \left[ -\frac{1}{2\ell_s^2} \sum_{i=0,1} \mathcal{K}_{ii} da^i \wedge *_4 da^i - \frac{1}{2\ell_s^2} \sum_{l=2,3} \mathcal{K}_{ll}(da^l - N_a s_a^l A_a) \wedge *_4(da^l - N_a s_a^l A_a) \right. \]

\[ + \frac{1}{8\pi^2} (r_a^2 a^2 + r_a^3 a^3) \text{Tr}(G_a \wedge G_a) + \frac{1}{8\pi^2} (r_a^2 a^2 + r_a^3 a^3) N_a F_a \wedge F_a \]

\[ + \frac{1}{8\pi^2} \left( \sum_{i=0}^{3} r_b^i a^i \right) (F_b \wedge F_b) \right]. \]

Put O6 along the even cycle

Focus
In the system with axions $a_2$ and $a_3$, the uneaten axion has an effective decay constant

$$f_{a_1} = \sqrt{\frac{u_2u_3}{u_1}} \frac{\sqrt{(u_2)^2(s_a)^2 + (u_3)^2(s_a)^2}}{|r_a^2s_a^3(u_3)^2 - r_a^3s_a^2(u_2)^2|} M_s$$

Can be large if

$$r_a^2s_a^3(u_3)^2 \simeq r_a^3s_a^2(u_2)^2, \quad u_i = \frac{R_2^{(i)}}{R_1^{(i)}}$$

**Tadpole condition** used to determine integer coefficients
String Theory Embedding

Explicit D6-brane model to realize super-Planckian excursion

2 stacks of D6’s wrapping *non-factorisable* 3-cycles

<table>
<thead>
<tr>
<th>sector</th>
<th>$SU(N_b)_{(Q_a,Q_b)}$</th>
<th>multiplicity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ab$</td>
<td>$(N_a)_{(1,-1)}$</td>
<td>$-s^2_a r^2_b - s^3_a r^3_b$</td>
</tr>
<tr>
<td>$ab'$</td>
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<td>$-s^2_a r^2_b - s^3_a r^3_b$</td>
</tr>
<tr>
<td>$aa'$</td>
<td>$(\text{Anti}<em>a)</em>{(2,0)}$</td>
<td>$-r^2_a s^2_a - r^3_a s^3_a$</td>
</tr>
<tr>
<td>$aa'$</td>
<td>$(\text{Sym}<em>a)</em>{(2,0)}$</td>
<td>$-r^2_a s^2_a - r^3_a s^3_a$</td>
</tr>
</tbody>
</table>

One solution $N_a = 3$.

$r^2_a = r^3_a = 1,$
$s^2_a = 2, s^3_a = 3,$
$r^0_b = 16, r^1_b = 0,$
$r^2_b = r^3_b = -3.$

chiral spectrum $15 \times (3)_{(1,-1)} + 15 \times (3)_{(1,1)} + 5 \times (3_A)_{(-2,0)} + 5 \times (\overline{6}_S)_{(-2,0)}$

$u_3/u_2$ asymptotes to the value $\sqrt{2}/\sqrt{3}$

$M_s \sim 10^{17} \text{ GeV}$

$f_{\alpha_1} \approx \frac{M_s}{3} \times 10^3 \sim 10M_{Pl}.$

Internal space $T^2_{(1)} \times T^2_{(2)} \times T^2_{(3)}.$